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THE DOMAIN OF ATTRACTION OF A NON-GAUSSIAN STABLE DISTRIBUTION IN A HILBERT SPACE

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Some characterization of the domain of attraction of a Gaussian measure in a Hilbert space has been given in papers [6] and [8].

The aim of this paper * is to obtain some necessary and sufficient conditions in order that a distribution belong to the domain of attraction of a non-Gaussian stable distribution in a Hilbert space. These conditions on the real line are given in the known theorem of Doeblin and Gnedenko (see [1] and [2]).

The theorem formulated in this paper is based on some results of Jajte (see [4] and [5]).

Let H be a separable real Hilbert space with the inner product $(\cdot, \cdot)$ and the norm $\|\cdot\|$. Denote by $\mathfrak{M}$ the set of all probability distributions in H, i.e., the set of normed regular measures defined on the σ-field $\mathcal{B}$ of all Borel subsets of H. $\mathfrak{M}$ is a complete space with the Lévy-Prokhorov metric (see [10], p. 188). Convergence in this metric space is equivalent to the weak convergence of distributions.

The convolution is a continuous operation in $\mathfrak{M}$. For any $p \in \mathfrak{M}$, we denote by p^{n*} the n-th convolution power of p.

The *characteristic functional* $\hat{p}$ of $p \in \mathfrak{M}$ is defined by the formula (see [9])

$$\hat{p}(h) = \int_H e^{i(x,h)} p(dx), \qquad h \in H,$$

and determines p uniquely.

A linear operator in H is called an *S-operator* if it is non-negative self-adjoint and has a finite trace (see [10], p. 193).

Denote by δ_x the distribution concentrated at a point $x \in H$. A sequence of distributions $\{p_n\}$ is called *shift-convergent* if there exists a sequence $\{x_n\}$ of elements of H such that the sequence $\{p_n * \delta_{x_n}\}$ is convergent in $\mathfrak{M}$.

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* The results presented here have been announced without proofs in [7].

For every positive a and every $p \in \mathfrak{M}$, we write

$$T_a p(A) = p\{x \in H: ax \in A\} \quad \text{for every } A \in \mathscr{B}.$$

A distribution p is said to be *stable* if, for every pair of positive numbers a and b, there exist a positive number c and an element $x \in H$ such that $T_a p * T_b p = T_c p * \delta_x$.

Let $p \in \mathfrak{M}$ and $f \in H$. By p^f we denote the distribution on the real line induced by the element f, i.e.,

$$p^f(A) = p\{x \in H: (x, f) \in A\} \quad \text{for every Borel set } A \text{ on the real line.}$$

LEMMA 1. *If, for a sequence of positive numbers $\{a_n\}$, the sequence of distributions $\{T_{a_n} p^{n*}\}$ is shift-convergent to a non-degenerate distribution q, then*

$$\lim_{n\to\infty} a_n = 0, \tag{1}$$

$$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 1 \tag{2}$$

(see Lemma 2 of [5]).

Proof. By the assumption, there exists a sequence $\{x_n\}$ of elements of H such that

$$\lim_{n\to\infty} T_{a_n} p^{n*} * \delta_{x_n} = q.$$

Thus, for an arbitrary $f \in H$, we have

$$\lim_{n\to\infty} T_{a_n} (p^f)^{n*} * \delta_{(x_n, f)} = q^f.$$

Since the distribution q is non-degenerate, it is easily seen that there exists an element $f_0 \in H$ such that the distribution q^{f_0} is non-degenerate. By the Lemma in Section 29 of [3], we get the assertion.

LEMMA 2. *A distribution q is stable if and only if there exist a sequence of positive numbers $\{a_n\}$ and a distribution p such that the sequence of distributions $\{T_{a_n} p^{n*}\}$ is shift-convergent to the distribution q.*

Proof. The proof is analogous to that for the real line (see [3], Section 33).

The set of all distributions p for which there exists a sequence of positive numbers $\{a_n\}$ such that the sequence of distributions $\{T_{a_n} p^{n*}\}$ is shift-convergent to a distribution q is called the *domain of attraction of the distribution q*.

We shall investigate the domain of attraction of a *non-Gaussian stable distribution*, i.e., of a distribution whose characteristic functional is of the form

$$\varphi(h) = \exp\Big[i(x_0, h) + \int_H K(x, h)M(dx)\Big], \qquad h \in H,$$

where $x_0 \in H$,

$$K(x, h) = e^{i(x,h)} - 1 - \frac{i(x, h)}{1 + \|x\|^2},$$

and M is a measure in H which is finite on the complement of every neighbourhood of zero in H and such that

$$\int_{\|x\| \leqslant 1} \|x\|^2 M(dx) < +\infty \tag{3}$$

and there exists a λ $(0 < \lambda < 2)$ with (see [5])

$$T_a M = a^\lambda M \quad \text{for every positive } a. \tag{4}$$

It is clear that the measure M is σ-finite on $H \setminus \{0\}$, and that if the measure M is non-degenerate, then $M(H) = +\infty$.

Denote by $\Delta^{(\lambda)}$ $(0 < \lambda < 2)$ the domain of attraction of the non-degenerate non-Gaussian stable distribution determined by the measure M defined above and satisfying (4).

Assign to a distribution $p \in \mathfrak{M}$ the distribution $\tilde{p}$ on the real line defined by the formula

$$\tilde{p}(A) = p\{x \in H : \|x\| \in A\} \quad \text{for every Borel set } A \text{ on the real line.} \tag{5}$$

LEMMA 3. *If a distribution p belongs to $\Delta^{(\lambda)}$, then the distribution $\tilde{p}$, defined by* (5), *is attracted by a stable distribution on the real line with the characteristic exponent λ.*

Proof. Let $\{a_n\}$ be a sequence of positive numbers such that the sequence of distributions $\{T_{a_n} p^{n*}\}$ is shift-convergent to the non-degenerate stable distribution determined by a measure M.

Put

$$M_n = nT_{a_n} p * \delta_{-z_n}, \quad \text{where } (z_n, f) = \int_{\|x\| \leqslant 1} (x, f) T_{a_n} p(dx). \tag{6}$$

Let U stand for an arbitrary neighbourhood of zero in H. It follows from Lemma 1 and from the Corollary of [4] that the sequence of measures M_n reduced to $H \setminus U$ converges weakly to the measure M (obviously, reduced to $H \setminus U$). Simultaneously (see [11], Lemma 7.1),

$$\lim_{n \to \infty} \|z_n\| = 0. \tag{7}$$

It follows from (7) that the sequence of measures $\{nT_{a_n}p\}$, reduced to $H\setminus U$, converges weakly to the measure M (reduced to $H\setminus U$). Hence and from (4), for every $u > 0$, we have

$$\lim_{n\to\infty} n \int_{\|x\|\geqslant u} T_{a_n}p(dx) = M\{x\in H\colon \|x\|\geqslant u\} = u^{-\lambda}M\{x\in H\colon \|x\|\geqslant 1\} = cu^{-\lambda}. \tag{8}$$

Observe that $c\neq 0$. Otherwise, the measure M would be concentrated at zero, i.e., the stable distribution determined by it would be degenerate, contrary to the assumption.

It follows easily from (8) that the conditions of Theorem 2 of Section 35 in [3] are satisfied (see the proof of this theorem). Thus the distribution $\tilde{p}$ is attracted by the stable distribution on the real line determined by the constants λ, 0 and c (see the Theorem of Section 34 in [3]).

LEMMA 4. *A non-degenerate distribution p belongs to $\Delta^{(\lambda)}$ if and only if there exists a sequence of positive numbers $\{a_n\}$ such that*

(a′) *for any neighbourhood U of zero in H, the sequence of measures $\{nT_{a_n}p\}$, reduced to $H\setminus U$, is weakly convergent to a non-degenerate measure M (reduced to $H\setminus U$) which is finite on $H\setminus U$ and satisfies conditions* (3) *and* (4);

(d′) *for every $\varepsilon > 0$,*

$$\lim_{N\to\infty}\sup_n \sum_{i=N}^{\infty}\int_{\|x\|\leqslant\varepsilon}(x,e_i)^2 nT_{a_n}p(dx) = 0, \quad \textit{where } \{e_i\} \textit{ is a basis in } H.$$

Proof. It follows from the Corollary of [4] that $p\in\Delta^{(\lambda)}$ if and only if there exists a sequence of positive numbers $\{a_n\}$ such that $\lim_{n\to\infty} a_n = 0$, and

(a) for any neighbourhood U of zero in H, the sequence of measures $M_n = nT_{a_n}p*\delta_{-z_n}$, reduced to $H\setminus U$, is weakly convergent to a non-degenerate measure M (reduced to $H\setminus U$) which is finite on $H\setminus U$ and satisfies conditions (3) and (4) (the sequence $\{z_n\}$ has been defined in (6)),

(b) for every $f\in H$,

$$\lim_{\varepsilon\to 0}\overline{\lim_{n\to\infty}}\int_{\|x\|\leqslant\varepsilon}(x,f)^2 M_n(dx) = 0,$$

(c) for every $\varepsilon > 0$,

$$\sup_n\sum_{i=1}^{\infty}\int_{\|x\|\leqslant\varepsilon}(x,e_i)^2 M_n(dx) < +\infty,$$

(d) for every $\varepsilon > 0$,

$$\lim_{N\to\infty}\sup_n\sum_{i=N}^{\infty}\int_{\|x\|\leqslant\varepsilon}(x,e_i)^2 M_n(dx) = 0,$$

where $\{e_i\}$ is a basis in H.

Observe that (a) is equivalent to (a′), since each of these conditions implies

$$\lim_{n\to\infty} a_n = 0,$$

and hence

$$\lim_{n\to\infty} \|z_n\| = 0.$$

Observe also that if we assume condition (a) or (a′), then, in order that conditions (c), (d) and (d′) be satisfied, it suffices that they be satisfied for some $\varepsilon > 0$.

It follows from (a) or (a′) that the distribution $\tilde{p}$ is attracted by a stable distribution on the real line with the characteristic exponent λ (see the proof of Lemma 3). Thus

$$\lim_{\varepsilon\to 0}\overline{\lim_{n\to\infty}} \int_{\|x\|\leqslant\varepsilon} \|x\|^2 nT_{a_n}p(dx) = 0 \tag{9}$$

and

$$\int_H \|x\|^2 p(dx) = +\infty \tag{10}$$

(see the proof of Theorem 2 of Section 35 in [3]).

Assume now that conditions (a′) and (d′) are satisfied for some sequence $\{a_n\}$. Let $\varepsilon\in(0, \frac{1}{2})$ and let $f\in H$. Since (7) follows from (a′), for n sufficiently large, we have

$$\begin{aligned}
\int_{\|x\|\leqslant\varepsilon} (x,f)^2 M_n(dx) &= \int_{\|x-z_n\|\leqslant\varepsilon} (x-z_n,f)^2 nT_{a_n}p(dx) \\
&\leqslant \int_{\|x\|\leqslant 2\varepsilon} (x-z_n,f)^2 nT_{a_n}p(dx) = \int_{\|x\|\leqslant 2\varepsilon} (x,f)^2 nT_{a_n}p(dx) + \\
&\quad + n(z_n,f)^2 - n(z_n,f)^2 \int_{\|x\|>2\varepsilon} T_{a_n}p(dx) - 2(z_n,f)\int_{\|x\|\leqslant 2\varepsilon} nT_{a_n}p(dx) \\
&\leqslant \int_{\|x\|\leqslant 2\varepsilon} (x,f)^2 nT_{a_n}p(dx) + n\Big[(z_n,f) - \int_{\|x\|\leqslant 2\varepsilon} (x,f)T_{a_n}p(dx)\Big]^2 \\
&= \int_{\|x\|\leqslant 2\varepsilon} (x,f)^2 nT_{a_n}p(dx) + n\Big[\int_{2\varepsilon<\|x\|\leqslant 1} (x,f)T_{a_n}p(dx)\Big]^2.
\end{aligned} \tag{11}$$

Simultaneously, we have

$$\lim_{n\to\infty} n\Big[\int_{2\varepsilon<\|x\|\leqslant 1} (x,f)T_{a_n}p(dx)\Big]^2 = 0.$$

Thus

$$\begin{aligned}
\overline{\lim_{n\to\infty}} \int_{\|x\|\leqslant\varepsilon} (x,f)^2 M_n(dx) &\leqslant \overline{\lim_{n\to\infty}} \int_{\|x\|\leqslant 2\varepsilon} (x,f)^2 nT_{a_n}p(dx) \\
&\leqslant \overline{\lim_{n\to\infty}} \|f\|^2 \int_{\|x\|\leqslant 2\varepsilon} \|x\|^2 nT_{a_n}p(dx).
\end{aligned} \tag{12}$$

Inequality (12) and equality (9) imply (b).

Observe now that

$$(13)\quad \overline{\lim_{n\to\infty}}\sum_{i=1}^{\infty} n\Big[\int\limits_{2\varepsilon<\|x\|\leqslant 1}(x, e_i)T_{a_n}p(dx)\Big]^2 = \overline{\lim_{n\to\infty}}\, n\|g_n^\varepsilon\|^2$$

$$\leqslant \lim_{n\to\infty}\|g_n^\varepsilon\|\, n\int\limits_{\|x\|>2\varepsilon}T_{a_n}p(dx) = 0,$$

where

$$(g_n^\varepsilon, f) = \int\limits_{2\varepsilon<\|x\|\leqslant 1}(x, f)T_{a_n}p(dx) \quad \text{for every } f\in H.$$

Condition (d) follows from (d′), (11) and (13). Finally, from (b) and (d) we obtain

$$\overline{\lim_{n\to\infty}}\sum_{i=1}^{\infty}\int\limits_{\|x\|\leqslant\varepsilon}(x, e_i)^2 M_n(dx) < +\infty$$

which is equivalent to (c).

Assume now that, for some sequence $\{a_n\}$, conditions (a), (b), (c) and (d) are satisfied. It remains to show how condition (d′) follows from them. Given $\varepsilon\in(0,1)$, for n sufficiently large we have

$$(14)\quad \int\limits_{\|x\|\leqslant\varepsilon}(x, e_i)^2 nT_{a_n}p(dx)$$

$$= \int\limits_{\|x\|\leqslant\varepsilon}(x-z_n, e_i)^2 nT_{a_n}p(dx) -$$

$$-(\bar{z}_n, e_i)^2\int\limits_{\|x\|\leqslant\varepsilon}nT_{a_n}p(dx) + 2(z_n, e_i)\int\limits_{\|x\|\leqslant\varepsilon}(x, e_i)nT_{a_n}p(dx)$$

$$\leqslant \int\limits_{\|x\|\leqslant 2\varepsilon}(x, e_i)^2 M_n(dx) + (z_n, e_i)^2\int\limits_{\|x\|>\varepsilon}nT_{a_n}p(dx) +$$

$$+n\Big[\int\limits_{\|x\|\leqslant\varepsilon}(x, e_i)T_{a_n}p(dx)\Big]^2 - n\Big[(z_n, e_i) - \int\limits_{\|x\|\leqslant\varepsilon}(x, e_i)T_{a_n}p(dx)\Big]^2$$

$$\leqslant \int\limits_{\|x\|\leqslant 2\varepsilon}(x, e_i)^2 M_n(dx) + \int\limits_{\|x\|>\varepsilon}(z_n, e_i)^2 nT_{a_n}p(dx) + n\Big[\int\limits_{\|x\|\leqslant\varepsilon}(x, e_i)T_{a_n}p(dx)\Big]^2.$$

It follows from (10) that

$$\lim_{n\to\infty}\frac{n\Big[\int\limits_{\|x\|\leqslant\varepsilon}\|x\|T_{a_n}p(dx)\Big]^2}{n\int\limits_{\|x\|\leqslant\varepsilon}\|x\|^2 T_{a_n}p(dx)} = 0$$

(see the proof of Theorem 1 of Section 35 in [3]), and whence, by (9), we have

$$(15)\quad \lim_{n\to\infty} n\Big[\int\limits_{\|x\|\leqslant\varepsilon}\|x\|T_{a_n}p(dx)\Big]^2 = 0.$$

It follows easily from (15) that

$$\lim_{n\to\infty}\sum_{i=1}^{\infty} n\Big[\int_{\|x\|\leqslant\varepsilon}(x,e_i)T_{a_n}p(dx)\Big]^2 = 0. \tag{16}$$

Simultaneously, we have

$$\lim_{n\to\infty}\sum_{i=1}^{\infty}(z_n,e_i)^2 n\int_{\|x\|>\varepsilon}T_{a_n}p(dx) = 0 \tag{17}$$

and, by (14), (16), (17) and assumption (d), we obtain (d′).

The proof of Lemma 4 is thus complete.

Assign to a non-degenerate distribution p the family of S-operators D_X defined by the bilinear form

$$(D_X g, h) = \frac{\int_{\|x\|\leqslant X}(x,g)(x,h)p(dx)}{\int_{\|x\|\leqslant X}\|x\|^2 p(dx)} \quad \text{for every } g, h\in H \tag{18}$$

and the family of measures m_X defined in H by the formula

$$m_X = \frac{T_{X^{-1}}p}{p\{x\in H:\ \|x\|\geqslant X\}}. \tag{19}$$

THEOREM. *A non-degenerate distribution p belongs to $\Delta^{(\lambda)}$ if and only if the following conditions are satisfied:*

(i) *for every $k>0$,*

$$\lim_{X\to+\infty}\frac{\int_{\|x\|\geqslant X}p(dx)}{\int_{\|x\|\geqslant kX}p(dx)} = k^{\lambda},$$

i.e., the distribution $\tilde{p}$, defined by (5), *is attracted by a stable distribution on the real line with the characteristic exponent λ* (see Theorem 2 of Section 35 in [3]);

(ii) *for any neighbourhood U of zero in H, the measures m_X, reduced to $H\setminus U$, are (as $X\to+\infty$) weakly convergent to a measure m (obviously, reduced to $H\setminus U$) satisfying the condition*

$$\int_{\|x\|\leqslant 1}\|x\|^2 m(dx) < +\infty;$$

(iii) *for a basis $\{e_i\}$ in H and a positive number X_0,*

$$\lim_{N\to\infty}\sup_{X>X_0}\sum_{i=N}^{\infty}(D_X e_i, e_i) = 0.$$

Then the measure m determines the measure defining the limit stable distribution uniquely up to a constant factor.

Proof. *Necessity.* Condition (i) immediately follows from Lemma 3. Let a sequence $\{a_n\}$ satisfy conditions (a′) and (d′) of Lemma 4. Let $\{x_n\}$ be an arbitrary sequence such that

$$\lim_{n\to\infty} x_n = +\infty.$$

It follows from Lemma 1 that, for every sufficiently large n, one can find a natural number $k(n)$ such that

$$a_{k(n)+1} \leqslant x_n^{-1} < a_{k(n)},$$

and thus such that

$$\lim_{n\to\infty} x_n a_{k(n)} = 1. \tag{20}$$

Hence the sequences of distributions $\{T_{x_n^{-1}} p^{k(n)*}\}$ and $\{T_{a_n} p^{n*}\}$ are shift-convergent to the same stable distribution determined by a certain measure M. Applying the Corollary of [4] to the sequence $\{T_{x_n^{-1}} p^{k(n)*}\}$ and then proceeding similarly as in the proof of Lemma 4, we obtain

(21) condition (a′) of Lemma 4 is satisfied for the sequence of measures $\{k(n) T_{x_n^{-1}} p\}$ (instead of $\{n T_{a_n} p\}$),

$$\lim_{N\to\infty} \sup_n \sum_{i=N}^{\infty} \int_{\|x\|\leqslant\varepsilon} (x, e_i)^2 k(n) T_{x_n^{-1}} p(dx) = 0 \quad \text{for every } \varepsilon > 0. \tag{22}$$

From Lemma 3 and from Theorem 4 of Section 25 in [3] we have

$$\lim_{n\to\infty} n T_{a_n} p\{x \in H: \|x\| \geqslant u\} = c u^{-\lambda} \quad \text{for every } u > 0, \tag{23}$$

where $c > 0$ (see the proof of Lemma 3).

The measures m_{x_n} can be written in the form

$$m_{x_n} = \frac{k(n) T_{x_n^{-1}} p}{k(n) T_{x_n^{-1}} p\{x \in H: \|x\| \geqslant 1\}}.$$

Thus it follows from (21) and (23) that the sequence of measures $\{m_{x_n}\}$, reduced to $H \setminus U$, converges weakly to the measure

$$m = \frac{M}{M\{x \in H: \|x\| \geqslant 1\}}$$

(obviously, reduced to $H \setminus U$) which has property (3).

Condition (iii) can be obtained from (22) if we notice that

$$\varliminf_{n\to\infty} k(n) \int_{\|x\|\leqslant 1} \|x\|^2 T_{x_n^{-1}} p(dx) > 0.$$

In fact, assuming the contrary, we have

$$0 = \overline{\lim_{n\to\infty}} k(n) \int\limits_{1/2\leqslant\|x\|\leqslant 1} \|x\|^2 T_{x_n^{-1}} p(dx) \geqslant \overline{\lim_{n\to\infty}} \frac{1}{4} k(n) \Big[\int\limits_{\|x\|\geqslant 1/2} T_{x_n^{-1}} p(dx) - $$

$$ - \int\limits_{\|x\|\geqslant 1} T_{x_n^{-1}} p(dx) \Big] = \frac{1}{4} (2^\lambda - 1) M\{x \in H\colon \|x\| \geqslant 1\} > 0.$$

Sufficiency. It follows from assumption (i) and from Theorem 4 of Section 25 in [3] that there exists a sequence of positive numbers $\{a_n\}$ satisfying condition (23). Setting $u = 1$ in (23) and applying assumption (ii), we infer that the sequence of measures $\{nT_{a_n} p\}$, reduced to $H \setminus U$, converges weakly to the measure cm (reduced to $H \setminus U$).

Let a be an arbitrary positive number. Then we have

$$m_{1/aa_n} = \frac{T_{aa_n} p}{p\{x \in H\colon \|x\| \geqslant 1/aa_n\}} = \frac{T_a(nT_{a_n} p)}{nT_{a_n} p\{x \in H\colon \|x\| \geqslant 1/a\}}.$$

Thus it follows from assumption (ii) and from (23) that the measure m, reduced to $H \setminus U$, has property (4).

The measure m is non-degenerate and finite on $H \setminus U$. This is a consequence of the fact that $0 \neq m\{x \in H\colon \|x\| \geqslant 1\} = 1 < +\infty$. Since $m(H) = +\infty$, $m(U) = +\infty$ and $m(\{0\}) = +\infty$, the measure m has property (4). We have shown that condition (a′) of Lemma 4, with $M = cm$, is satisfied.

From (iii) we obtain

$$\limsup_{N\to\infty} \sum_{i=N}^{\infty} \frac{n \int\limits_{\|x\|\leqslant\varepsilon} (x, e_i)^2 T_{a_n} p(dx)}{n \int\limits_{\|x\|\leqslant\varepsilon} \|x\|^2 T_{a_n} p(dx)} = 0 \quad \text{for every } \varepsilon > 0. \tag{24}$$

Condition (a′) implies (9) (see the proof of Lemma 4). Thus, for ε sufficiently small, we have

$$\overline{\lim_{n\to\infty}} n \int\limits_{\|x\|\leqslant\varepsilon} \|x\|^2 T_{a_n} p(dx) < +\infty. \tag{25}$$

Condition (d′) of Lemma 4 follows now easily from (24) and (25).

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