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CONTINUA WHICH CANNOT BE MAPPED ONTO ANY NON-PLANER CIRCLE-LIKE CONTINUUM

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Fort [3] in 1959 proved that the dyadic solenoid was not the continuous image of any plane continuum. His result was generalized by McCord [7] (see also [5], p. 248) and this in turn by Ingram [5] who showed that a circle-like continuum M is the continuous image of a plane continuum if and only if M is a plane continuum. Taking P to be the class of all connected and simply connected polyhedra, we prove here(1) that no P-like continuum can be mapped onto a nonplaner circle-like continuum.

Using the theorems of Mardešić and Segal [6] together with the characterization of non-planer circle-like continua given by Bing [1] this result will be proved in the following restated form.

THEOREM. *Suppose that M is the inverse limit $\{P_i, \pi_{ij}\}$ each P_i is a connected and simply connected polyhedron. Further suppose that N is the inverse limit $\{S_i, \lambda_{ij}\}$ where each S_i is a circle and the bonding maps are such that if $e > 0$ then there is an integer n such that $|\deg \lambda_{1n}| > e$. Then there is no map of M onto N.*

Here $\deg \lambda_{kn}$ is the topological degree of the map λ_{kn} (see [6], ch. II). The notation for inverse limits will be that used in [6] and the directed sets will be always the positive integers. S will denote the unit circle in the complex plane and φ the map from the real numbers R to S defined by $\varphi(x) = e^{2\pi i x}$.

The theorem is a consequence of the following two lemmas.

LEMMA 1. *If for each map k from a continuum Q to S there is a map $\hat{k}$ from Q to R (a lift of k) such that $\varphi\hat{k} = k$, then there is no map of Q onto N.*

Indication of proof of Lemma 1.

Suppose there is a map f of Q onto N. Then by hypothesis there is a map $\widehat{h_1 f}$ such that $h_1 f = \varphi \widehat{h_1 f}$. h_i will denote the natural map from N

(1) The author enjoyed discussing this result with P. Roy.

to S_i. Let e be an integer greater than the diameter of $\widehat{h_1 f}(Q)$. By the original assumption on the bonding maps for N there is an integer n such that the $|\deg\lambda_{ln}| > e$. Now $h_1 = \lambda_{ln}h_n$ and $h_n f$ maps Q onto S. Since λ_{ln} maps S more than e times around S it follows that $\widehat{h_1 f}(Q)$ must have a diameter at least as large as $|\deg\lambda_n|$. However, this diameter was assumed to be less than e.

LEMMA 2. *If k is a map of M to S then there is a map $\hat{k}$ from M to R such that $k = \varphi\hat{k}$.*(2)

By Lemma 1 of Mioduszewski [8] there is an integer n and a map w from P_n to S such that $|wg_n(x) - k(x)| < \frac{1}{3}$ for each $x \in M$. Consider the following diagram, where g is the natural map from M to P_n.

$$\begin{tikzcd}
P_n \arrow[d, "\hat{w}"'] \arrow[dr, "w"] & M \arrow[l, "g_n"'] \arrow[d, "k"] \arrow[dl, dashed, "\hat{k}"'] \\
R \arrow[r, "\varphi"] & S
\end{tikzcd}$$

Since P_n is simply connected there is a map $\hat{w}$ from P_n to R such that $\varphi\hat{w} = w$. Define $\hat{k}(x)$ to be the point of $\varphi^{-1}k(x)$ closest to $\hat{w}\ g(x)$. $\hat{k}$ is well defined since φ^{-1} of a point is geometrically congruent to the set of integers in R.

Remark. A plane continuum which separates the plane is not the inverse limit of simple connected polyhedra, thus our result does not include Ingrams.

Question 1. Characterize the class W of continua which cannot be mapped onto a non-planer circle-like continuum. (**P 742**)

Theorem 4 of [5] implies that $A \in W$ if and only if A cannot be mapped onto any non-planer solenoid.

Question 2. What is the relation between W above and the class C of continua which cannot be mapped onto a non-planer solenoid by an upper semi-continuous map whose point inverses are continua (cf. [2]). (**P 743**)

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[1] R. H. Bing, *Embedding circle-like continua*, Canadian Journal of Mathematics 14 (1967), p. 113-128.

[2] H. Cook, *Upper semi-continuous continuum-valued mappings onto circle-like continua*, Fundamenta Mathematicae 60 (1967), p. 233-239.

(2) The author feels that this Lemma should be in the literature, but being unable to find it, has included a proof for completeness.

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