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REMARK ON FOURIER-STIELTJES TRANSFORMS OF CONTINUOUS MEASURES

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In a paper by Hartman and Ryll-Nardzewski [2], the following question is asked: can a continuous measure μ have its Fourier-Stieltjes transform $\hat{\mu}$ such that $|\hat{\mu}| \geqslant \varepsilon > 0$ on a set H which is not a Sidon set. The answer was shown by Kaufman [3] to be yes. Here* an explicit example is constructed.

Let $E = \{n_k\}_{k=1}^{\infty}$ be a lacunary subset of the non-negative integers with $n_{k+1}/n_k \geqslant 3$, $k = 1, 2, \ldots$ Let $F = \{n_i \pm n_j\colon i > j,\ i, j = 1, 2, \ldots\}$, and $H = F \cup -F$. A simple argument (see [4], p. 621, or [1], p. 52) shows that the characteristic function χ_H of H has the properties:

(1) $\chi_H \in \mathrm{cl}\big(M(T)\hat{}\,\big)$ (closure in sup-norm, T the circle group),

(2) the von Neumann mean $\mathscr{M}(\chi_H)$ of χ_H is zero, and

(3) H is not a Sidon set.

Indeed, letting $\mu \in M(T)$ be the continuous measure given by the Riesz product

$$\prod_{k=1}^{\infty} (1 + \cos n_k x),$$

we infer that

$$(2\hat{\mu} - 2\hat{m}_T)^n \xrightarrow{n} \chi_{E \cup -E}$$

(m_T denotes the Haar measure of T); and given $\varepsilon > 0$, there exist n and m such that

$$\| [(4\hat{\mu} - 4\hat{m}_T) - 2(2\hat{\mu} - 2\hat{m}_T)^n]^m - \chi_H \|_\infty < \varepsilon.$$

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REFERENCES

[1] C. Dunkl and D. Ramirez, *Topics in harmonic analysis*, New York 1971.

[2] S. Hartman and C. Ryll-Nardzewski, *Quelques résultats et problèmes en algèbre des mesures continues*, Colloquium Mathematicum 22 (1971), p. 271-277.

[3] R. Kaufman, *Remark on Fourier-Stieltjes transforms of continuous measures*, ibidem 22 (1971), p. 279-280.

[4] D. Ramirez, *Uniform approximation by Fourier-Stieltjes coefficients*, Proceedings of the Cambridge Philosophical Society 64 (1968), p. 615-623.

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