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TRANSFINITE NILPOTENCE DEFINES A TRIVIAL RADICAL

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For a ring R (associative) the (*right*) *transfinite powers* R^α are defined as follows:

$$R^1 = R, \qquad R^{\alpha+1} = R^\alpha R \qquad \text{and} \qquad R^\beta = \bigcap_{\alpha<\beta} R^\alpha \qquad \text{if } \beta \text{ is a limit ordinal;}$$

R is said to be (*right*) *transfinitely nilpotent* if $R^\mu = 0$ for some μ. This property was first discussed by Baer [1]. Let $\mathscr{M}$ denote the class of all transfinitely nilpotent rings, and $\mathscr{M}_\omega$ the class of rings R with $R^\omega = 0$.

In a recent paper, Szász [2] asks whether $L(\mathscr{M})$, the lower radica class defined by $\mathscr{M}$, is special, and attributes an upper bound to this class (Theorem 38). The latter result is invalid and, in fact, we have

$$L(\mathscr{M}_\omega) = L(\mathscr{M}) = \textit{the class of all rings}.$$

Proof. Let F be a free ring, $0 \neq x = n_1 w_1 + \ldots + n_k w_k \in F$ and let m be the maximum of the lengths of the words $w_1, \ldots, w_k$. Then $x \notin F^{m+1}$. Thus $F \in \mathscr{M}_\omega$ and the homomorphic closure of $\mathscr{M}_\omega$ is the class of all rings.

Note. There is also a misprint in Definition 35 of [2]: $\sum$ should be replaced by $\bigcap$ in the definition of R^β at limit ordinals.

REFERENCES

[1] R. Baer, *Radical ideals*, American Journal of Mathematics 65 (1943), p. 537-568.

[2] F. A. Szász, *On some weakly supernilpotent radicals of rings*, Colloquium Mathematicum 28 (1973), p. 195-201.

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