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APPROXIMATELY ADDITIVE SET FUNCTIONS

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The following question was communicated by F. Galvin during his visit to Wrocław in the spring of 1981.

Let n be a positive integer and A a set of cardinality n. Define $K(n)$ as the least number with the following property: given any approximately additive set function $f\colon P(A) \to R$, i.e., $f(\emptyset) = 0$ and

$$|f(B)+f(C)-f(B \cup C)| \leqslant 1$$

whenever B and C are disjont, there exists a signed measure μ such that

$$|\mu(B)-f(B)| \leqslant K(n) \quad (B \subseteq A).$$

Obviously, the sequence $\{K(n)\}_{n=1}^{\infty}$ is increasing.

What is the value of

$$K = \lim_{n\to\infty} K(n)?$$

The number K is called the *Kalton constant.*

The above question is important because of its connection with the Maharam problem. The first step in this direction was the author's result (proved below and also quoted in the paper of Kalton and Roberts([1]). Later, Kalton and Roberts showed that the Kalton constant is finite and $K \leqslant 45$ (op. cit.).

Let $A = X \cup Y$, where X and Y are disjoint sets of k elements. If X' and Y' denote nonempty proper subsets of X and Y, respectively, then we put

$$\begin{array}{lll} f(\emptyset \cup \emptyset) = 0, & f(Y' \cup \emptyset) = -1, & f(Y \cup \emptyset) = -3, \\ f(\emptyset \cup X') = 1, & f(Y' \cup Y') = 0, & f(Y \cup X') = -1, \\ f(\emptyset \cup X) = 3, & f(Y' \cup X) = 1, & f(Y \cup X) = 0. \end{array}$$

([1]) N. J. Kalton and J. W. Roberts, *Uniformly exhaustive submeasures and nearly additive set functions* (to appear).

It is easy to check that

$$|f(B)+f(C)-f(B\cup C)| \leqslant 1$$

for all disjoint B and C contained in A.

Let μ be a measure such that

$$\max_{B\subseteq A}|f(B)-\mu(B)| \leqslant \max_{B\subseteq A}|f(B)-\nu(B)|$$

for any measure ν. Now we choose a pair of points $x\in X$ and $y\in Y$ for which the value of $|\mu(\{x\})|+|\mu(\{y\})|$ is minimal and we put $C=(X-\{x\})\cup\{y\}$. Since $f(X)=3$ and $f(C)=0$, we have

$$K(2k) \geqslant (3-|\mu(\{x\})|-|\mu(\{y\})|)/2,$$

whence the Kalton constant is not smaller than 3/2.

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