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CERTAIN TYPES OF AFFINE MOTION IN A FINSLER MANIFOLD. III

BY

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1. Introduction. In a series of papers [7]–[9] the author discussed the necessary and sufficient conditions for contra, concurrent, special concircular, recurrent, concircular, torse forming and birecurrent vector fields to generate an affine motion in a Finsler manifold. The present paper, which is the last one of the above series, deals with the same problem for a vector field $v^i(x^j)$ whose Berwald's covariant derivative $\mathscr{B}_k v^i$ is recurrent, i.e, $\mathscr{B}_j \mathscr{B}_k v^i = u_j \mathscr{B}_k v^i$, u_j being a non-zero covariant vector field. This paper also presents an elegant generalization of theorems due to the author [4], Kumar [1], Misra and Meher [2] and Takano [11].

2. Preliminaries. Let $F_n(F, g, G)$ be an n-dimensional Finsler manifold of class at least C^7 equipped with a metric function F(1) satisfying the required conditions [10], the corresponding symmetric metric tensor g and the Berwald's connection G. The coefficients of Berwald's connection G, denoted by G^i_{jk}, satisfy

$$\text{(2.1)} \qquad \text{(a) } G^i_{jk} = G^i_{kj}, \quad \text{(b) } G^i_{jk}\dot{x}^k = G^i_j, \quad \text{(c) } \dot{\partial}_k G^i_j = G^i_{jk},$$

where $\dot{\partial}_k$ means the partial derivative with respect to $\dot{x}^k$. The partial derivatives $\dot{\partial}_h G^i_{jk}$ of the connection coefficients G^i_{jk} constitute a tensor whose components are denoted by G^i_{jkh}. This tensor is symmetric in its lower indices and satisfies

$$\text{(2.2)} \qquad G^i_{jkh}\dot{x}^h = G^i_{khj}\dot{x}^h = G^i_{hjk}\dot{x}^h = 0.$$

The covariant derivative $\mathscr{B}_k T^i_j$ of an arbitrary tensor T^i_j for the connection G is given by

$$\text{(2.3)} \qquad \mathscr{B}_k T^i_j = \partial_k T^i_j - (\dot{\partial}_r T^i_j) G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk},$$

(1) Unless otherwise stated, all the geometric objects used in the paper are supposed to be functions of line elements $(x^i, \dot{x}^i)$. The indices $i, j, k, \ldots$ take positive integral values from 1 to n.

where $\partial_k \equiv \partial/\partial x^k$. The operator $\mathcal{B}_k$ commutes with the operator $\dot{\partial}_k$ and itself according to

$$(2.4) \qquad \dot{\partial}_j \mathcal{B}_k T^i_h - \mathcal{B}_k \dot{\partial}_j T^i_h = T^r_h G^i_{jkr} - T^i_r G^r_{jkh},$$

$$(2.5) \qquad \mathcal{B}_j \mathcal{B}_k T^i_h - \mathcal{B}_k \mathcal{B}_j T^i_h = T^r_h H^i_{jkr} - T^i_r H^r_{jkh} - (\dot{\partial}_r T^i_h) H^r_{jk},$$

where H^i_{jkh} constitute Berwald's curvature tensor. This tensor is skew-symmetric in first two lower indices, and positively homogeneous of degree zero in $\dot{x}^i$. It should be noted that H^i_{jkh} coincides with H^i_{hkj} of Rund [10]. The tensor H^i_{jk} appearing in (2.5) is connected with the curvature tensor by

$$(2.6) \qquad \text{(a)}\ H^i_{jkh}\dot{x}^h = H^i_{jk}, \qquad \text{(b)}\ \dot{\partial}_h H^i_{jk} = H^i_{jkh}.$$

This tensor is related with the deviation tensor H^i_j by

$$(2.7) \qquad \text{(a)}\ H^i_{jk}\dot{x}^k = H^i_j, \qquad \text{(b)}\ \tfrac{1}{3}(\dot{\partial}_k H^i_j - \dot{\partial}_j H^i_k) = H^i_{jk}.$$

The associate vector y_i of $\dot{x}^i$ satisfies the relations (see [6] and [10])

$$(2.8) \qquad \text{(a)}\ y_i \dot{x}^i = F^2, \qquad \text{(b)}\ y_i H^i_{jk} = 0, \qquad \text{(c)}\ g_{ik} H^i_{mj} + y_i H^i_{mjk} = 0,$$

where g_{ij} are components of the metric tensor g.

Let us consider the infinitesimal transformation

$$(2.9) \qquad \bar{x}^i = x^i + \varepsilon v^i(x^j)$$

generated by a vector $v^i(x^j)$, ε being an infinitesimal constant. The Lie derivatives of an arbitrary tensor T^i_j and the connection coefficients G^i_{jk} with respect to (2.9) are given by (see [12])

$$(2.10) \qquad \pounds T^i_j = v^r \mathcal{B}_r T^i_j - T^r_j \mathcal{B}_r v^i + T^i_r \mathcal{B}_j v^r - (\dot{\partial}_r T^i_j)\mathcal{B}_s v^r \dot{x}^s,$$

$$(2.11) \qquad \pounds G^i_{jk} = \mathcal{B}_j \mathcal{B}_k v^i + H^i_{mjk} v^m + G^i_{jkr} \mathcal{B}_s v^r \dot{x}^s.$$

The operator $\pounds$ commutes with the operators $\mathcal{B}_k$ and $\dot{\partial}_k$ according to

$$(2.12) \qquad (\pounds \mathcal{B}_k - \mathcal{B}_k \pounds) T^i_j = T^r_j \pounds G^i_{rk} - T^i_r \pounds G^r_{jk} - (\dot{\partial}_r T^i_j) \pounds G^r_k,$$

$$(2.13) \qquad (\dot{\partial}_k \pounds - \pounds \dot{\partial}_k)\Omega = 0,$$

where Ω is a vector, tensor or connection coefficient. The necessary and sufficient condition for the vector $v^i(x^j)$ to generate an affine motion is given by (see [12])

$$(2.14) \qquad \pounds G^i_{jk} = 0.$$

3. Affine motion in a Finsler manifold.

THEOREM 3.1. *A vector field $v^i(x^j)$ which satisfies any two of the following conditions must satisfy the third one:*

$$(A) \qquad v^m H^i_{mjk} = w_j \mathcal{B}_k v^i,$$

(B) $$\mathscr{B}_j \mathscr{B}_k v^i = u_j \mathscr{B}_k v^i,$$

(C) $$\text{(i) } \pounds G^i_{jk} = 0, \quad \text{(ii) } G^i_{jkr} \mathscr{B}_s v^r \dot{x}^s = 0,$$

where u_j and w_j are non-zero covariant vector fields.

Proof. Let us consider a vector field $v^i(x^j)$ which satisfies (A) and (B). The Lie derivative of G^i_{jk} with respect to an infinitesimal transformation generated by the vector field $v^i(x^j)$ is given by (2.11), which, in view of (A) and (B), may be written as

$$\pounds G^i_{jk} = (u_j + w_j) \mathscr{B}_k v^i + G^i_{jkr} \mathscr{B}_s v^r \dot{x}^s. \tag{3.1}$$

Since G^i_{jk} and G^i_{jkr} are symmetric with respect to the indices j and k in (3.1), $(u_j + w_j) \mathscr{B}_k v^i$ must be symmetric, i.e.,

$$(u_j + w_j) \mathscr{B}_k v^i = (u_k + w_k) \mathscr{B}_j v^i. \tag{3.2}$$

This equation implies at least one of the following:

$$\text{(a) } \mathscr{B}_k v^i = 0, \quad \text{(b) } u_j + w_j = 0, \quad \text{(c) } \mathscr{B}_k v^i = (u_k + w_k) X^i \tag{3.3}$$

for some non-zero vector field X^i. If (3.3a) holds, equation (3.1) reduces to $\pounds G^i_{jk} = 0$. Also, in this case, $G^i_{jkr} \mathscr{B}_s v^r \dot{x}^s = 0$ holds identically. Thus, condition (C) holds for the vector $v^i(x^j)$. If (3.3b) holds, equation (3.1) reduces to

$$\pounds G^i_{jk} = G^i_{jkr} \mathscr{B}_s v^r \dot{x}^s. \tag{3.4}$$

Transvecting (3.4) by $\dot{x}^k$, and using (2.1b) and (2.2), we have

$$\pounds G^i_j = 0. \tag{3.5}$$

Differentiating (3.5) partially with respect to $\dot{x}^k$, utilizing the commutation formula (2.13) and using equation (2.1c), we have $\pounds G^i_{jk} = 0$, and hence (3.4) gives $G^i_{jkr} \mathscr{B}_s v^r \dot{x}^s = 0$. Thus, condition (C) holds. In case of (3.3c), condition (A) becomes

$$v^m H^i_{mjk} = w_j (u_k + w_k) X^i. \tag{3.6}$$

Transvecting (3.6) by $\dot{x}^k$ and using (2.6a), we have

$$v^m H^i_{mj} = w_j (u_k + w_k) \dot{x}^k X^i. \tag{3.7}$$

Transvecting (3.7) by y_i and using (2.8b), we have at least one of the following:

$$\text{(a) } (u_k + w_k) \dot{x}^k = 0, \quad \text{(b) } y_i X^i = 0, \tag{3.8}$$

since $w_j \neq 0$. In case of (3.8a), equation (3.7) gives $v^m H^i_{mj} = 0$, which after partial differentiation with respect to $\dot{x}^k$ implies $v^m H^i_{mjk} = 0$. Also from (3.8b) and (3.6) we have $v^m y_i H^i_{mjk} = 0$. Transvecting (2.8c) by v^m and using $v^m y_i H^i_{mjk} = 0$, we have $g_{ik} H^i_{mj} v^m = 0$. Transvecting $g_{ik} H^i_{mj} v^m = 0$ by g^{kl} and using $g^{kl} g_{ik} = \delta^l_i$, we get $H^l_{mj} v^m = 0$, which implies $v^m H^i_{mjk} = 0$. Thus, both the conditions given by (3.8a) and (3.8b) imply $v^m H^i_{mjk} = 0$ separately. In view of this equation and $w_j \neq 0$, condition (A) gives $\mathscr{B}_k v^i = 0$, which is nothing but (3.3a), a condition already discussed. Thus, we conclude that each of the conditions given by (3.3) implies (C). Therefore, (A) and (B) imply (C).

Let us consider a vector field $v^i(x^j)$ which satisfies (A) and (C). In this case, equation (2.11) may be written as

$$\mathscr{B}_j \mathscr{B}_k v^i = u_j \mathscr{B}_k v^i,$$

where we have put $-w_j = u_j$. Thus, (B) holds for the vector field $v^i(x^j)$.

Again, if a vector field $v^i(x^j)$ satisfies (B) and (C), equation (2.11) gives (A), where $w_j = -u_j$. This completes the proof.

Let us consider a vector field $v^i(x^j)$ satisfying condition (B). Since any contra vector field $v^i(x^j)$ satisfies (B) trivially, we shall exclude this case from our discussion. Differentiating (B) partially with respect to $\dot{x}^h$ and utilizing the commutation formula exhibited by (2.4), we have

$$\mathscr{B}_j (G^i_{hkr} v^r) + G^i_{jhr} \mathscr{B}_k v^r - G^r_{jhk} \mathscr{B}_r v^i = (\dot{\partial}_h u_j) \mathscr{B}_k v^i + u_j G^i_{hkr} v^r. \tag{3.9}$$

Transvecting (3.9) by $\dot{x}^k$ and using (2.2), we get

$$G^i_{jhr} \dot{x}^k \mathscr{B}_k v^r = (\dot{\partial}_h u_j) \dot{x}^k \mathscr{B}_k v^i. \tag{3.10}$$

If condition (Cii) holds, (3.10) gives at least one of the following:

$$\text{(a)}\ \dot{\partial}_h u_j = 0, \qquad \text{(b)}\ \dot{x}^k \mathscr{B}_k v^i = 0. \tag{3.11}$$

If (3.11b) holds, its partial differentiation with respect to $\dot{x}^h$ and the use of (2.2) imply $\mathscr{B}_h v^i = 0$, a trivial case. Hence, for a non-trivial case, (3.11a) must hold, i.e., the covariant vector field u_k is independent of $\dot{x}^i$. Conversely, if the vector field u_k is independent of $\dot{x}^i$, equation (3.10) gives (Cii). Thus, we conclude

THEOREM 3.2. *If a vector field $v^i(x^j)$ satisfies condition* (B), *then the necessary and sufficient condition for the covariant vector field u_k to be independent of $\dot{x}^i$ is given by* (Cii).

Let us consider a veetor field $v^i(x^j)$ which satisfies (B) and the vector field u_k which is independent of $\dot{x}^i$. In view of Theorems 3.1 and 3.2, we may conclude that conditions (A) and (Ci) are equivalent. Since (Ci) is the necessary and sufficient condition for the vector field $v^i(x^j)$ to generate an affine motion, we have

THEOREM 3.3. *Condition* (A) *is necessary and sufficient for a vector field $v^i(x^j)$ satisfying condition* (B) *where u_k is independent of $\dot{x}^i$ to generate an affine motion.*

4. Affine motion in special Finsler manifolds. Takano [11] considered a non-Riemannian manifold of recurrent curvature and proved that a vector field v^i generates an affine motion in a non-Riemannian manifold of recurrent curvature, characterized by

$$\nabla_m B^i_{jkh} = \lambda_m B^i_{jkh}, \tag{4.1}$$

if

$$£\lambda_m = 0, \tag{4.2}$$

$$LB^i_{jkh} = A_{kh} \nabla_j v^i, \tag{4.3}$$

$$L = \lambda_m v^m \neq \text{const}, \tag{4.4}$$

$$A_{kl} = 2\nabla_{[l}\lambda_{k]} \quad (2\Omega_{[ij]} = \Omega_{ij} - \Omega_{ji}), \tag{4.5}$$

where ∇_m, B^i_{jkh} and λ_m are the operator for covariant differentiation, the curvature tensor and the recurrence vector, respectively.

Kumar [1], Misra and Meher [2] extended this theorem to Finsler manifolds of recurrent curvature and proved that a vector field v^i generates an affine motion in a recurrent Finsler manifold, characterized by

$$\mathscr{B}_m H^i_{jkh} = \lambda_m H^i_{jkh}, \tag{4.6}$$

if

$$LH^i_{jkh} = A_{jk} \mathscr{B}_h v^i, \tag{4.7}$$

$$£\lambda_m = 0, \tag{4.8}$$

$$L = \lambda_m v^m \neq \text{const}, \tag{4.9}$$

$$A_{jk} = 2\mathscr{B}_{[j}\lambda_{k]}, \tag{4.10}$$

$$G^i_{jkh} = 0. \tag{4.11}$$

The author [4] generalized this result by relaxing the condition (4.11). Now, we propose an elegant generalization of this theorem in the following form:

THEOREM 4.1. *A vector field $v^i(x^j)$ generates an affine motion in a recurrent Finsler manifold if $\alpha H^i_{jkh} = a_{jk} \mathscr{B}_h v^i$, α and a_{jk} being any non-zero scalar and tensor fields, respectively.*

Proof. Let us consider a vector field $v^i(x^j)$ satisfying $\alpha H^i_{jkh} = a_{jk} \mathscr{B}_h v^i$ in a recurrent Finsler manifold characterized by (4.6). Dividing the equation $\alpha H^i_{jkh} = a_{jk} \mathscr{B}_h v^i$ by α and putting $a_{jk}/\alpha = \bar{a}_{jk}$, we have

$$H^i_{jkh} = \bar{a}_{jk} \mathscr{B}_h v^i. \tag{4.12}$$

Differentiating (4.12) covariantly with respect to x^m and using (4.6), we have

$$(\lambda_m \bar{a}_{jk} - \mathscr{B}_m \bar{a}_{jk}) \mathscr{B}_h v^i = \bar{a}_{jk} \mathscr{B}_m \mathscr{B}_h v^i. \tag{4.13}$$

Since the tensor field $\bar{a}_{jk}$ is non-vanishing, we may choose a tensor field f^{jk} such that $\bar{a}_{jk} f^{jk} = 1$. Transvecting (4.13) by f^{jk}, we have condition (B), where

$$u_m = (\lambda_m \bar{a}_{jk} - \mathscr{B}_m \bar{a}_{jk}) f^{jk}.$$

Transvecting (4.12) by v^j, we get condition (A), where $w_k = \bar{a}_{jk} v^j$. In view of (2.11) and conditions (A) and (B), the Lie derivative of G^i_{jk} with respect to an infinitesimal transformation generated by the vector field $v^i(x^j)$ is given by

$$\text{(4.14)} \qquad £G^i_{jk} = (u_j + w_j) \mathscr{B}_k v^i + G^i_{jkr} \mathscr{B}_s v^r \dot{x}^s.$$

Since G^i_{jk} and G^i_{jkr} are symmetric in the indices j and k, equation (4.14) shows the symmetry of $(u_j + w_j) \mathscr{B}_k v^i$ in j and k. Hence

$$\text{(4.15)} \qquad (u_j + w_j) \mathscr{B}_k v^i = (u_k + w_k) \mathscr{B}_j v^i,$$

which implies at least one of the following:

$$\text{(4.16)} \qquad \text{(a) } \mathscr{B}_k v^i = 0, \quad \text{(b) } u_j + w_j = 0, \quad \text{(c) } \mathscr{B}_k v^i = (u_k + w_k) X^i,$$

where X^i is some non-zero vector field. (4.16a) cannot hold as it, in view of (4.12), leads to $H^i_{jkh} = 0$, a contradiction to the fact that the curvature tensor H^i_{jkh} of a recurrent Finsler manifold is non-vanishing. If (4.16c) holds, (4.12) may be written as

$$H^i_{jkh} = f_{jkh} v^i, \qquad \text{where } f_{jkh} = \bar{a}_{jk}(u_h + w_h).$$

Thus, the curvature tensor H^i_{jkh} is written as the product of a tensor and a vector. This contradicts the Theorem of [6] which states that the curvature tensor of a non-flat Finsler manifold cannot be written as the product of a tensor and a vector. Therefore, (4.16c) does not hold. Hence we have (4.16b). This reduces (4.14) to

$$\text{(4.17)} \qquad £G^i_{jk} = G^i_{jkr} \mathscr{B}_s v^r \dot{x}^s.$$

Transvecting (4.17) by $\dot{x}^k$ and using (2.1b) and (2.2), we have $£G^i_j = 0$. Differentiating $£G^i_j = 0$ partially with respect to $\dot{x}^k$ and using the commutation formula (2.13), we get $£\dot{\partial}_k G^i_j = 0$, which, in view of (2.1c), gives $£G^i_{jk} = 0$. Thus, the vector field $v^i(x^j)$ generates an affine motion.

Proceeding in a similar way, we may prove

THEOREM 4.2. *A vector field $v^i(x^j)$ generates an affine motion in a symmetric Finsler manifold, characterized by $\mathscr{B}_m H^i_{jkh} = 0$, if $\alpha H^i_{jkh} = a_{jk} \mathscr{B}_h v^i$, where α and a_{jk} are non-zero scalar and tensor fields, respectively.*

Let us consider a vector field $v^i(x^j)$ satisfying $\alpha H^i_{jkh} = a_{jk} \mathscr{B}_h v^i$ in a recurrent Finsler manifold. According to Theorem 4.1, $v^i(x^j)$ generates an affine motion. Since every affine motion is a curvature collineation, we have

$\pounds H^i_{jkh} = 0$, which, in view of (2.10) and (4.6), can be written as

(4.18)
$$LH^i_{jkh} - H^r_{jkh}\mathscr{B}_r v^i + H^i_{rkh}\mathscr{B}_j v^r + H^i_{jrh}\mathscr{B}_k v^r + H^i_{jkr}\mathscr{B}_h v^r + (\dot{\partial}_r H^i_{jkh})\mathscr{B}_s v^r \dot{x}^s = 0,$$

where $L = \lambda_r v^r$. Transvecting (4.18) by a_{lm} and using $\alpha H^i_{jkh} = a_{jk}\mathscr{B}_h v^i$, we have

(4.19)
$$La_{lm} H^i_{jkh} - \alpha\{H^r_{jkh} H^i_{lmr} - H^i_{rkh} H^r_{lmj} - H^i_{jrh} H^r_{lmk} - H^i_{jkr} H^r_{lmh} - (\dot{\partial}_r H^i_{jkh}) H^r_{lm}\} = 0.$$

Differentiating (4.6) covariantly with respect to x^l and taking skew-symmetric part with respect to the indices l and m, we get

$$2\mathscr{B}_{[l}\mathscr{B}_{m]} H^i_{jkh} = 2(\mathscr{B}_{[l}\lambda_{m]}) H^i_{jkh},$$

which, in view of the commutation formula exhibited by (2.5) and (4.10), gives

(4.20) $$H^r_{jkh} H^i_{lmr} - H^i_{rkh} H^r_{lmj} - H^i_{jrh} H^r_{lmk} - H^i_{jkr} H^r_{lmh} - (\dot{\partial}_r H^i_{jkh}) H^r_{lm} = A_{lm} H^i_{jkh}.$$

From (4.19) and (4.20) we get

(4.21) $$La_{lm} = \alpha A_{lm},$$

since $H^i_{jkh} \neq 0$. Since a_{lm} and α are non-zero tensor and scalar fields, respectively, it is obvious from (4.21) that the vanishing of L implies and is implied by the vanishing of the tensor A_{lm}. Thus, we have

THEOREM 4.3. *If a recurrent Finsler manifold admits a vector field $v^i(x^j)$ satisfying $\alpha H^i_{jkh} = a_{jk}\mathscr{B}_h v^i$, then the vector field v^i is orthogonal to the recurrence vector λ_m if and only if the tensor field A_{lm} vanishes identically.*

Let us consider a vector field $v^i(x^j)$ satisfying $\alpha H^i_{jkh} = a_{jk}\mathscr{B}_h v^i$. This vector field, in view of Theorem 4.1, generates an affine motion. Since the recurrence vector is a Lie invariant under an affine motion, we have $\pounds\lambda_m = 0$, which, in view of (2.10) and (4.10), may be written as

(4.22) $$v^r A_{rm} + \mathscr{B}_m L = 0,$$

the recurrence vector λ_m being independent of $\dot{x}^i$ (see [5]). From (4.22) it is clear that the covariant derivative of the scalar L vanishes if and only if $v^r A_{rm} = 0$. We claim that $v^r A_{rm} = 0$ if and only if $A_{rm} = 0$. Suppose $v^r A_{rm} = 0$. If $A_{rm} \neq 0$, then the tensor A_{jk} satisfies (see [3])

$$\lambda_m A_{jk} + \lambda_j A_{km} + \lambda_k A_{mj} = 0,$$

which after transvection by v^m and then summing for m, takes the form

(4.23) $$LA_{jk} + \lambda_j A_{km} v^m + \lambda_k A_{mj} v^m = 0.$$

In view of $v^r A_{rm} = 0$ and skew-symmetry of A_{jk}, (4.23) gives $LA_{jk} = 0$, whence $L = 0$, which contradicts Theorem 4.3. Thus, $v^r A_{rm} = 0$ implies $A_{jk} = 0$. Conversely, if $A_{jk} = 0$, we have $v^r A_{rm} = 0$ identically. Hence the conditions $A_{jk} = 0$ and $v^r A_{rm} = 0$ are equivalent. Thus we may conclude

THEOREM 4.4. *For a vector field $v^i(x^j)$ satisfying $\alpha H^i_{jkh} = a_{jk} \mathscr{B}_h v^i$ in a recurrent Finsler manifold, the following conditions are equivalent*:

$$\text{(i) } \mathscr{B}_m L = 0, \quad \text{(ii) } v^r A_{rm} = 0, \quad \text{(iii) } A_{jk} = 0.$$

If $L \neq 0$, we have $A_{jk} \neq 0$ and both satisfy (4.21). Putting the value of a_{lm} from (4.21) in $\alpha H^i_{jkh} = a_{jk} \mathscr{B}_h v^i$ and bearing in mind that $\alpha \neq 0$, we have (4.7). Hence, in this case, Theorem 4.1 takes the form

THEOREM 4.5. *If a vector field v^i which is not orthogonal to the recurrence vector ($L \neq 0$) satisfies $LH^i_{jkh} = A_{jk} \mathscr{B}_h v^i$, then it generates an affine motion in the recurrent Finsler manifold.*

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