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GENERALIZED CURVATURE TENSORS ON CONFORMALLY SYMMETRIC MANIFOLDS

BY

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1. Introduction. An n-dimensional ($n > 3$) Riemannian manifold (not necessarily of definite metric form) is said to be *conformally symmetric* [1] if its Weyl's conformal curvature tensor

$$C^h{}_{ijk} = R^h{}_{ijk} - \frac{1}{n-2}(g_{ij}R^h{}_k - g_{ik}R^h{}_j + \delta^h_k R_{ij} - \delta^h_j R_{ik}) + \tag{1}$$

$$+\frac{R}{(n-1)(n-2)}(\delta^h_k g_{ij} - \delta^h_j g_{ik})$$

satisfies the condition

$$C^h{}_{ijk,l} = 0, \tag{2}$$

where the comma indicates covariant differentiation with respect to the metric.

It follows easily from (1) and (2) that every conformally flat ($n > 3$) as well as every locally symmetric Riemannian manifold ($n > 3$) is necessarily conformally symmetric. The converse of this is, in general, not true ([4], Theorem 1).

Let M be a Riemannian manifold of class C^∞ with not necessarily definite metric form, $\dim M \geqslant 3$. A (1, 3)-tensor B of class C^∞ (with components $B^h{}_{ijk}$) will be called a *generalized curvature tensor* on M (see [3] and [7]) if

$$B^h{}_{ijk} + B^h{}_{jki} + B^h{}_{kij} = 0 \quad \text{(the first Bianchi identity)}, \tag{3}$$

$$B^h{}_{ijk} = -B^h{}_{ikj}, \qquad B_{hijk} = B_{jkhi}, \tag{4}$$

where $B_{hijk} = g_{rh}B^r{}_{ijk}$.

The tensor B is said to be *proper* if it satisfies the second Bianchi identity

$$B^h{}_{ijk,l} + B^h{}_{ikl,j} + B^h{}_{ilj,k} = 0.$$

For every generalized curvature tensor B there is a natural decomposition

$$B(1) + B(2) + B(3) = B,$$

where

$$B(1)^h{}_{ijk} = \frac{1}{n(n-1)} S(g_{ij}\delta^h_k - g_{ik}\delta^h_j),$$

$$B(2)^h{}_{ijk} = \frac{1}{n-2}(B_{ij}\delta^h_k - B_{ik}\delta^h_j + g_{ij}B^h{}_k - g_{ik}B^h{}_j) + \\ + \frac{2}{n(n-2)} S(g_{ik}\delta^h_j - g_{ij}\delta^h_k),$$

$$B(3)^h{}_{ijk} = B^h{}_{ijk} + \frac{1}{n-2}(\delta^h_j B_{ik} - \delta^h_k B_{ij} + g_{ik}B^h{}_j - g_{ij}B^h{}_k) + \\ + \frac{1}{(n-1)(n-2)} S(g_{ij}\delta^h_k - g_{ik}\delta^h_j),$$

and where $B_{ij} = B^r{}_{ijr}$ are the components of the Ricci tensor $\mathrm{Ric}(B)$, and $S = S(B) = B^r{}_r$ is the scalar curvature of B. $B(3)$ is called the *Weyl conformal curvature tensor* of B.

One can easily verify that for a proper generalized curvature tensor B the relations

$$(5) \qquad B^r{}_{ijk,r} = B_{ij,k} - B_{ik,j}, \quad B^r{}_{j,r} = \frac{1}{2} S_{,j}$$

hold.

Tanno proved, generalizing a result of Głodek ([2], Theorem 2), the following remarkable

THEOREM A ([8], Theorem 6). *A connected conformally symmetric manifold (not necessarily of definite metric form) is conformally flat or its scalar curvature is constant.*

The author obtained the following results:

THEOREM B ([5], Theorem 1). *Let M be a connected conformally symmetric manifold (not necessarily of definite metric form), $\dim M > 4$. Then its Weyl's conformal curvature tensor C is null (i.e. $\langle C, C\rangle = 0$) on M or M is locally symmetric.*

THEOREM C ([5], Theorem 2). *Let M be a connected Riemannian manifold (not necessarily of definite metric form) whose Ricci tensor satisfies the condition*

$$(6) \qquad R_{ij,k} - R_{ik,j} = \frac{1}{2(n-1)}(R_{,k}g_{ij} - R_{,j}g_{ik}).$$

If B is a parallel generalized curvature tensor on M, then the scalar curvature of M is constant or $B = B(1)$ and $B(2) = B(3) = 0$.

The main aim of this paper is a generalization of Theorem B. More precisely, we shall prove the following

THEOREM. *Let M be a connected conformally symmetric manifold (not necessarily of definite metric form),* $\dim M > 4$. *If B is a parallel generalized curvature tensor on M, then M is locally symmetric or*

$$\langle B-B(1), B-B(1)\rangle = 0 \quad \text{on } M.$$

All Riemannian manifolds under consideration are assumed to be connected and of class C^∞. Their metric forms, unless stated otherwise, are not necessarily definite.

2. Preliminary results. We start with the well-known lemma.

LEMMA 1. *The Weyl conformal curvature tensor satisfies the relations*

$$C_{hijk} = -C_{ihjk} = -C_{hikj} = C_{jkhi}, \quad C^r{}_{ijr} = C^r{}_{irk} = C^r{}_{rjk} = 0, \tag{7}$$

$$C^h{}_{ijk} + C^h{}_{jki} + C^h{}_{kij} = 0, \tag{8}$$

$$C^r{}_{ijk,r} = \frac{n-3}{n-2}\left[(R_{ij,k} - R_{ik,j}) - \frac{1}{2(n-1)}(R_{,k}g_{ij} - R_{,j}g_{ik})\right]. \tag{9}$$

LEMMA 2. *Let M be a conformally symmetric manifold of constant scalar curvature,* $\dim M > 4$. *If B is a parallel generalized curvature tensor on M, then the relations*

$$R_{rm,p}B^r{}_k = \frac{1}{n}SR_{km,p}, \tag{10}$$

$$R^{rs}{}_{,p}B_{rijs} = \frac{1}{n(1-n)}SR_{ij,p} \tag{11}$$

hold.

Proof. Since B is parallel on M, we have

$$B_{hijk,lm} - B_{hijk,ml} = 0,$$

whence, because of the Ricci identity,

$$B_{rijk}R^r{}_{hlm,p} + B_{hrjk}R^r{}_{ilm,p} + B_{hirk}R^r{}_{jlm,p} + B_{hijr}R^r{}_{klm,p} = 0. \tag{12}$$

The last equation, in view of

$$R^h{}_{ijk,p} = \frac{1}{n-2}(g_{ij}R^h{}_{k,p} - g_{ik}R^h{}_{j,p} + \delta^h_k R_{ij,p} - \delta^h_j R_{ik,p}), \tag{13}$$

which is a consequence of (2) and $R = \text{const}$, implies

$$\begin{aligned} &g_{hl}R_{rm,p}B^r{}_{ijk} - g_{hm}R_{rl,p}B^r{}_{ijk} + R_{hl,p}B_{mijk} - R_{hm,p}B_{lijk} + \\ &+ g_{il}R_{rm,p}B^r{}_{hkj} - g_{im}R_{rl,p}B^r{}_{hkj} + R_{il,p}B_{mhkj} - R_{im,p}B_{lhkj} + \\ &+ g_{jl}R_{rm,p}B^r{}_{khi} - g_{jm}R_{rl,p}B^r{}_{khi} + R_{jl,p}B_{mkhi} - R_{jm,p}B_{lkhi} + \\ &+ g_{kl}R_{rm,p}B^r{}_{jih} - g_{km}R_{rl,p}B^r{}_{jih} + R_{kl,p}B_{mjih} - R_{km,p}B_{ljih} = 0. \end{aligned} \tag{14}$$

Contracting (14) with g^{hl} and making use of (3), we get

$$(15)\quad (n-2)R_{rm,p}B^{r}{}_{ijk}+R_{ri,p}B^{r}{}_{mjk}+R_{rj,p}B^{r}{}_{imk}+R_{rk,p}B^{r}{}_{ijm}+ \\ +g_{jm}R^{rs}{}_{,p}B_{rkis}-g_{km}R^{rs}{}_{,p}B_{rjis}+R_{jm,p}B_{ki}-R_{km,p}B_{ij}=0.$$

The last relation, in view of

$$(16)\quad R_{ij,k}=R_{ik,j},$$

which follows easily from (2), (9) and $R=\text{const}$, can be written as

$$(17)\quad (n-2)R_{rp,m}B^{r}{}_{ijk}+R_{rp,i}B^{r}{}_{mjk}+R_{rp,j}B^{r}{}_{imk}+R_{rk,p}B^{r}{}_{ijm}+ \\ +g_{jm}R_{rp,s}B^{r}{}_{ki}{}^{s}-g_{km}R_{rp,s}B^{r}{}_{ji}{}^{s}+R_{pj,m}B_{ki}-R_{km,p}B_{ij}=0.$$

Contracting now (17) with g^{pk} and using (16), we obtain

$$(18)\quad (n-3)R^{rs}{}_{,m}B_{rijs}+R^{rs}{}_{,i}B_{rmjs}+R^{rs}{}_{,j}B_{rims}+R_{rj,m}B^{r}{}_{i}=0,$$

whence

$$(n-3)(R^{rs}{}_{,m}B_{rijs}-R^{rs}{}_{,j}B_{rims})+R^{rs}{}_{,j}B_{rims}-R^{rs}{}_{,m}B_{rijs}=0.$$

Hence

$$(19)\quad R^{rs}{}_{,m}B_{rijs}=R^{rs}{}_{,j}B_{rims}$$

which, together with (18), implies

$$(20)\quad R^{rs}{}_{,m}B_{rijs}=\frac{1}{1-n}R_{rj,m}B^{r}{}_{i}.$$

But the last equation yields

$$(21)\quad R_{rj,m}B^{r}{}_{i}=R_{ri,m}B^{r}{}_{j},\qquad R_{rs,m}B^{rs}=0.$$

Contracting now (15) with g^{ij} and substituting (21), we obtain (10). Equation (11) follows immediately from (10) and (20).

LEMMA 3. *Let M be a conformally symmetric manifold of constant scalar curvature, $\dim M>4$. If B is a parallel generalized curvature tensor on M, then the relation*

$$(22)\quad (n-1)R_{rm,p}B^{r}{}_{ijk}+R_{ri,p}B^{r}{}_{mjk}+B_{ik}R_{jm,p}-B_{ij}R_{km,p}+ \\ +\frac{1}{n(1-n)}S(g_{jm}R_{ik,p}-g_{km}R_{ij,p})=0$$

holds.

Proof. Substituting (11) into (15), we get

$$(23)\quad (n-2)R_{rm,p}B^{r}{}_{ijk}+R_{ri,p}B^{r}{}_{mjk}+R_{rj,p}B^{r}{}_{imk}+R_{rk,p}B^{r}{}_{ijm}+ \\ +\frac{1}{n(1-n)}Sg_{jm}R_{ik,p}-\frac{1}{n(1-n)}Sg_{km}R_{ij,p}+B_{ik}R_{jm,p}-B_{ij}R_{km,p}=0.$$

Computing the cyclic sum of (23) with respect to m, j, and k, and using (3), we obtain

$$R_{rm,p}B^r{}_{ijk} + R_{rj,p}B^r{}_{ikm} + R_{rk,p}B^r{}_{imj} = 0$$

which, together with (23), leads to (22). The lemma is proved.

LEMMA 4. *Let M be a conformally symmetric manifold of constant scalar curvature, $\dim M > 4$. If the Ricci tensor of a parallel generalized curvature tensor on M is of the form*

$$B_{ij} = \frac{1}{n} S g_{ij},$$

then the relation

$$R_{rm,p}B^r{}_{ijk} = \frac{1}{n(n-1)} S(g_{ij}R_{km,p} - g_{ik}R_{jm,p}) \tag{24}$$

holds.

Proof. As a consequence of (22) we get

$$-(n-1)R_{rp,m}B^r{}_{ijk} - R_{ri,m}B^r{}_{pjk} - B_{ik}R_{jp,m} + B_{ij}R_{kp,m} - \\ - \frac{1}{n(1-n)} S(g_{jp}R_{ik,m} - g_{kp}R_{ij,m}) = 0$$

which, together with (22) and (16), yields

$$R_{ri,p}B^r{}_{mjk} - R_{ri,m}B^r{}_{pjk} + \\ + \frac{1}{n(1-n)} S(g_{jm}R_{ik,p} + g_{kp}R_{ij,m} - g_{km}R_{ij,p} - g_{jp}R_{ik,m}) = 0. \tag{25}$$

But (25), in view of (16), implies

$$R_{rp,i}B^r{}_{mjk} - R_{rm,i}B^r{}_{pjk} + \\ + \frac{1}{n(1-n)} S(g_{jm}R_{ik,p} + g_{kp}R_{ij,m} - g_{km}R_{ij,p} - g_{jp}R_{ik,m}) = 0. \tag{26}$$

Replacing in (26) p by m, i by p, and m by i, we obtain

$$R_{rm,p}B^r{}_{ijk} - R_{ri,p}B^r{}_{mjk} + \\ + \frac{1}{n(1-n)} S(g_{ij}R_{pk,m} + g_{km}R_{pj,i} - g_{ki}R_{pj,m} - g_{jm}R_{pk,i}) = 0. \tag{27}$$

Adding now (27) and (22) and using (16), we get

$$nR_{rm,p}B^r{}_{ijk} + B_{ik}R_{jm,p} - B_{ij}R_{km,p} + \frac{1}{n(1-n)} S(g_{ij}R_{pk,m} - g_{ki}R_{pj,m}) = 0.$$

The last relation, together with $B_{ij} = (1/n)Sg_{ij}$ and (16), leads immediately to (24). Thus the lemma is proved.

LEMMA 5. *Let M be a conformally symmetric manifold of constant scalar curvature,* $\dim M > 4$. *If B is a parallel generalized curvature tensor on M such that*

$$T_{ij} = B_{ij} - \frac{1}{n}Sg_{ij} \neq 0$$

at some point, then M is locally symmetric or there exists a point $q \in M$ for which (24) *is satisfied with* $R_{ij,p} \neq 0$.

Proof. Contracting (14) with g^{hk} and making use of (10), we get

$$(28)\qquad \frac{1}{n}S(g_{il}R_{jm,p} - g_{im}R_{jl,p} + g_{jl}R_{im,p} - g_{jm}R_{il,p}) + \\ + R_{il,p}B_{mj} - R_{im,p}B_{lj} + R_{jl,p}B_{mi} - R_{jm,p}B_{li} = 0.$$

A cyclic permutation of i, l, and j gives

$$(29)\qquad \frac{1}{n}S(g_{lj}R_{im,p} - g_{lm}R_{ij,p} + g_{ij}R_{lm,p} - g_{im}R_{lj,p}) + \\ + R_{lj,p}B_{mi} - R_{lm,p}B_{ji} + R_{ij,p}B_{ml} - R_{im,p}B_{jl} = 0$$

and, furthermore,

$$(30)\qquad -\frac{1}{n}S(g_{ji}R_{lm,p} - g_{jm}R_{li,p} + g_{li}R_{jm,p} - g_{lm}R_{ji,p}) - \\ - R_{ji,p}B_{ml} + R_{jm,p}B_{il} - R_{li,p}B_{mj} + R_{lm,p}B_{ij} = 0.$$

Summing up (28)-(30), we obtain

$$T_{mi}R_{jl,p} = T_{jl}R_{mi,p}.$$

But the last relation, in view of $T_{ij} = T_{ji}$ and (16), yields

$$T_{mi}R_{jl,p} = T_{jl}R_{mi,p} = T_{jl}R_{pm,i} = T_{pm}R_{jl,i} = T_{mp}R_{jl,i}.$$

Hence

$$(31)\qquad T_{mi}R_{jl,p} = T_{mp}R_{jl,i}.$$

If now $R_{ij,p} = 0$ everywhere, then, in view of (13), M is locally symmetric. Otherwise, there exists a point q of M such that at q the condition $R_{ij,p} \neq 0$ holds.

On the other hand, since T_{ij} is parallel and non-zero at some point by assumption, T_{ij} is non-zero at every point of M. Therefore, there exists a vector v^j such that the equation $v^r v^s T_{rs} = e$ ($e = \pm 1$) holds at q.

Transvecting now (31) with $v^m v^i$ and putting $A_j = v^r T_{rj}$ and $S_{ij} = v^r R_{ij,r}$, we find at q

$$R_{jl,p} = eA_p S_{jl}. \tag{32}$$

But it follows easily from (16) that $v^r R_{rj,k} = S_{jk}$.
Therefore, transvecting (32) with v^j, we get

$$S_{lp} = eA_p Q_l, \quad \text{where } Q_l = v^r S_{rl}. \tag{33}$$

Moreover, as a consequence of (33) we obtain $Q_p = eQA_p$ $(Q = v^r Q_r)$.
But the last result, together with (33) and (32), yields

$$R_{ij,k} = DA_i A_j A_k, \quad \text{where } D \neq 0. \tag{34}$$

Substituting now (34) into (31), we obtain easily

$$A_p T_{mi} - A_i T_{mp} = 0,$$

whence $T_{mi} = eA_m A_i$ and, consequently, both equations (34) and

$$B_{mi} = \frac{1}{n} Sg_{mi} + eA_m A_i \tag{35}$$

hold at q.

Substituting (34) and (35) into (22), we obtain

$$(n-1)A_m A_r B^r{}_{ijk} + A_i A_r B^r{}_{mjk} + \frac{1}{n} S(A_j A_m g_{ik} - A_k A_m g_{ij}) + \\ + \frac{1}{n(1-n)} S(A_i A_k g_{jm} - A_i A_j g_{km}) = 0. \tag{36}$$

Suppose that $A_t \neq 0$ at q. Then (36) with $i = m = t$ yields

$$A_r B^r{}_{tjk} = \frac{1}{n(n-1)} S(A_k g_{tj} - A_j g_{tk}).$$

Putting now $m = t$ in (36) and making use of the last relation, we get

$$A_r B^r{}_{ijk} = \frac{1}{n(n-1)} S(A_k g_{ij} - A_j g_{ik})$$

which, in view of (34), leads immediately to (24). The lemma is proved.

3. Proof of the Theorem. It follows easily from (2) and (9) that, for a conformally symmetric manifold, condition (6) is satisfied and, therefore, Theorem C works.

Since B is parallel on M by assumption, Theorem C yields $B = B(1)$ or the scalar curvature of M is constant. If $B = B(1)$, the condition $\langle B - B(1), B - B(1) \rangle = 0$ holds. Suppose, therefore, that $R = \text{const}$ and

$n > 4$. Then Lemmas 2-5 work. Hence M is locally symmetric or equation (24) with $R_{ij,p} \neq 0$ is satisfied at some point q.

Transvecting now (14) with B^{mijk} and using (24), we get at q

$$\begin{aligned}\langle B, B\rangle R_{hl,p} + \frac{1}{n(n-1)} S[g_{hl}(g_{ij}R_{km,p} - g_{ik}R_{jm,p})B^{mijk} - \\ - g_{hm}(g_{ij}R_{kl,p} - g_{ik}R_{jl,p})B^{mijk} + g_{il}(g_{hk}R_{jm,p} - g_{hj}R_{km,p})B^{mijk} + \\ + g_{jl}(g_{hk}R_{im,p} - g_{ki}R_{hm,p})B^{mijk} - g_{jm}(g_{kh}R_{il,p} - g_{ki}R_{hl,p})B^{mijk} + \\ + g_{kl}(g_{ji}R_{hm,p} - g_{jh}R_{im,p})B^{mijk} - g_{km}(g_{ji}R_{hl,p} - g_{jh}R_{il,p})B^{mijk}] - \\ - R_{mh,p}B^{mijk}B_{lijk} + R_{il,p}B^{imkj}B_{mhkj} + R_{jl,p}B^{jkmi}B_{mkhi} - \\ - R_{jm,p}B^{jkmi}B_{lkhi} + R_{kl,p}B^{kjim}B_{mjih} - R_{mk,p}B^{mijk}B_{ljih} = 0.\end{aligned}$$

Applying (24) and (10) to the last equation and using the second equation of (21), we obtain

$$\left(\langle B, B\rangle - \frac{2}{n(n-1)} S^2\right) R_{hl,p} = 0,$$

whence

$$\langle B, B\rangle - \frac{2}{n(n-1)} S^2 = 0 \quad \text{at } q.$$

On the other hand, since B is parallel on M,

$$\langle B, B\rangle - \frac{2}{n(n-1)} S^2 = \text{const}.$$

Hence

$$\langle B, B\rangle - \frac{2}{n(n-1)} S^2 = 0 \quad \text{on } M.$$

Our assertion follows now from the fact that

$$\langle B - B(1), B - B(1)\rangle = \langle B, B\rangle - \frac{2}{n(n-1)} S^2.$$

The Theorem is proved.

COROLLARY 1. *Let M be a conformally symmetric manifold of dimension $n > 4$. If B is a parallel generalized curvature tensor on M and its scalar curvature vanishes, then M is locally symmetric or $\langle B, B\rangle = 0$.*

Remark 1. It follows easily from Lemma 1 that, on a conformally symmetric manifold, Weyl's conformal curvature tensor C is a parallel generalized curvature tensor with $S(C) = 0$. Therefore, Theorem B is an immediate consequence of Corollary 1.

As a consequence of Theorem B we have

COROLLARY 2 (see [4], Theorem 2). *Let M be a conformally symmetric manifold with positive definite metric form*, $\dim M > 4$. *Then M is locally symmetric or M is conformally flat.*

Since every conformally flat manifold of dimension $n > 3$ is conormally symmetric, we get

COROLLARY 3. *Let M be a conformally flat manifold with positive definite metric form*, $\dim M > 4$. *If M is not locally symmetric, then each parallel generalized curvature tensor B on M satisfies $B = B(1)$ and $B(2) = B(3) = 0$.*

Since scalar curvatures of $B(2)$ as well as of $B(3)$ are zero, Corollary 3 yields

COROLLARY 4. *Let B be a generalized curvature tensor on a conformally flat manifold with positive definite metric form*, $\dim M > 4$. *If M is not locally symmetric and $B(2)$ is parallel on M, then*

$$B(2) = 0 \quad \text{and} \quad B_{ij} = \frac{1}{n} S g_{ij}.$$

COROLLARY 5. *Let B be a generalized curvature tensor on a conformally flat manifold with positive definite metric form*, $\dim M > 4$. *If M is not locally symmetric and $B(3)$ is parallel on M, then $B(3) = 0$.*

Tanno obtained ([8], Theorem 2) the following remarkable result:

Let M be a Riemannian manifold with positive definite metric form. If its Weyl's conformal curvature tensor has the vanishing k-th covariant derivative, i.e., $C_{hijl,t_1\ldots t_k} = 0$ for some integer $k \geqslant 1$, then $C_{hijl,p} = 0$.

Using Tanno's result and Corollary 2, we have

COROLLARY 6. *Let M be a Riemannian manifold with positive definite metric form*, $\dim M > 4$. *If its Weyl's conformal curvature tensor C satisfies $C_{hijl,t_1\ldots t_k} = 0$ for some integer $k \geqslant 1$, then M is conformally flat or locally symmetric.*

Remark 2. Suppose that $a_{ij} \neq c g_{ij}$ ($c = \text{const}$) is symmetric and parallel on a not locally symmetric conformally symmetric manifold M. Then, as has been proved (see [6], p. 229), equation (31) holds. Therefore, each point of M has a neighbourhood (see the proof of Lemma 5) in which $R_{ij,p}$ can be expressed in the form

$$R_{jk,p} = D Q_j Q_k Q_p. \tag{37}$$

On the other hand, since M is not locally symmetric by assumption, there exist a point q and a neighbourhood U of q such that in U the relation

$$a_{ij} = \frac{1}{n} a^r{}_r g_{ij} + e Q_i Q_j \tag{38}$$

holds. Therefore, in U both equations (37) and (38) are satisfied.

Since the parallel tensor $\bar{a}_{ij} - C_1 g_{ij} - C_2 a_{ij}$ vanishes in U (see [6], p. 231), Theorem 1 of [6] remains true without assuming the analyticity of M.

Remark 3. Let M be a not locally symmetric conformally flat manifold with positive definite metric form, $\dim M > 3$. If a_{ij} is symmetric and parallel on M, then a_{ij} is a multiple of g_{ij}.

Otherwise, a_{ij} would be of form (38) in some neighbourhood U. But (38) yields $Q^r Q_r = 0$. Hence

$$a_{ij} = \frac{1}{n} a^r{}_r g_{ij},$$

a contradiction.

Added in proof. Recently, it has been proved that Corollary 2 as well as Corollary 6 are valid also for $\dim M = 4$ (A. Derdziński and W. Roter, *On conformally symmetric manifolds with metrics of indices* 0 *and* 1, Tensor, New Series, 31 (1977), p. 255-259).

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