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REMARKS ON DYADIC SPACES, II

BY

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Let $D = \{0, 1\}$ denote the two-point discrete space. For any cardinal number $\mathfrak{m}$ by the $\mathfrak{m}$-*Cantor set*, we mean the Cartesian product $D^{\mathfrak{m}}$ of $\mathfrak{m}$ copies of D. The $\aleph_0$-Cantor set is the well-known Cantor perfect set on the real line. It is known (see e. g. [20], vol. II, p. 13) that every compact metrizable space is a continuous image of $D^{\aleph_0}$. In [1] P. S. Alexandroff defined a dyadic space as a space which, for some cardinal number $\mathfrak{m}$, is a continuous image of $D^{\mathfrak{m}}$. Marczewski in [21], answering a question raised by P. S. Alexandroff, showed that there exist compact spaces which are not dyadic. The class of dyadic spaces was investigated also by Šanin [28], Esenin-Volpin [12], Alexandroff and Ponomarev [2], Efimov [6], [7], [8] and Engelking and Pełczyński [10].

This note begins with simple proofs of the four known theorems on dyadic spaces, in particular, of those of Šanin and Esenin-Volpin. By Šanin theorem a dyadic space cannot be represented as a union of its nowhere dense subsets, which form an increasing transfinite sequence (this is a reinforcement of the classical Baire category theorem). The theorem due to Esenin-Volpin deals with the relation between character at every point and weight of dyadic space (for definitions see footnote (2) on p. 185). In the second part, devoted to dense and pseudocompact subsets of the Cartesian product of compact metrizable spaces, we show that the product is the Čech-Stone compactification of each such subset. This is a slight generalization of a result by Corson and also an auxiliary theorem for the results of the part three. This part is devoted to the irreducible dyadic spaces (i. e. the images of $D^{\mathfrak{m}}$ under irreducible continuous mappings) and their dense subspaces, which are called *irreducible subdyadic spaces*. We prove that a metric compact space is irreducible dyadic if and only if it is dense in itself. In connection with the Theorem 3 from [10], which claims that if the Čech-Stone compactification of the space X is dyadic, then X is pseudocompact, we show that the Čech-Stone compactification of any irreducible subdyadic pseudocompact space is dyadic. It turns out that the class of irreducible subdyadic pseudocompact spaces is closed with respect to the Cartesian multiplication. Two last parts present some complements to an earlier paper on dyadic spaces written by A. Pełczyński and the second of the present authors. These parts are concerned with the dyadicity of subsets of basically disconnected spaces and with extending of mappings from closed subsets of $D^{\mathfrak{m}}$.

By *space* we always mean a completely regular T_1-space, by *mapping* we mean a continuous function. The Čech-Stone compactification of a space X is denoted by βX. It is characterized among all compactifications of X (to within a homeomorphism keeping X pointwise fixed) by the fact that every mapping $f: X \to Z$ into a compact space Z has a continuous extension over βX. The Cartesian product (with the Tychonoff topology) of a family $\{X_s\}_{s \in S}$ of spaces will be denoted by $\underset{s \in S}{P} X_s$. For every $S_0 \subset S$ the projection $p_{S_0}: \underset{s \in S}{P} X_s \to \underset{s \in S_0}{P} X_s$ is defined, which is a mapping from the whole product $\underset{s \in S}{P} X_s$ onto $\underset{s \in S_0}{P} X_s$. In the particular case of $S_0 = \{s_0\}$ the mapping $p_{s_0} = p_{\{s_0\}}$ is called the *projection onto s_0-axis*. A subset $K = \underset{s \in S}{P} K_s$, where $K_s \subset X_s$ for $s \in S$, of the Cartesian product $\underset{s \in S}{P} X_s$ will be called the *cube* in $\underset{s \in S}{P} X_s$. A cube $K = \underset{s \in S}{P} K_s$, where K_s is a non-empty open subset of X_s and the set $\{s \in S: K_s \neq X_s\}$ is finite, is called a *basic cube* in $\underset{s \in S}{P} X_s$. The definitions of all notions from general topology which are not defined here can be found in [19], [14] or [20].

1. Simple proofs of some known theorems on dyadic spaces. Let $\{A_t\}_{t \in T}$ be a family of sets. A subset T_0 of T is said to be a *Δ-set for the family* $\{A_t\}_{t \in T}$ or shortly a *Δ-set*, if $A_{t_1} \cap A_{t_2} = \bigcap_{t \in T_0} A_t$ for every $t_1, t_2 \in T_0$ with $t_1 \neq t_2$.

From a theorem by Erdös and Rado [11], proved also in an elegant manner by Michael [23], it follows at once that the following Lemma 1 holds under the assumption that $\mathfrak{m}$ is regular cardinal number (¹) of the form $\aleph_{\alpha+1}$. Presented here proof of the general case is obtained by an insignificant modification of Michael's proof from [23].

LEMMA 1 (Šanin [28], p. 24). *For every family $\{A_t\}_{t \in T}$ of finite sets, where $\overline{\overline{T}} = \mathfrak{m} > \aleph_0$ is a regular cardinal number, there exists a Δ-set $T_0 \subset T$ of power $\mathfrak{m}$.*

Proof (Michael [23]). Assume that every Δ-set contained in T is of power less than $\mathfrak{m}$. In order to obtain the contradiction it is enough

(¹) i. e. $\mathfrak{m}$ cannot be expressed as a sum of less than $\mathfrak{m}$ cardinal numbers, each of which is less than $\mathfrak{m}$.

to define a sequence of sets $T_1, T_2, \ldots$ satisfying conditions

(1) $$\overline{\overline{T}}_i < \mathfrak{m} \quad \text{for} \quad i = 1, 2, \ldots,$$

(2) $$T = \bigcup_{i=1}^{\infty} T_i.$$

Put $T_0 = 0$ and suppose that the sets $T_0, T_1, \ldots, T_{k-1}$ of power less than $\mathfrak{m}$ have been already defined. Let

$$F_k = \bigcup_{i=1}^{k-1} \bigcup_{t \in T_i} A_t,$$

and let $\mathfrak{F}_k$ denote the family of all finite subsets of F_k. Since the property of being a Δ-set is of finite character, then for every $B \in \mathfrak{F}_k$ the set

$$T(B) = \{t \in T\colon A_t \cap F_k = B\}$$

contains a maximal Δ-set $T^*(B)$. Moreover, we can assume that

(3) if $A_{t_1} \cap A_{t_2} = B$ for some $t_1, t_2 \in T(B)$, where $t_1 \neq t_2$, then $\bigcap_{t \in T^*(B)} A_t = B$.

Since $\overline{\overline{T_0 \cup T_1 \cup \ldots \cup T_{k-1}}} < \mathfrak{m}$, we have $\overline{\overline{F}}_k < \mathfrak{m}$ and also $\overline{\overline{\mathfrak{F}}}_k < \mathfrak{m}$. By our assumption, $\overline{\overline{T^*(B)}} < \mathfrak{m}$ for every $B \in \mathfrak{F}_k$, whence by the regularity of $\mathfrak{m}$ the power of the set

$$T_k = \bigcup_{B \in \mathfrak{F}_k} T^*(B)$$

is less than $\mathfrak{m}$. We shall show that the sets $T_1, T_2, \ldots$ defined in this manner satisfy (2).

Suppose that there exists $t_0 \in T \setminus \bigcup_{i=1}^{\infty} T_i$. For $B_i = A_{t_0} \cap F_i \in \mathfrak{F}_i$ we have

(4) $$t_0 \in T(B_i) \setminus T^*(B_i) \quad \text{for} \quad i = 1, 2, \ldots$$

For every i there exists $t_i \in T^*(B_i)$ such that

$$A_{t_0} \cap (A_{t_i} \setminus F_i) \neq 0.$$

Indeed, supposing the contrary, we would have $A_{t_0} \cap A_t = B_i$ for $t \in T^*(B_i)$. Thus, by (3) and (4), $T^*(B_i) \cup \{t_0\} \subset T(B_i)$ would be a Δ-set containing $T^*(B_i)$ as a proper subset, which is impossible by the maximality of $T^*(B_i)$.

Let us choose a point $x_i \in A_{t_0} \cap (A_{t_i} \setminus F_i)$ for $i = 1, 2, \ldots$ The points $x_1, x_2, \ldots$ are all in the set A_{t_0}, and, as $x_i \in A_{t_i} \subset F_{i+1} \subset F_j$ for $i < j$, and $x_j \notin F_j$ they are also different, but this is impossible, because the set A_{t_0} is finite.

LEMMA 2. *Every family* $\{K_t\}_{t\in T}$, *where* $\overline{\overline{T}} = \mathfrak{m} > \aleph_0$ *is a regular cardinal number, of basic cubes in* $D^{\mathfrak{n}} = \mathop{P}_{s\in S} D_s$ *contains a subfamily* $\{K_t\}_{t\in T_0}$ *such that* $\overline{\overline{T}}_0 = \mathfrak{m}$ *and* $\bigcap_{t\in T_0} K_t \neq 0$.

Proof. The set $A_t = \{s\in S: p_s(K_t) \neq D_s\}$ is finite for every $t\in T$. By Lemma 1 there exists a subset $T_1 \subset T$ which is a Δ-set for the family $\{A_t\}_{t\in T}$ and such that $\overline{\overline{T}}_1 = \mathfrak{m}$. If $\bigcap_{t\in T_1} A_t = 0$, then the sets of the family $\{A_t\}_{t\in T_1}$ are disjoint and the intersection $\bigcap_{t\in T_1} K_t$ is non-empty. Thus we can suppose that $0 \neq \bigcap_{t\in T_1} A_t = \{s_1, s_2, \ldots, s_k\}$. For some sequence $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k$, where $\varepsilon_i = 0$ or 1 for $i = 1, 2, \ldots, k$, one can find a subset $T_0 \subset T_1$ of power $\mathfrak{m}$ such that

$$p_{s_i}(K_t) = \varepsilon_i \quad \text{for} \quad t\in T_0 \quad \text{and} \quad i = 1, 2, \ldots, k.$$

Since $A_{t_1} \cap A_{t_2} \setminus \bigcap_{t\in T_0} A_t = 0$ for $t_1, t_2 \in T_0$, with $t_1 \neq t_2$, the intersection $\bigcap_{t\in T_0} K_t$ is non-empty.

From Lemma 2 we obtain

THEOREM 1 (Šanin [28], p. 83). *Every family of regular power* $\mathfrak{m} > \aleph_0$, *composed of non-empty open subsets of a dyadic space, contains a subfamily of power* $\mathfrak{m}$ *which has a non-empty intersection.*

THEOREM 2 (Šanin [28], p. 83). *Every decreasing transfinite sequence* $G_1 \supset G_2 \supset \ldots \supset G_\xi \supset \ldots$, $\xi < \alpha$, *of type* α *not cofinal with* ω_0, *composed of non-empty open subsets of a dyadic space, has a non-empty intersection.*

Proof. Theorem 2 follows at once from Theorem 1 if we remark that the power of the smallest ordinal number cofinal with α is a regular cardinal number greater than $\aleph_0$.

COROLLARY 1. *Dyadic space cannot be decomposed into the union of its proper closed subsets which form an increasing transfinite sequence* $F_1 \subset F_2 \subset \ldots \subset F_\xi \subset \ldots$, $\xi < \alpha$ *of type* α, *not cofinal with* ω_0.

THEOREM 3 (Šanin [28], p. 84). *The intersection of every family of dense open subsets of a dyadic space* X *which form a decreasing transfinite sequence* $G_1 \supset G_2 \supset \ldots \supset G_\xi \ldots$, $\xi < \alpha$, *is dense in* X.

Proof. If α is cofinal with ω_0, then the theorem follows from the Baire category theorem, which is valid in arbitrary compact space. Suppose that α is not cofinal with ω_0 and let U be an arbitrary non-empty open subset of X. By Theorem 2, applied to the transfinite sequence $U \cap G_1 \supset U \cap G_2 \supset \ldots \supset U \cap G_\xi \supset \ldots$, $\xi < \alpha$, it follows that the intersection of our family meets U, i. e. that the intersection is dense in X.

COROLLARY 2. *Dyadic space cannot be decomposed into the union of its nowhere dense subsets, which form an increasing transfinite sequence* $F_1 \subset F_2 \subset \ldots \subset F_\xi \subset \ldots, \xi < a$.

THEOREM 4 (Esenin-Volpin [12]). *If the character at every point* (²) *of dyadic space X is not greater than $\mathfrak{m} \geqslant \aleph_0$, then the weight of the space X is also not greater than $\mathfrak{m}$* (³).

Proof. Let $f\colon D^{\mathfrak{n}} \to X$ be a mapping of the $\mathfrak{n}$-Cantor set $D^{\mathfrak{n}} = \underset{s \in S}{P} D_s$ onto X. For every $x \in X$ the counter-image $f^{-1}(x)$ can be represented as the intersection of at most $\mathfrak{m}$ open subsets of $D^{\mathfrak{n}}$:

(5) $$f^{-1}(x) = \bigcap_{t \in T(x)} U_t, \quad \text{where} \quad \overline{\overline{T(x)}} \leqslant \mathfrak{m}.$$

Let us choose for every $x \in X$ a point $x' \in f^{-1}(x)$ and, for every $t \in T(x)$, a basic cube $\underset{s \in S}{P} D_s^t$, where $D_s^t = D_s$ for $s \notin S(t)$ and $\overline{\overline{S(t)}} < \aleph_0$, such that

(6) $$x' \in \underset{s \in S}{P} D_s^t \subset U_t.$$

For

$$S(x) = \bigcup_{t \in T(x)} S(t) \quad \text{and} \quad D_s^x = \bigcap_{t \in T(x)} D_s^t$$

we have by (5) and (6)

(7) $$x' \in K(x) = \underset{s \in S}{P} D_s^x \subset f^{-1}(x), \text{ where } D_s^x = D_s \text{ for } s \notin S(x) \text{ and}$$

$$\overline{\overline{S(x)}} \leqslant \mathfrak{m}.$$

Since $\{K(x)\}_{x \in X}$ is the family of non-empty pairwise disjoint cubes each of which has at most $\mathfrak{m}$ faces different from the factors of the Cartesian product $\underset{s \in S}{P} D_s$, it follows from Theorem 6 of [9] (or from Theorem 1.2 of [23]) that $\overline{\overline{X}} \leqslant 2^{\mathfrak{m}}$.

We may confine our attention to the $2^{\mathfrak{m}}$-Cantor set $D^{2^{\mathfrak{m}}} = \underset{s \in S}{P} D_s'$, where $D_s' = D_s$ for $s \in \bigcup_{x \in X} S(x)$ and $D_s' = \{0\}$ for $s \in S \setminus \bigcup_{x \in X} S(x)$, and to the mapping $f' = f|D^{2^{\mathfrak{m}}}\colon D^{2^{\mathfrak{m}}} \to X$, which maps $D^{2^{\mathfrak{m}}}$ onto X. Since $D^{2^{\mathfrak{m}}}$ contains a dense subset of the power $\mathfrak{m}$ (see [17], [21] or [9]), there exists in X a dense subset X_0 such that $\overline{\overline{X_0}} \leqslant \mathfrak{m}$. Let $D^{\mathfrak{m}} = \underset{s \in S}{P} D_s'' \subset D^{2^{\mathfrak{m}}}$, where $D_s'' = D_s$ for $s \in \bigcup_{x \in X_0} S(x)$ and $D_s'' = \{0\}$ for $s \in S \setminus \bigcup_{x \in X_0} S(x)$. The

(²) By the *character at a point* x of a topological space X we mean the smallest cardinality of local bases at x. By the *weight* of a space X we mean the smallest cardinality of bases of X.

(³) For some generalizations of this theorem see [6] and [7].

mapping $f'' = f|D^{\mathfrak{m}}: D^{\mathfrak{m}} \to X$ maps $D^{\mathfrak{m}}$ onto X. Indeed, the image $f(D^{\mathfrak{m}})$ contains the dense subset X_0 of X and, as a compact set, it is also closed in X. Now theorem follows from the fact that the weight of a continuous image of a compact space is not greater than the weight of this space.

2. Dense and pseudocompact subsets of products. The following lemma is given without proof in [18] and attributed to Corson. The proof can be obtained by a slight modifications of the proof either of Theorem 2 of [4] or of Theorem 2.1 of [5]. Those proofs are based upon the theorem of Bockstein [3] (see also [27] and [9]), which claims that for every pair U, V of disjoint open subsets of the Cartesian product $\underset{s\in S}{P} X_s$ of a number of spaces with countable bases there exists a countable subset $S_0 \subset S$ such that $p_{S_0}(U)$ and $p_{S_0}(V)$ are disjoint.

LEMMA 3. *For every continuous real-valued function f defined on a dense subset M of the Cartesian product $\underset{s\in S}{P} X_s$ of spaces with countable bases there exists a countable subset $S_0 \subset S$ such that f is constant on each set $M \cap p_{S_0}^{-1}(x)$, where $x \in \underset{s\in S_0}{P} X_s$.*

LEMMA 4. *A subset M of the Cartesian product $\underset{s\in S}{P} X_s$ of a number of compact metrizable spaces is dense and pseudocompact* (4) *if and only if for every countable subset $S_0 \subset S$ we have $p_{S_0}(M) = \underset{s\in S_0}{P} X_s$.*

Proof. If $M \subset \underset{s\in S_0}{P} X_s$ is dense and pseudocompact then, for any countable $S_0 \subset S$, the image $p_{S_0}(M)$ is dense in $\underset{s\in S_0}{P} X_s$ and pseudocompact. Since for the metrizable space $\underset{s\in S_0}{P} X_s$ pseudocompactness is equivalent with countable compactness and compactness, we have $p_{S_0}(M) = \underset{s\in S_0}{P} X_s$.

Suppose now that $p_{S_0}(M) = \underset{s\in S_0}{P} X_s$ for every countable subset $S_0 \subset S$. Clearly M is dense in $\underset{s\in S}{P} X_s$. Let f be an arbitrary continuous real-valued function defined on M and let S_0 be as in Lemma 3. We have then $f = f_0 p_{S_0}|M$, where f_0 is a real-valued function defined on $\underset{s\in S_0}{P} X_s$ by the formula $f_0(x) = f(M \cap p_{S_0}^{-1}(x))$. Since the set $p_{S_0}^{-1}(x) \cap M$ meets every non-empty open set V such that $x \in p_{S_0}(V)$, the function f_0 is continuous. Thus the function f is bounded, because $f(M) = f_0(\underset{s\in S_0}{P} X_s)$ is a compact subset of the set of real numbers.

(4) By *pseudocompact* space we mean a space X such that every continuous real-valued function defined on X is bounded. For normal spaces pseudocompactness coincides with countable compactness.

THEOREM 5. *The Cartesian product* $\underset{s\in S}{P} X_s$ *of compact metrizable spaces is the Čech-Stone compactification of every dense and pseudocompact subspace.*

Proof. It suffices to show that every mapping from a dense and pseudocompact subspace M of $\underset{s\in S}{P} X_s$ into the closed interval $[0, 1]$ can be extended over $\underset{s\in S}{P} X_s$. But this follows immediately from the proof of Lemma 4.

Theorem 5 presents a slight generalization of the Theorem 2 from [4], where some particular dense and pseudocompact subsets of the Cartesian product of compact metrizable spaces are examined. Let us remark that the converse theorem is also true. Indeed, the Cartesian product $\underset{s\in S}{P} X_s$ of compact metrizable spaces is dyadic and every $M \subset \underset{s\in S}{P} X_s$ such that $\beta M = \underset{s\in S}{P} X_s$ is pseudocompact by Theorem 3 from [10].

The following theorem follows at once from Lemma 4.

THEOREM 6. *Let* $\{P_s\}_{s\in S}$ *be a family of Cartesian products of compact metrizable spaces and let* M_s *be a subspace of* P_s. *The Cartesian product* $\underset{s\in S}{P} M_s$ *is dense in* $\underset{s\in S}{P} P_s$ *and pseudocompact if and only if* M_s *is dense in* P_s *and pseudocompact for every* $s \in S$.

3. Irreducible dyadic and subdyadic spaces. We recall that a mapping $f\colon X \to Y$ of a space X onto a space Y is said to be *irreducible* provided that $f(X_0) \neq Y$ for every proper closed subset $X_0 \subset X$. It is easy to see that for an irreducible mapping $f\colon X \to Y$ of a compact space X we have

(8) $\qquad \operatorname{Int} f(U) \neq 0$ for any non-empty and open $U \subset X$,

(9) $\qquad \operatorname{Int} f(U) \cap \operatorname{Int} f(X \setminus U) = 0$ for any open $U \subset X$.

Moreover, a mapping $f\colon X \to Y$ from a space X onto a space Y, which satisfies (8) and (9), is irreducible.

A space X is said to be *irreducible dyadic* provided that for some $\mathfrak{m}$ there exists an irreducible mapping $f\colon D^{\mathfrak{m}} \to X$ of the $\mathfrak{m}$-Cantor set onto X. A space X is said to be (*irreducible*) *subdyadic* provided that there exists a (irreducible) dyadic space which contains a homeomorph of X as a dense subspace ([5]).

It is clear that the image of a dense in itself space (i. e. of the space without isolated points) by an irreducible mapping is dense in itself.

([5]) This class of spaces was investigated first by Alexandroff and Ponomarev in [2] and [25].

Hence there exist dyadic metrizable spaces which are not irreducible dyadic. However we have

THEOREM 7. *Every dense in itself metrizable compact space is irreducible dyadic.*

Moreover, in the space of all mappings of the Cantor set onto such a space the irreducible mappings form a dense G_δ-set.

Proof. Let X be a dense in itself metrizable compact space and let ϱ be a metric for X. The space $X^{D^{\aleph_0}}$ of all mappings of the Cantor set $D^{\aleph_0}$ into X with the metric $\bar{\varrho}$ defined by the formula

$$\bar{\varrho}(f, g) = \sup_{x \in D^{\aleph_0}} \varrho\big(f(x), g(x)\big)$$

is a complete metric space. It is easy to see that the non-empty (see [20], vol. II, p. 13) set

$$\mathfrak{F} = \{f \in X^{D^{\aleph_0}} : f(D^{\aleph_0}) = X\}$$

is closed in $X^{D^{\aleph_0}}$ (cf. [20], vol. II, p. 30) and hence it is also complete.

Let $\{G_i\}_{i=1}^{\infty}$ be a countable base of $D^{\aleph_0}$ composed of non-empty closed-and-open sets (which are homeomorphic to $D^{\aleph_0}$) and let

$$\mathfrak{F}_i = \{f \in \mathfrak{F} : f(D^{\aleph_0} \setminus G_i) = X\}.$$

The subset

$$N = \bigcup_{i=1}^{\infty} \mathfrak{F}_i$$

of $\mathfrak{F}$ coincides with the set of all reducible mappings from $D^{\aleph_0}$ to X. Since $\mathfrak{F}_i$ is closed for $i = 1, 2, \ldots$ if suffices to prove, by the Baire category theorem, that the sets $\mathfrak{F} \setminus \mathfrak{F}_i$ are dense in $\mathfrak{F}$.

Consider an integer k, a mapping $f \in \mathfrak{F}$ and an $\varepsilon > 0$. It is enough to show that there exists $f^* \in \mathfrak{F}$ such that

$$f^*(D^{\aleph_0} \setminus G_k) \neq X \tag{10}$$

and

$$\bar{\varrho}(f, f^*) < \varepsilon. \tag{11}$$

Let $\{U_i\}_{i=1}^{m}$ be an irreducible (i. e. such that $X \setminus \bigcup_{i \neq i_0} U_i \neq 0$ for every $i_0 \leqslant m$) covering of X by open sets of diameter less then ε, and let $\{F_i\}_{i=1}^{m}$ be a closed covering of X such that $F_i \subset U_i$ for $i = 1, 2, \ldots, m$ (see [20], vol. I, p. 124). Since $D^{\aleph_0}$ is zero-dimensional (6) there exist

(6) A space X is said to be *zero-dimensional* if for any two disjoint zero-sets (i. e. counter-images of closed subsets of the real line by a continuous real-valued function defined on X) A, B there exists closed-and-open set $E \subset X$ such that $A \subset E$ and $B \subset X \setminus E$. For compact X, and for metrizable X with countable base, this is equivalent to the existence of a base composed of closed-and-open sets.

(see [20], vol. I, p. 167) closed-and-open subsets $C_1, C_2, \ldots, C_m$ of $D^{\aleph_0}$ satisfying

$$D^{\aleph_0} = \bigcup_{i=1} C_i, \quad C_i \cap C_j = 0 \quad \text{for} \quad i \neq j \quad \text{and} \quad f(C_i) \subset U_i.$$

It follows from the irreducibility of the covering $\{U_i\}_{i=1}^m$ that $C_i \neq 0$ for $i = 1, 2, \ldots, m$. Consider $i_0 \leqslant m$ which satisfies $C_{i_0} \cap G_k \neq 0$; let $f^*\colon C_i^* \to F_i$ be, for every $i \neq i_0$, a mapping of the Cantor set C_i onto F_i and let f'_{i_0} be a mapping of the Cantors et $C_{i_0} \cap G_k$ onto F_{i_0}. The formula

$$f^*_{i_0}(x) = \begin{cases} f'_{i_0}(x) & \text{for} \quad x \in C_{i_0} \cap G_k, \\ x_0 & \text{for} \quad x \in C_{i_0} \setminus G_k, \end{cases}$$

where x_0 is a fixed point of $X \setminus \bigcup_{i \neq i_0} F_i \subset F_{i_0}$, defines a mapping $f^*_{i_0}\colon C_{i_0} \to F_{i_0}$.

For the mapping $f^*\colon D^{\aleph_0} \to X$ defined by the formula

$$f^*(x) = f^*_i(x), \quad \text{where} \quad x \in C_i,$$

we have

$$f^*(D^{\aleph_0}) = f^*\Big(\bigcup_{i=1}^{m} C_i\Big) = \bigcup_{i=1}^{m} f^*_i(C_i) = \bigcup_{i=1}^{m} F_i = X.$$

Thus $f^* \in \mathfrak{F}$. However,

$$f^*(D^{\aleph_0} \setminus G_k) \subset \bigcup_{i \neq i_0} F_i \cup \{x_0\}$$

and since $X \setminus \bigcup_{i \neq i_0} F_i$ is a non-empty open set in X, (10) is satisfied. It is easy to verify that (11) is satisfied too.

The first part of Theorem 7 was known (see for example [26], p. 91), but the proof, to our best knowledge, has been never published. The proof of the first part is much simpler and can be carried as follows. For any compact metrizable space X there is a mapping $f\colon D^{\aleph_0} \to X$ of the Cantor set onto X. By the Brouwer reduction theorem (see [20], vol. II, p. 27) there exists closed set $F_0 \subset D$ such that $f(F_0) = X$ and $f(F) \neq X$ for $F = \overline{F} \nsubseteq F_0$. Hence $f_0 = f|F_0\colon F_0 \to X$ is irreducible. If X is dense in itself, then so is F_0 which, being zero-dimensional, is homeomorphic to $D^{\aleph_0}$ (see [20], vol. II, p. 58).

COROLLARY 3. *Every dense in itself, metrizable space with countable base is an irreducible subdyadic space.*

LEMMA 5. *Let two families $\{X_s\}_{s \in S}$ and $\{Y_s\}_{s \in S}$ of spaces and a family $\{f_s\}_{s \in S}$ of mappings, where $f_s\colon X_s \to Y_s$, be given. If the mappings of the family $\{f_s\}_{s \in S}$ are irreducible, then the mapping $f = \underset{s \in S}{\mathrm{P}} f_s\colon \underset{s \in S}{\mathrm{P}} X_s \to \underset{s \in S}{\mathrm{P}} Y_s$, defined by the formula $f(\{x_s\}) = \{f_s(x_s)\}$, is also irreducible.*

Proof. Consider an arbitrary closed proper subset X_0 of $X = \underset{s \in S}{P} X_s$. Then there exist a finite set $S_0 = \{s_1, s_2, \ldots, s_k\} \subset S$ and non-empty open subsets $W_i \subset X_{s_i}$ for $i = 1, 2, \ldots, k$ such that

$$W = \bigcap_{i=1}^{k} p_{s_i}^{-1}(W_i) \subset X \setminus X_0.$$

From the irreducibility of f_{s_i} it follows that there exists a point

$$y_{s_i} \in Y_{s_i} \setminus f_{s_i}(X_{s_i} \setminus W_i) \qquad \text{for} \qquad i = 1, 2, \ldots, k.$$

Let y_s be an arbitrary point of Y_s for $s \in S \setminus S_0$. Consider the point $y = \{y_s\} \in \underset{s \in S}{P} Y_s$ and an arbitrary point $x = \{x_s\} \in \underset{s \in S}{P} X_s$ such that $f(x) = y$. We have then $y_{s_i} = f_{s_i}(x_{s_i})$ and $x_{s_i} \in W_i$ for $i = 1, 2, \ldots, k$, i. e.

$$x = \{x_s\} \in W \subset X \setminus X_0 \qquad \text{and} \qquad y \notin f(X_0) \neq f(X).$$

The following theorem follows from Lemma 5:

THEOREM 8. *The Cartesian product of irreducible dyadic (subdyadic) spaces is irreducible dyadic (subdyadic) space.*

Since the Cartesian product of an infinite family of topological spaces, each of which contains at least two points, is dense in itself, by Theorems 7 and 8 we have

COROLLARY 4. *The Cartesian product of an infinite family of metrizable compact spaces, each of which contains at least two points, is irreducible dyadic space.*

Similarly, by Corollary 3 and Theorem 8 we have

COROLLARY 5. *Every dense subset of the Cartesian product of an infinite family of metrizable spaces with countable base, each of which contains at least two points, is an irreducible subdyadic space.*

Let us remark that the Corollary of Lemma 2 in [10] (see also [8]) gives

THEOREM 9. *If f: $X \to Y$ is an irreducible mapping from a dyadic space X onto the space Y, then the weight of Y is equal to the weight of Y.*

From Theorem 9 it follows at once that the disjoint union of $D^{\mathfrak{m}}$ and $D^{\mathfrak{n}}$, where $\mathfrak{m}, \mathfrak{n} \geqslant \aleph_0$ and $\mathfrak{m} \neq \mathfrak{n}$, is reducible dyadic (see also Theorem 5 from [8]). Thus there exist reducible dyadic spaces without isolated points. There is, however, an open question raised by P. S. Alexandroff whether a dyadic space X which has the fixed character, $\mathfrak{m}_0 > \aleph_0$ at every point $x \in X$ is irreducible.

In the proof of Lemma 6 below we shall use the following criterion for pseudocompactness (see Lemma 9.13 in [14], p. 134):

In order that a space X be pseudocompact it is necessary and sufficient that for any decreasing sequence $U_1 \supset U_2 \supset \ldots$ of non-empty open subsets of X the intersection $\bigcap_{i=1}^{\infty} \overline{U}_i$ be non-empty.

LEMMA 6. *If $f\colon X \to Y$ is an irreducible mapping from a compact space X onto a space Y, then for every dense and pseudocompact subspace $M \subset Y$, the counter-image $f^{-1}(M)$ is pseudocompact and dense in X.*

Proof. From (8) it follows at once that $f^{-1}(M)$ is dense in X. To prove that $f^{-1}(M)$ is pseudocompact, consider a decreasing sequence $U_1 \supset U_2 \supset \ldots$ of non-empty open subsets of $f^{-1}(M)$. Denote by $V_1, V_2, \ldots$ open subsets of X satisfying

$$(12) \qquad V_1 \supset V_2 \supset \ldots \quad \text{and} \quad U_i = V_i \cap f^{-1}(M) \text{ for } i = 1, 2, \ldots$$

By (8) and the fact that M is pseudocompact and dense in Y there exists a point

$$(13) \qquad y \in M \cap \bigcap_{i=1}^{\infty} \overline{M \cap \operatorname{Int} f(V_i)} \neq 0.$$

For any open set $V \subset X$ which contains $f^{-1}(y)$ we have $y \notin f(X \setminus V) = \overline{f(X \setminus V)} \supset \overline{Y \setminus f(V)}$, i. e. $y \in \operatorname{Int} f(V)$, because the image $f(X \setminus V)$ of the compact subspace $X \setminus V$ of X is closed. Thus, by (13), $y \in \operatorname{Int} f(V) \cap \overline{\operatorname{Int} f(V_i)}$ and $\operatorname{Int} f(V) \cap \operatorname{Int} f(V_i) \neq 0$ for $i = 1, 2, \ldots$

From (9) it follows then that

$$0 \neq V \cap V_i \subset V \cap \overline{V}_i \quad \text{for} \quad i = 1, 2, \ldots$$

and, by the normality of X, that

$$(14) \qquad \overline{V}_i \cap f^{-1}(y) \neq 0 \quad \text{for} \quad i = 1, 2, \ldots$$

Since X is compact we have, by (12), (14) and the fact that $f^{-1}(M)$ is dense in X,

$$0 \neq f^{-1}(y) \cap \bigcap_{i=1}^{\infty} \overline{V}_i = f^{-1}(y) \cap \bigcap_{i=1}^{\infty} \overline{f^{-1}(M) \cap V_i} \subset f^{-1}(M) \cap \bigcap_{i=1}^{\infty} \overline{U}_i,$$

which implies the pseudocompactness of $f^{-1}(M)$.

THEOREM 10. *The Čech-Stone compactification of any irreducible subdyadic pseudocompact space is an irreducible dyadic space.*

Proof. Let X be a pseudocompact space which has an irreducible dyadic compactification cX and let $f\colon D^{\mathfrak{m}} \to cX$ be an irreducible mapping from $D^{\mathfrak{m}}$ onto cX. By Lemma 6 the counter-image $M = f^{-1}(X)$ is dense in $D^{\mathfrak{m}}$ and pseudocompact, whence by Theorem 5 we have $\beta M = D^{\mathfrak{m}}$.

Consider a mapping $g\colon \beta X \to cX$ which is the identity on X and an extension $\bar{f}'\colon D^{\mathfrak{m}} \to \beta X$ of the mapping $f' = f|M\colon M \to X$. Since $f' = g(\bar{f}'|M)$, we have $f = g\bar{f}'$. Hence for arbitrary closed subset $F \subset D^{\mathfrak{m}}$ such that $\bar{f}'(F) = \beta X$ we have

$$f(F) = g\bar{f}'(F) = g(\beta X) = cX$$

and, by the irreducibility of f, $F = D^{\mathfrak{m}}$. Thus $\bar{f}'\colon D^{\mathfrak{m}} \to \beta X$ is irreducible.

COROLLARY 6. *The continuous image of any irreducible subdyadic pseudocompact space under irreducible closed* (7) *mapping is pseudocompact and irreducible subdyadic.*

Proof. Let X be pseudocompact and irreducible subdyadic and let $f\colon X \to Y$ be an irreducible closed mapping from X onto Y. The extension $f'\colon \beta X \to \beta Y$ of f to βX is irreducible. Indeed, if $f'(F) = \beta Y$ and $F \neq \beta X$, then there would exist $x \in \beta X \setminus F$ and open sets U, $V \subset \beta X$ such that $F \subset U$, $x \in V$ and $U \cap V = 0$. Hence we would have

$$f(\overline{U} \cap X) = \overline{f(\overline{U} \cap X)} = Y \cap f'(\overline{\overline{U} \cap X}) \supset Y \cap f'(\overline{U \cap X}) \supset Y \cap f'(F) = Y,$$

which is impossible since $0 \neq V \cap X \subset X \setminus \overline{U}$ and f is irreducible. The Corollary follows now from the fact that the composition $f'g\colon D^{\mathfrak{m}} \to \beta Y$, where $g\colon D^{\mathfrak{m}} \to \beta X$ is an irreducible mapping which exists by Theorem 10, is irreducible.

THEOREM 11. *The Cartesian product of a number of irreducible subdyadic pseudocompact spaces is a irreducible subdyadic pseudocompact space*

Proof. Consider a family $\{X_s\}_{s \in S}$ of irreducible subdyadic pseudocompact spaces. Let, for every $s \in S$, cX_s be an irreducible dyadic compactification of X_s and let $f_s\colon D^{\mathfrak{m}_s} \to cX_s$ be an irreducible mapping from $D^{\mathfrak{m}_s}$ onto cX_s. By Theorem 8, it suffices to show that $\mathop{P}_{s \in S} X_s$ is pseudocompact.

By Lemma 6 the counter-image $M_s = f_s^{-1}(X_s)$ is pseudocompact and dense in $D^{\mathfrak{m}_s}$. It follows from Theorem 6 that the Cartesian product $\mathop{P}_{s \in S} M_s$, and hence also its continuous image $\mathop{P}_{s \in S} X_s$, is pseudocompact.

We do not know, whether (**P 497**) Theorems 10 and 11 are valid if the word „irreducible" is omitted in their formulation.

Let us remark also that in the proof of the theorem by Mardešić and Papić ([22] Theorem 14) asserting that a dyadic space Y is the continuous image of an ordered compact space if and only if Y is metriz-

(7) A mapping $f\colon X \to Y$ is called *closed* if the image of every closed subset of X is closed in Y.

able, the irreducible mappings can be used instead of light quasi-open mappings. Indeed, let $f\colon X \to Y$ be a mapping from an ordered compact space X onto Y; we can find, as in the proof of Lemma 2 of [22], a closed subspace $X_1 \subset X$ such that $f_1 = f|X_1\colon X_1 \to Y$ is irreducible. It follows from (8) and (9) on the basis of the well-known fact that every family of non-empty pairwise disjoint open sets in dyadic space is countable (see [21]), that the same is true in X_1. By a reasoning similar to that in the proof of Lemma 5 of [22] it appears that every open set in X_1 is F_σ-set. Hence every closed set in X_1 is G_δ-set and so is every point in Y. Metrizability of Y follows then from Theorem 4.

4. Dyadic subspaces of basically disconnected spaces. It is shown in [10] that there is no infinite extremally disconnected ([8]) dyadic space. As R. Sikorski has remarked (see [10], footnote ([8])), in a similar way one can prove that every basically disconnected dyadic space is finite. Since the basic disconnectedness is not hereditary with respect to closed subsets (see [14], Problem 6 W. 4, p. 100), the fact that every dyadic subspace of a basically disconnected space is finite requires a special proof. This proof is a modification of the proof of Theorem 4 from [10].

We begin with a lemma which follows also from some results in [14] (see Theorem 14.25 p. 208 and Problem 14 N. 4, p. 215).

LEMMA 7. *In a basically disconnected compact space Y the closure of every set $X = \bigcup_{i=1}^{\infty} V_i$, where V_i are closed-and-open in Y and disjoint, is homeomorphic to βX.*

Proof. It suffices to prove that every continuous function f from X into the closed interval $I = [0,1]$ can be extended over $\overline{X}$.

Consider two disjoint closed sets $A, B \subset I$. For every $i = 1, 2, \ldots$

$$V_i \cap f^{-1}(A) \quad \text{and} \quad V_i \cap f^{-1}(B)$$

are disjoint zero-sets in Y. Hence there exist (see footnote ([6])) closed-and-open sets $E_i \subset Y$ such that

$$V_i \cap f^{-1}(A) \subset E_i \quad \text{and} \quad V_i \cap f^{-1}(B) \subset X \setminus E_i \text{ for } i = 1, 2, \ldots$$

The sets $U = \overline{\bigcup_{i=1}^{\infty} V_i \cap E_i}$ and $W = \overline{\bigcup_{i=1}^{\infty} V_i \setminus E_i}$ are closed-and-open

([8]) A space X is called *extremally disconnected* if every open set has an open closure. X is *basically disconnected* if every cozero-set (i. e. a complement of a zero-set) has an open closure. Hence every extremally disconnected space is basically disconnected. It is easy to see that X is basically disconnected if and only if X is zero-dimensional and every set which is a countable sum of closed-and-open subsets of X has an open closure.

and disjoint. Since we have

$$f^{-1}(A) \subset U \quad \text{and} \quad f^{-1}(B) \subset W,$$

the sets $f^{-1}(A)$ and $f^{-1}(B)$ have disjoint closures in X, which implies, by a theorem from [29], that f can be extended over $\bar{X}$.

The following Lemma is a generalization of Theorem 4.1 from [13]; it can be also obtained by a modification of the proof of this theorem.

LEMMA 8. *Every infinite compact subspace X of a basically disconnected space Y can be continuously mapped onto the Čech-Stone compactification βN of the space N of positive integers.*

Proof. Since the Čech-Stone compactification of a basically disconnected space is basically disconnected (see [14], Problem 6 M. 1, p. 96), we can suppose that Y is compact. Because X is infinite and Y has a base composed from closed-and-open sets, there exist in Y closed-and-open sets $V_1, V_2, \ldots$ such that

$$X \cap V_i \neq 0 \quad \text{and} \quad V_i \cap V_j = 0 \text{ for } i \neq j. \tag{15}$$

By Lemma 7 the mapping $f\colon X_0 = \bigcup_{i=1}^{\infty} V_i \to \beta N$ defined by the condition

$$f(V_i) = i \quad \text{for} \quad i = 1, 2, \ldots$$

can be extended over $\bar{X}_0$. Since $\bar{X}_0$ is closed-and-open subset of Y, it can be also extended over the whole Y. From the first part of (15) it follows that $N \subset f(X)$, whence, X being compact, we have $f(X) = \beta N$.

COROLLARY 7 (Gillman and Jerison [14]). *Every infinite compact subspace of a basically disconnected space has the power greater than or equal to $2^{2^{\aleph_0}}$.*

In [14] this Corollary is deduced from the fact (Problem 9 H. 2, p. 137) that every infinite compact subspace X of a basically disconnected space contains a homeomorph of βN. To prove this let us remark that the set $M = \bigcup_{i=1}^{\infty} \{x_i\}$, where $x_i \in X \cap V_i$ (see (15)), is closed in the space X_0 (which is normal as an F_σ-set in the normal space Y) and homeomorphic to N, whence (see [14], p. 89)

$$\beta N = \beta M = \bar{M} \cap \beta X_0 = \bar{M} \subset X.$$

The following theorem follows from Lemma 8 and Theorem 3 of [10]:

THEOREM 12. *Every dyadic subspace of a basically disconnected space is finite.*

A. Pełczyński has remarked that Theorem 12 can be also proved by the „function space method" as follows. If X is a closed subset of a basically disconnected compact space Y, then there exists (see [14], p. 141) an epimorphism (of rings) $\varphi\colon C(Y)\to C(X)$. If X is infinite then X can be continuously mapped onto the one-point compactification of the discrete space of power $\aleph_0$, because X has a base composed of closed-and-open subsets. Hence if X is dyadic, there exists a linear mapping ψ from $C(X)$ onto its subspace isomorphic to the space c of all convergent sequences of real numbers (see Lemma 6 in [10]). Therefore $\psi\varphi$ is a linear mapping from $C(Y)$ onto c. But by an argument of Grothendieck (see p. 168 of [15]), there is no linear mapping from $C(Y)$ onto c for any basically disconnected compact space Y.

Theorem 12 follows also from the theorem of M. Katětov quoted in the footnote (2) of [10].

In the theory of Boolean Algebras (see § 31 of [16]) from this theorem follows that any projective Boolean Algebra which is a homomorphic image of σ-complete Boolean Algebra is finite.

5. A theorem on extending of mappings from closed subsets of $D^{\mathfrak{m}}$. We prove now

THEOREM 13. *For every mapping $f\colon A\to X$ from a non-empty closed subset $A\subset D^{\mathfrak{m}}$ into a metrizable separable space X there exists an extension $\bar{f}\colon D^{\mathfrak{m}}\to X$ such that $\bar{f}(D^{\mathfrak{m}})=f(A)$.*

Proof. We can suppose that $X=I^{\aleph_0}$ is the Hilbert cube; by Tietze-Urysohn extension theorem there exists an extension $f'\colon D^{\mathfrak{m}}=\mathop{P}\limits_{s\in S} D_s\to I^{\aleph_0}$ of f. From Lemma 3 (or Lemma 2 in [10]) it follows that there exists a countable subset $S_0\subset S$ such that $f'=f''p_{S_0}$ where $f''\colon D^{\aleph_0}=\mathop{P}\limits_{s\in S_0} D_s\to I^{\aleph_0}$. The set $p_{S_0}(A)$ is a closed subset of $\mathop{P}\limits_{s\in S_0} D_s$ and therefore there exists (see [20], vol. I, p. 169) a retraction (9) $r\colon \mathop{P}\limits_{s\in S_0} D_s\to p_{S_0}(A)$. The mapping $\bar{f}=f''rp_{S_0}\colon D^{\mathfrak{m}}\to I^{\aleph_0}$ satisfies the required conditions because we have

$$\bar{f}(a)=f''rp_{S_0}(a)=f''p_{S_0}(a)=f'(a)=f(a)\quad\text{for}\quad a\in A$$

and

$$\bar{f}(D^{\mathfrak{m}})=f''r\big(p_{S_0}(D^{\mathfrak{m}})\big)=f''r(\mathop{P}\limits_{s\in S_0} D_s)=f''\big(p_{S_0}(A)\big)=f'(A)=f(A).$$

COROLLARY 8. *For every compact metrizable subspace X of $D^{\mathfrak{m}}$ there exists a retraction of $D^{\mathfrak{m}}$ onto X.*

Let us recall that for every closed G_δ-set X in $D^{\mathfrak{m}}$ there exists also a retraction of $D^{\mathfrak{m}}$ onto X (see [10], p. 57).

(9) By a *retraction* we mean a mapping $r\colon X\to A$ of the space X onto its subspace A such that $r(a)=a$ for $a\in A$.

The last Corollary is a special case of a general theorem which follows easily from Theorem 17 of [16] and which asserts that if a space X is a retract of $D^{\mathfrak{n}}$ for some $\mathfrak{n}$, then it is also a retract of every containing it $\mathfrak{m}$-Cantor set $D^{\mathfrak{m}}$.

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