

ADDENDUM TO
 "ČECH AND STEENROD HOMOTOPY THEORY
 WITH APPLICATIONS TO GEOMETRIC TOPOLOGY"

BY

HAROLD M. HASTINGS (HEMPSTEAD, NEW YORK)

The proof of Lemma (3.3.32) of Edwards and the author's paper [2] is based on the earlier Lemma (3.3.31) in [2] which is incorrect. In this note* we give a new proof of Lemma (3.3.32) in [2], which allows us to remove one of the conditions in [2], i.e.

CONDITION N3. *Every object in C is cofibrant or every object is fibrant, for pro- C to be a closed model category.*

Thus, the new proof shows that pro-maps (C) are a closed model category for C being simplicial sets, topological spaces, simplicial groups, etc.

PROPOSITION (Lemma (3.3.32), [2]). *Let $f: X \rightarrow Y$ be a trivial fibration and let $g: Y \rightarrow Z$ be a trivial cofibration. Then the composite $g \circ f$ is a weak equivalence.*

Proof. By Proposition (3.3.36) in [2] there is a levelwise trivial cofibration $g': Y' \rightarrow Z'$ (in some C^J) isomorphic to g (in maps pro- C). By Propositions (3.3.15) and (3.3.26) in [2] (Quillen's [4] Axiom M6 for pro- C and Axiom M6 for fibrations in pro- C), the composite mapping

$$f': X \xrightarrow{f'} Y \xrightarrow{\cong} Y'$$

is a trivial fibration. Reindex f' ([2], Section 2.1, see also Appendix in [1], and [3]) to obtain a level map $f'': X'' \rightarrow Y''$ in some C^K , where K is a cofinite, strongly directed set. Extend this new indexing to obtain a sequence

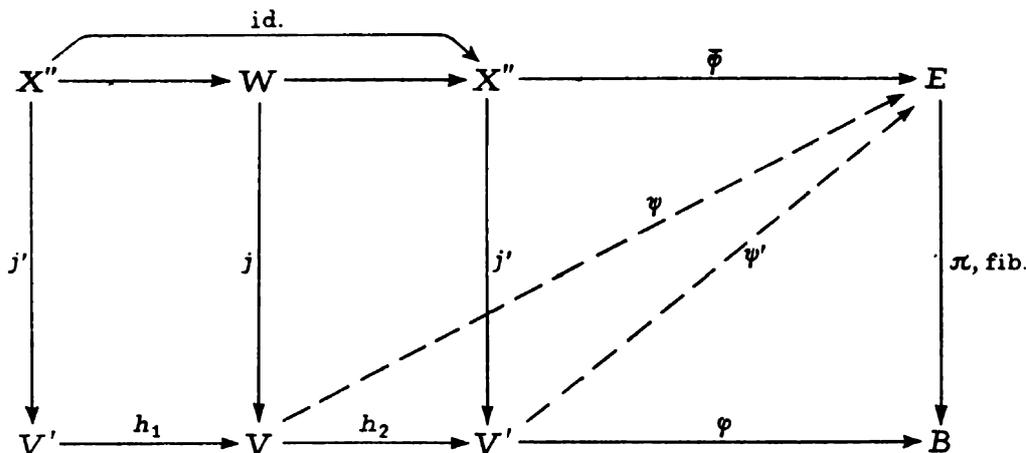
$$X'' \xrightarrow{f''} Y'' \xrightarrow{g''} Z''$$

in C^K . Note that f'' is a trivial fibration in pro- C and g'' is a levelwise trivial cofibration in C^K . By Proposition (3.2.24) in [2] (Quillen's [4] Axiom M2 for C^K), f factors as

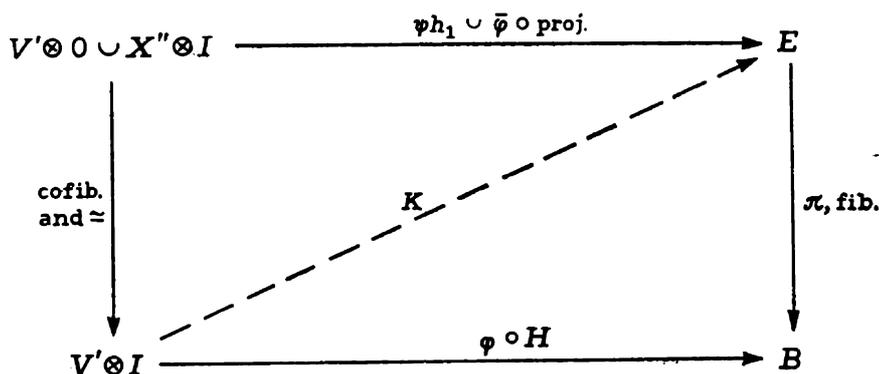
$$X'' \xrightarrow{i} W \xrightarrow{p} Y''$$

* Partially supported by National Science Foundation, U.S.A., grant MCS 77-01628.

For the second claim, consider a commutative solid-arrow diagram



A filler ψ exists by Proposition (3.3.26) in [2]. Then ψh_1 extends $\bar{\varphi}$ and covers φ . Use H to deform ψh_1 into the required filler ψ' as follows: ψ' is the "1-end" of the filler K in the diagram



This completes the proof.

REFERENCES

- [1] M. Artin and B. Mazur, *Etale homotopy theory*, Lecture Notes in Mathematics 100 (1969).
- [2] D. A. Edwards and H. M. Hastings, *Čech and Steenrod homotopy theory with applications to geometric topology*, ibidem 542 (1976).
- [3] S. Mardešić, *On the Whitehead theorem in shape theory*, Fundamenta Mathematicae 91 (1976), p. 51-64.
- [4] D. G. Quillen, *Homotopical algebra*, Lecture Notes in Mathematics 43 (1967).

DEPARTMENT OF MATHEMATICS
 HOFSTRA UNIVERSITY
 HEMPSTAD, NEW YORK

Reçu par la Rédaction le 10. 10. 1977