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GENERALIZED P_1- AND P_2-LATTICES OF ORDER ω^+

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In 1963, Traczyk [7] introduced the notion of a P_0-lattice. Epstein and Horn [1] investigated the theory of P_0-lattices in detail and used this concept in searching for some new important generalizations of Post algebras of finite order. They discovered P_1- and P_2-lattices in this way.

On the other hand, the notion of generalized Post algebras of order ω^+ was introduced by Rasiowa [4]. In the previous paper by Traczyk and the author [8], generalized P_0-lattices of order ω^+ have been examined. In particular, some conditions for a P_0-lattice to be a Heyting algebra or a B-algebra (definitions are in Section 1) have been found. The present paper deals with generalized P_1- and P_2-lattices of order ω^+, i.e. P_0-lattices being Heyting algebras or partial B-algebras.

1. Preliminaries. Let A be a distributive lattice which is bounded, that is, it has the least element 0 and the greatest element 1; $x \cup y$ and xy denote the join and the meet, respectively, of two elements $x, y \in A$. The interval $\{z \in A: x \leqslant z \leqslant y\}$ is denoted by $[x, y]$. The center $B(A)$ of A is the Boolean sublattice of all complemented elements of A; the complement of an element $b \in B(A)$ is denoted by $\bar{b}$.

The greatest element $z \in A$ $(z \in B(A))$ such that $xz \leqslant y$, if it exists, is denoted by $x \to y$ $(x \Rightarrow y$, respectively). In particular, $x \to 0$ is denoted by x^*, and $1 \Rightarrow x$ by $!x$. If $x \to y$ $(x \Rightarrow y$, respectively) exists for any $x, y \in A$, then A is called a *Heyting algebra* (a *B-algebra*). An element x is *dense* in a Heyting algebra if $x^* = 0$. For the properties of Heyting algebras, see [5]; in particular, the following property will frequently be used: if an infinite join $\bigcup_{t \in T} a_t$ exists in a Heyting algebra A and $a \in A$, then $\bigcup_{t \in T} aa_t$ exists and (see [5], I, 11.2)

$$a \bigcup_{t \in T} a_t = \bigcup_{t \in T} aa_t. \tag{1.1}$$

A B-algebra is called a *P-algebra* provided that the identity

$$(x \Rightarrow y) \cup (y \Rightarrow x) = 1$$

or, equivalently,

$$z \Rightarrow (x \cup y) = (z \Rightarrow x) \cup (z \Rightarrow y) \tag{1.2}$$

is satisfied in it (see [2] for the definition and properties of P-algebras).

Let A be a bounded distributive lattice with center B. A (countable) chain

$$0 = e_0 \leqslant e_1 \leqslant \ldots \leqslant e_i \leqslant e_{i+1} \leqslant \ldots \leqslant e_\omega = 1$$

is called a *chain base* for A if for every element $x \in A$ there exists a descending sequence $x_1 \geqslant x_2 \geqslant \ldots \geqslant x_i \geqslant \ldots$ in B such that

$$x = \bigcup_{i=1}^{\infty} x_i e_i. \tag{1.3}$$

Then (1.3) is called a *monotonic representation* of x. The lattice A is called a *P_0-lattice of order ω^+* if it has a chain base (see [8]).

2. P_1-lattices of order ω^+.

Definition 2.1. A *P_1-lattice $\langle A, (e_i)_{0\leqslant i \leqslant \omega}\rangle$ of order ω^+* is a P_0-lattice A with a distinguished chain base $(e_i)_{0\leqslant i\leqslant\omega}$ such that A is a Heyting algebra and $e_{i+1} \to e_i = e_i$ for $i = 0, 1, 2, \ldots$ It follows that $e_j \to e_i = e_i$ for $j > i$.

LEMMA 2.1. *If $x = \bigcup_{k=1}^{\infty} x_k e_k$ is a monotonic representation of x in a P_1-lattice $\langle A, (e_i)_{0\leqslant i\leqslant\omega}\rangle$, then*

$$x_{i+1} \cup e_i = (x \to e_i) \to e_i \qquad \textit{for } i = 0, 1, \ldots$$

Proof. By Lemma 3.1 (i) of [8] and the definition of a P_1-lattice,

$$x \to e_i = \bigcap_{k=1}^{\infty} [\bar{x}_k \cup (e_k \to e_i)] = \bigcap_{k=i+1}^{\infty} (\bar{x}_i \cup e_i) = \bar{x}_{i+1} \cup e_i.$$

Therefore,

$$(x \to e_i) \to e_i = \bar{x}_{i+1} \to e_i = x_{i+1} \cup e_i$$

(see [1], Lemma 2.6, or [5] for the computational properties of the operation $\to$).

THEOREM 2.1 (see [1], Theorem 3.4). *Let $\langle A, (e_i)_{0\leqslant i\leqslant\omega}\rangle$ be a P_1-lattice. Then, for each $i = 0, 1, \ldots$, e_{i+1} is the smallest dense element in the interval $[e_i, 1]$. Thus $(e_i)_{0\leqslant i\leqslant\omega}$ is the unique chain such that $\langle A, (e_i)_{0\leqslant i\leqslant\omega}\rangle$ is a P_1-lattice of order ω^+.*

Proof. Let us assume that $x = \bigcup_{k=1}^{\infty} x_k e_k$ (a monotonic representation) is dense in $[e_i, 1]$, i.e. $x \geqslant e_i$ and $x \to e_i = e_i$. We have to show that $x \geqslant e_{i+1}$. Indeed, as $x \geqslant e_i$, we may assume that $x_1 = \ldots = x_i = 1$. Moreover,

$x_{i+1} \cup e_i = 1$ by Lemma 2.1. By Theorem 2.1 of [8], we obtain

$$x = \bigcap_{k=1}^{\infty} (x_k \cup e_{k-1}) = \bigcap_{k=i+2}^{\infty} (x_k \cup e_{k-1}) \geqslant e_{i+1}.$$

Note. The property of the chain base in P_1-lattices mentioned in Theorem 2.1 is analogous to the corresponding property of "the chain of the smallest dense elements" in Stone algebras of order n (see [3] for Stone algebras, and [1] for connections with P_1-lattices of order n). An application of our Lemma 2.1 and Theorem 2.1 of [8] yields the following representation of an element x in a P_1-lattice:

$$x = \bigcap_{i=0}^{\infty} [(x \to e_i) \to e_i].$$

Observe that $y_i = (x \to e_i) \to e_i \in [e_i, 1]$, and if y^{*i} denotes $y \to e_i$, then $y_i^{*i*i} = y_i$. Compare with the representation

$$x = \bigcap_{i=0}^{n-1} z_i, \quad \text{where } z_i \in [e_i, 1] \text{ and } z_i^{*i*i} = z_i,$$

in a Stone algebra of order n [3].

3. P_2-lattices of order ω^+.

Definition 3.1. A P_1-lattice $\langle A, (e_i)_{0 \leqslant i < \omega} \rangle$ of order ω^+ is called a *P_2-lattice of order* ω^+ if $e_i \Rightarrow x$ exists for every $x \in A$ $(i = 1, 2, \ldots)$. Then we write

$$D_i(x) = e_i \Rightarrow x \quad \text{for } i = 1, 2, \ldots \tag{3.1}$$

LEMMA 3.1. *The following conditions are equivalent for a P_2-lattice $\langle A, (e_i)_{0 \leqslant i < \omega} \rangle$ with center B:*

(i) *an infinite meet $\bigcap_{i=1}^{\infty}{}^{(B)} D_i(x)$ exists in B for every $x \in A$;*

(ii) *A is a B-algebra;*

(iii) *A is a P-algebra.*

Proof. The equivalence of (i) and (ii) follows from Lemma 3.2 (i) of [8] and from the fact that $y \Rightarrow z = !(y \to z)$ for $y, z \in A$. The equivalence of (ii) and (iii) was proved in [8], Theorem 3.1 (i).

Definition 3.2. $x = \bigcup_{i=1}^{\infty} x_i e_i$ is said to be the *highest monotonic representation* of x provided that $x_i \geqslant y_i$ for any monotonic representation

$$x = \bigcup_{i=1}^{\infty} y_i e_i.$$

THEOREM 3.1. *Let $\langle A, (e_i)_{0 \leqslant i \leqslant \omega}\rangle$ be a P_2-lattice with center B. Then*

(i) *every $x \in A$ has the highest monotonic representation*

$$x = \bigcup_{i=1}^{\infty} D_i(x) e_i$$

with the following properties $(i, j = 1, 2, \ldots)$:

(ii) $D_i(x \cup y) = D_i(x) \cup D_i(y)$;

(iii) $D_i(xy) = D_i(x) D_i(y)$;

(iv) $D_i(b) = b$ *for* $b \in B$;

(v) $D_i(x \to y) = \big(D_1(x) \to D_1(y)\big) \ldots \big(D_i(x) \to D_i(y)\big)$;

(vi) $D_i(x^*) = \big(D_1(x)\big)^* = \overline{D_1(x)}$;

(vii) $D_i(x) \cup \big(D_i(x)\big)^* = 1$;

(viii) $D_i(e_j) = 1$ *for* $i \leqslant j$, *and* $D_i(e_j) = {!e_j}$ *for* $i > j$;

(ix) $D_i\big(D_j(x)\big) = D_j(x)$.

Proof. (i) was proved in [8], Theorem 3.2, under the assumption that A is a B-algebra; however, only the existence of $e_i \Rightarrow x$ was used.

(iii) follows immediately from (3.1) and from the properties of $\Rightarrow$ (see [1], Lemma 2.6).

(iv) is verified directly by the definition of $\Rightarrow$ and of a P_1-lattice.

(vi) follows from (v) and (iv).

(vii) is obvious.

(viii) follows from the fact that $x \Rightarrow y = {!(x \to y)}$, and from the definition of a P_1-lattice.

(ix) is a consequence of (iv).

It remains to show (ii) and (v).

(ii) Let $x = \bigcup x_j e_j$ and $y = \bigcup y_k e_k$ be monotonic representations. By Lemma 3.2 (ii) of [8], we obtain

$$D_i(x) = e_i \Rightarrow x = \bigcap_{j=1}^{\infty}{}^{(B)} [x_j \cup (e_i \Rightarrow e_{j-1})], \tag{3.2}$$

and analogously for y and $x \cup y = \bigcup (x_k \cup y_k) e_k$ instead of x. Applying twice the dual of (1.1) (B — Boolean algebra), we obtain

$$D_i(x) \cup D_i(y) = \bigcap_{j,k=1}^{\infty}{}^{(B)} [x_j \cup y_k \cup (e_i \Rightarrow e_{j-1}) \cup (e_i \Rightarrow e_{k-1})].$$

But for $j \leqslant k$ the expression under the meet sign is not less than

$$x_k \cup y_k \cup (e_i \Rightarrow e_{k-1}) \geqslant D_i(x \cup y)$$

by an analogue of (3.2) with $x \cup y$ instead of x. Since the same applies to $j > k$, we get

$$D_i(x) \cup D_i(y) \geqslant D_i(x \cup y).$$

The converse inequality is obvious. Note that if A is a B-algebra, then (ii) follows immediately from Lemma 3.1 and (1.2).

(v) Since $e_i \Rightarrow (x \to y) = xe_i \Rightarrow y$ (easy to verify), using [6], p. 203, and Lemma 3.2 (i) of [8] we obtain

$$D_i(x \to y) = xe_i \Rightarrow y = \Big(\bigcup_{j=1}^{i} D_j(x)e_j\Big) \Rightarrow y$$
$$= \bigcap_{j=1}^{i} [\overline{D_j(x)} \cup (e_j \Rightarrow y)] = \bigcap_{j=1}^{i} [\overline{D_j(x)} \cup D_j(y)].$$

But $\overline{D_j(x)} \cup D_j(y) = D_j(x) \to D_j(y)$ (see [1], Lemma 2.6 (iii), or [5], I, 12.4).

For applications of P_0-lattices of finite order (see references in [1]), a so-called disjoint representation is of great importance. Here we give the following

Definition 3.3. A *disjoint representation* is a representation of the form

$$x = \bigcup_{1 \leqslant i \leqslant \omega} c_i e_i$$

(written $\bigcup_{i=1}^{\infty} c_i e_i \cup c_\omega$ in the sequel), where $c_i \in B$ and $c_i c_j = 0$ for $i \neq j$, $1 \leqslant i \leqslant \omega$, $1 \leqslant j \leqslant \omega$.

THEOREM 3.2. *Let $\langle A, (e_i)_{0 \leqslant i \leqslant \omega}\rangle$ be a P_2-lattice with center B, such that A is a B-algebra. Then every element $x \in A$ has the disjoint representation*

$$x = \bigcup_{i=1}^{\infty} C_i(x) e_i \cup C_\omega(x), \tag{3.3}$$

where

$$C_i(x) = D_i(x)\overline{D_{i+1}(x)} \quad \textit{for } i = 1, 2, \ldots, \tag{3.4}$$

and

$$C_\omega(x) = !x = \bigcap_{i=1}^{\infty}{}^{(B)} D_i(x)$$

(see Lemma 3.1). *Moreover, the following equality holds:*

$$D_i(x) = \bigcup_{j=i}^{\infty}{}^{(B)} C_j(x) \cup C_\omega(x). \tag{3.5}$$

Proof. Let us write, for simplicity, $c_i = C_i(x)$ and $d_i = D_i(x)$ (x — fixed). Obviously, $c_i e_i \leqslant x$ for $1 \leqslant i \leqslant \omega$. Let $z \in A$, and $c_i e_i \leqslant z$ for $1 \leqslant i \leqslant \omega$. In order to prove (3.3), we have only to show that $x \leqslant z$.

By an easy calculation based on (3.4), we obtain

$$d_i e_i \leqslant z \cup d_i e_i = z \cup d_{i+1} e_i = \ldots = z \cup d_j e_i \quad \text{for all } j \geqslant i.$$

Therefore, using Theorem 3.1, we have

$$d_i \leqslant [e_i \Rightarrow (z \cup d_j e_i)] = D_i(z \cup d_j e_i) = (e_i \Rightarrow z) \cup d_j,$$

and

$$d_i \overline{(e_i \Rightarrow z)} \leqslant d_j \qquad \text{for } j \geqslant i.$$

Hence

$$d_i \overline{(e_i \Rightarrow z)} \leqslant \bigcap_{j=i}^{\infty}{}^{(B)} d_j = c_\omega = !c_\omega \leqslant !z,$$

and

$$D_i(x) = d_i \leqslant !z \cup (e_i \Rightarrow z) = !z \cup D_i(z) = D_i(z) \qquad \text{for } i = 1, 2, \ldots$$

Thus $x \leqslant z$, and (3.3) is proved. The proof of (3.5) is analogous and will be omitted.

4. Obtaining P_2-lattices from P_0-lattices. Epstein and Horn (see [1], Theorems 3.3 and 4.4) have proved that if a P_0-lattice of order n is a Heyting algebra, then it has a chain base $(h_i)_{0 \leqslant i \leqslant n-1}$ such that $\langle A, (h_i)_{0 \leqslant i \leqslant n-1}\rangle$ is a P_1-lattice of order n. If, moreover, A is a B-algebra, then the resulting P_1-lattice is a P_2-lattice. In this section, imposing rather strong conditions on a P_0-lattice of order ω^+, we construct a chain base $(h_i)_{0 \leqslant i \leqslant \omega}$ such that $\langle A, (h_i)_{0 \leqslant i \leqslant \omega}\rangle$ is a P_2-lattice.

Throughout this section we assume that A is a P_0-lattice with a chain base $(e_i)_{0 \leqslant i \leqslant \omega}$, such that

(A) A is a σ-complete lattice.

(B) The center B of A is a σ-regular sublattice of A.

(C) A is a Heyting algebra and a B-algebra.

LEMMA 4.1. *Under assumptions* (A), (B), *and* (C), *if*

$$x = \bigcup_{i=1}^{\infty} b_i e_i, \quad \textit{where } b_i \in B \ (i = 1, 2, \ldots)$$

(*not necessarily a monotonic representation*), *then x has a monotonic representation*

$$x = \bigcup_{i=1}^{\infty} x_i e_i, \quad \textit{where } x_i = \bigcup_{k=i}^{\infty} b_k \ (i = 1, 2, \ldots).$$

Proof. Applying (1.1), we obtain

$$b_i e_i \leqslant x_i e_i = \Big(\bigcup_{k=i}^{\infty} b_k\Big) e_i = \bigcup_{k=i}^{\infty} b_k e_i \leqslant \bigcup_{k=i}^{\infty} b_k e_k \leqslant x.$$

Thus

$$x = \bigcup_{i=1}^{\infty} b_i e_i \leqslant \bigcup_{i=1}^{\infty} x_i e_i \leqslant x.$$

LEMMA 4.2. *Let n be an integer, $n \geqslant 0$. Under assumptions* (A), (B), *and* (C), *there exists a chain base $(f_i)_{0 \leqslant i \leqslant \omega}$ such that*

$$f_i = e_i \quad \text{for } i = 0, 1, 2, \ldots, n, \tag{4.1}$$

$$f_{n+1} \to f_n = f_n. \tag{4.2}$$

If $x \in A$ has a (not necessarily monotonic) representation

$$x = \bigcup_{i=1}^{\infty} x_i e_i, \tag{4.3}$$

then there exists a (not necessarily monotonic) representation

$$x = \bigcup_{i=1}^{\infty} y_i f_i \tag{4.4}$$

such that

$$y_i = x_i \quad \text{for } i = 1, 2, \ldots, n-1, \tag{4.5}$$

and

$$\bigcup_{i=1}^{k} y_i f_i \geqslant \bigcup_{i=1}^{k} x_i e_i \quad \text{for } k = 1, 2, \ldots \tag{4.6}$$

Proof. Let $f_i = e_i$ for $i = 0, 1, 2, \ldots, n$, let f_{n+1} have a monotonic representation

$$f_{n+1} = \bigcup_{i=1}^{\infty} (e_{i-1} \Rightarrow e_n) e_i, \tag{4.7}$$

and let $f_i = f_{n+1} \cup e_i$ for $i = n+2, n+3, \ldots$

By Lemma 3.3 (ii) of [8] we have

$$e_k \to e_n = e_k \cup (e_k \Rightarrow e_n) \quad \text{for } k, n = 0, 1, \ldots$$

Therefore,

$$[\overline{(e_k \Rightarrow e_n)} \cup (e_{k+1} \Rightarrow e_n)](e_k \to e_n) = [\overline{(e_k \Rightarrow e_n)} \cup (e_{k+1} \to e_n)][e_n \cup (e_k \Rightarrow e_n)]$$
$$\leqslant e_n \cup (e_{k+1} \Rightarrow e_n) = e_{k+1} \to e_n.$$

Applying this, one can prove the inequality

$$\bigcap_{i=1}^{k} [\overline{(e_{i-1} \Rightarrow e_n)} \cup (e_i \to e_n)] \leqslant e_k \to e_n \quad \text{for } k = 1, 2, \ldots \tag{4.8}$$

by induction over k. By (4.7), Lemma 3.1 (i) of [8], (4.8), and Lemma 3.1 (i) of [8] again, we obtain

$$f_{n+1} \to e_n = \bigcap_{i=1}^{\infty} [\overline{(e_{i-1} \Rightarrow e_n)} \cup (e_i \to e_n)]$$
$$\leqslant \bigcap_{k=1}^{\infty} (e_k \to e_n) = \Big(\bigcup_{k=1}^{\infty} e_k\Big) \to e_n = 1 \to e_n = e_n.$$

Since the converse inequality is obvious, (4.2) is proved.

Now, the following equality can be shown:

$$f_i[e_n \cup (\overline{e_i \Rightarrow e_n})] = e_i \quad \text{for } i \geqslant n+1. \tag{4.9}$$

It is verified by a substitution $f_i = f_{n+1} \cup e_i$, where f_{n+1} is given by (4.7), by an application of (1.1) and by an easy observation that

$$e_i \leqslant e_n \cup (\overline{e_i \Rightarrow e_n})$$

(since $e_i(e_i \Rightarrow e_n) \leqslant e_n$).

Let $x \in A$ have representation (4.3). Set

$$y_i = \begin{cases} x_i & \text{for } i = 1, 2, \ldots, n-1, \\ \bigcup_{j=n}^{\infty} x_j & \text{for } i = n, \\ x_i(\overline{e_i \Rightarrow e_n}) & \text{for } i = n+1,\ n+2, \ldots \end{cases} \tag{4.10}$$

Then inequality (4.6) is obvious for $k \leqslant n$, and it is easily obtained from (4.9) for $k > n$. On the other hand, we have $y_i f_i \leqslant x$ for all $i = 1, 2, \ldots$ (apply (1.1) for $i = n$ and (4.9) for $i > n$). The last observation together with (4.6) yields (4.4). The fact that $(f_i)_{0 \leqslant i \leqslant \omega}$ is a chain base follows now from Lemma 4.1.

THEOREM 4.3. *Under assumptions* (A), (B), *and* (C), *there exists a chain base* $(h_i)_{0 \leqslant i \leqslant \omega}$ *for* A *such that* $h_{i+1} \to h_i = h_i$ $(i = 0, 1, 2, \ldots)$. *Thus* $\langle A, (h_i)_{0 \leqslant i \leqslant \omega} \rangle$ *is a* P_2*-lattice.*

Proof. Let $e_{i,0} = e_i$. By induction we construct the sequence $E_0, E_1, E_2, \ldots$ of chain bases,

$$E_k = (e_{i,k})_{0 \leqslant i \leqslant \omega} \quad \text{for } k = 0, 1, 2, \ldots$$

Namely, E_{k+1} is obtained by an application of Lemma 4.2 to the chain base E_k, where the integer $n = k$. At the same time, for given

$$x = \bigcup_{i=1}^{\infty} x_{i,0} e_{i,0},$$

we obtain the sequence of representations

$$x = \bigcup_{i=1}^{\infty} x_{i,k} e_{i,k}$$

of x in E_k (with the properties as in Lemma 4.2). We have, by (4.1),

$$e_{i,i} = e_{i,i+1} = e_{i,i+2} = \ldots$$

and, by (4.5),

$$x_{i,i+1} = x_{i,i+2} = \ldots$$

Set $h_i = e_{i,i}$ and $z_i = x_{i,i+1}$ for $i = 1, 2, \ldots$ It follows easily from (4.2) that $h_{i+1} \to h_i = h_i$. By (4.6), we have

$$\bigcup_{i=1}^{l} z_i h_i = \bigcup_{i=1}^{l} x_{i,l+1} e_{i,l+1} \geqslant \bigcup_{i=1}^{l} x_{i,0} e_{i,0}.$$

On the other hand, $z_i h_i = x_{i,i+1} e_{i,i+1} \leqslant x$. Thus

$$\bigcup_{i=1}^{\infty} z_i h_i = x. \tag{4.11}$$

In view of Lemma 4.1, the proof is complete.

Remark. The last application of Lemma 4.1 is, in fact, superfluous. Namely, using (4.10) and calculating effectively $x_{i,1}, x_{i,2}, \ldots$ and, finally, $x_{i,i+1} = z_i$, we obtain the following formula:

$$z_i = \bigcup_{j=i}^{\infty} x_{j,0} \overline{(e_j \Rightarrow e_0)}\, \overline{(e_j \Rightarrow e_1)} \ldots \overline{(e_j \Rightarrow e_{i-1})}.$$

Thus $z_1 \geqslant z_2 \geqslant \ldots$ irrespective of the initial representation

$$x = \bigcup x_{i,0} e_i.$$

5. Post algebras of order ω^+.

THEOREM 5.1. *Let $\langle A, (e_i)_{0 \leqslant i \leqslant \omega} \rangle$ be a P_2-lattice of order ω^+. Then the following properties are equivalent*:

(i) *each element $x \in A$ has a unique monotonic representation*;

(ii) $D_i(e_j) = 0$ *for* $j < i$;

(iii) $!e_i = 0$ *for* $i = 0, 1, 2, \ldots$;

(iv) $e_{i+1} \Rightarrow e_i = 0$ *for* $i = 0, 1, 2, \ldots$

Proof. Obviously, (i) implies (ii), and (ii) is equivalent to (iii) (Theorem 3.1 (viii)); (iv) is a particular case of (ii); (iv) implies (i) (see [6], Lemma 1.3).

Definition 5.2. A P_2-lattice is called a *generalized Post algebra of order* ω^+ if it satisfies (i)-(iv) of Theorem 5.1.

It is easy to see that this definition is equivalent to the definition of [4].

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