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ADDENDUM TO "ČECH AND STEENROD HOMOTOPY THEORY WITH APPLICATIONS TO GEOMETRIC TOPOLOGY"

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The proof of Lemma (3.3.32) of Edwards and the author's paper [2] is based on the earlier Lemma (3.3.31) in [2] which is incorrect. In this note* we give a new proof of Lemma (3.3.32) in [2], which allows us to remove one of the conditions in [2], i.e.

CONDITION N3. *Every object in C is cofibrant or every object is fibrant,* for pro-C to be a closed model category.

Thus, the new proof shows that pro-maps (C) are a closed model category for C being simplicial sets, topological spaces, simplicial groups, etc.

PROPOSITION (Lemma (3.3.32), [2]). *Let $f\colon X \to Y$ be a trivial fibration and let $g\colon Y \to Z$ be a trivial cofibration. Then the composite $g \circ f$ is a weak equivalence.*

Proof. By Proposition (3.3.36) in [2] there is a levelwise trivial cofibration $g'\colon Y' \to Z'$ (in some C^J) isomorphic to g (in maps pro-C)). By Propositions (3.3.15) and (3.3.26) in [2] (Quillen's [4] Axiom M6 for pro-C and Axiom M6 for fibrations in pro-C), the composite mapping

$$f'\colon X \xrightarrow{f} Y \xrightarrow{\cong} Y'$$

is a trivial fibration. Reindex f' ([2], Section 2.1, see also Appendix in [1], and [3]) to obtain a level map $f''\colon X'' \to Y''$ in some C^K, where K is a cofinite, strongly directed set. Extend this new indexing to obtain a sequence

$$X'' \xrightarrow{f''} Y'' \xrightarrow{g''} Z''$$

in C^K. Note that f'' is a trivial fibration in pro-C and g'' is a levelwise trivial cofibration in C^K. By Proposition (3.2.24) in [2] (Quillen's [4] Axiom M2 for C^K), f factors as

$$X'' \xrightarrow{i} W \xrightarrow{p} Y''$$

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For the second claim, consider a commutative solid-arrow diagram

$$\begin{tikzcd}
X'' \arrow[r] \arrow[d, "j'"'] \arrow[rr, bend left, "\mathrm{id.}"] & W \arrow[r] \arrow[d, "j"] & X'' \arrow[r, "\bar{\varphi}"] \arrow[d, "j'"] & E \arrow[d, "\pi,\ \mathrm{fib.}"] \\
V' \arrow[r, "h_1"] & V \arrow[r, "h_2"] \arrow[urr, dashed, "\psi"] & V' \arrow[r, "\varphi"] \arrow[ur, dashed, "\psi'"'] & B
\end{tikzcd}$$

A filler ψ exists by Proposition (3.3.26) in [2]. Then ψh_1 extends $\bar{\varphi}$ and covers φ. Use H to deform ψh_1 into the required filler ψ' as follows: ψ' is the "1-end" of the filler K in the diagram

$$\begin{tikzcd}
V'\otimes 0 \cup X''\otimes I \arrow[r, "\psi h_1 \cup \bar{\varphi} \circ \mathrm{proj.}"] \arrow[d, "\substack{\mathrm{cofib.} \\ \mathrm{and} \simeq}"'] & E \arrow[d, "\pi,\ \mathrm{fib.}"] \\
V'\otimes I \arrow[r, "\varphi \circ H"] \arrow[ur, dashed, "K"] & B
\end{tikzcd}$$

This completes the proof.

REFERENCES

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