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SUMS AND LIMITS OF ALMOST CONTINUOUS FUNCTIONS

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Fast [4] has shown that if T is a collection of c-many real functions, then there exists a function g such that $g+t$ is Darboux for each t in T. From this theorem, Fast infers, as corollaries, that each real function is the sum of two Darboux functions and the pointwise limit of a sequence of Darboux functions. In the present note we show that Fast's results hold if „Darboux" is replaced by „almost continuous".

Fast was not the first to prove that each real function is the sum of two Darboux functions and the pointwise limit of a sequence of Darboux functions. Several different proofs of these facts have appeared. For a survey of these, see [2].

Unless otherwise stated, all functions considered are real-valued with domain the set real numbers R. No distinction is made between a function and its graph. The function f is said to be *Darboux* if $f(C)$ is connected whenever C is a connected subset of the domain of f. If each open set containing f also contains a continuous function with the same domain as f, then f is *almost continuous*. It is clear that if $f\colon R\to R$ is almost continuous, then f is Darboux (this is not true for functions of several variables). Suppose $f\colon A\to B$. The statement that K is a *minimal blocking set of f in $A\times B$* means that K is a closed subset of $A\times B$, K contains no point of f, K intersects each continuous function $g\colon A\to B$, and no proper subset of K has the preceding properties. If $A\times B$ is the plane, we simply say that K is a *minimal blocking set of f*. If D is a subset of the plane, the X-projection of D is denoted by $(D)_X$. The letter I denotes the interval $[0,1]$, and c denotes the cardinality of R.

LEMMA 1. *Suppose $f\colon I\to R$. If f is not almost continuous, then there exists a minimal blocking set K of f in $I\times R$, and $(K)_X$ is non-degenerate and connected. Also K is a perfect set.*

Proof. The proof of the existence of K is essentially the same as that given in [6]. That $(K)_X$ is non-degenerate is obvious.

Assume that $(K)_X$ is not connected. Then some number z in I separates $(K)_X$. By the minimality of K, there exist continuous functions

[2] A. M. Bruckner and J. G. Ceder, *Darboux continuity*, Jahresbericht der Deutschen Mathematiker Vereinigung 67 (1965), p. 93-117.

[3] — and Max Weiss, *Uniform limits of Darboux functions*, Colloquium Mathematicum 15 (1966), p. 65-77.

[4] H. Fast, *Une remarque sur la propriété de Weierstrass*, ibidem 7 (1959), p. 75-77.

[5] J. B. Jones, *Connected and disconnected plane sets and the functional equation* $f(x)+f(y) = f(x+y)$, Bulletin of the American Mathematical Society 48 (1942), p. 115-120.

[6] K. R. Kellum and B. D. Garrett, *Almost continuous real functions*, Proceedings of the American Mathematical Society 33 (1972), p. 181-184.

[7] D. Phillips, *Real functions having graphs connected and dense in the plane*, Fundamenta Mathematicae 75 (1972), p. 47-49.

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