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ON COMPLETE SURFACES WITH GAUSSIAN CURVATURE ZERO IN 3-SPHERE

BY

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1. Introduction. At the symposium on Differential Geometry held at Nara last September, R. Takagi told us the following question:

"Is there any complete flat surface (i.e., a complete surface whose Gaussian curvature with respect to the induced Riemannian metric is everywhere zero) in S^3 other than the Clifford tori?"

In this paper I shall give an answer to this problem. Main theorems are Theorems 7 and 8. The geometric construction given in Theorem 8 and the remark after it may be seen to correspond to Theorem I in Massey [4] which characterizes complete flat surfaces in Euclidean space E^3.

Let us consider the unit hypersphere S^3

$$x_1^2+x_2^2+x_3^2+x_4^2 = 1 \tag{1.1}$$

in Euclidean space E^4. Then, by identifying diametral points of S^3 with the induced Riemannian metric, we get an elliptic (non-Euclidean) space $\mathscr{E}^3$. In this $\mathscr{E}^3$, a Clifford surface Q is given in the standard form by

$$\sin^2\frac{\theta}{2}(x_1^2+x_2^2)-\cos^2\frac{\theta}{2}(x_3^2+x_4^2) = 0. \tag{1.2}$$

It has two families of generators such that any two lines of a family are left parallel and any two lines of another family are right parallel and lines from different families intersect at the constant angle θ. It is homeomorphic with a torus and is flat, i.e. its Gaussian curvature is everywhere zero.

The Clifford surface was first found by W. K. Clifford in 1873 and was the source of the problem of Clifford-Klein space forms (cf. Klein [3]).

We denote the natural projection of S^3 onto $\mathscr{E}^3$ by φ. Then, the covering surface $\varphi^{-1}(Q)$ of Q in S^3 is given by equations

$$x_1^2+x_2^2 = \cos^2\frac{\theta}{2},\qquad x_3^2+x_4^2 = \sin^2\frac{\theta}{2}. \tag{1.3}$$

We call this surface as *Clifford torus* in S^3, since it is flat and is homeomorphic with a torus. On the Clifford torus there are two families of great circles. Any two great circles in a same family do not intersect each other, but any two great circles in different families intersect at two points. For any small quadrilateral, which is made of these great circles, opposite sides have equal lengths and adjacent sides intersect at the constant angle θ. Clifford torus with $\theta = \pi/2$ is a minimal surface in S^3 (cf. Chern [2]).

2. Reduction of the fundamental forms. Let $f: M^2 \to S^3$ be an isometric immersion of a 2-dimensional flat Riemannian manifold $M^2 \equiv M$ into S^3. We take a coordinate neighborhood $\mathscr{U}$ of M with local coordinates (u^1, u^2) and denote the natural frame $(\partial/\partial u^1, \partial/\partial u^2)$ by (e_1, e_2). We put $X_a = f(e_a)$ and denote the canonical Riemannian metric of S^3 by G; then

$$g_{ab} \equiv g\left(\frac{\partial}{\partial u^a}, \frac{\partial}{\partial u^b}\right) = G(X_a, X_b) \tag{2.1}$$

are components in $\mathscr{U}$ of the induced Riemannian metric g of M in S^3. We denote the unit normal vector field over $f(\mathscr{U})$ by N; then the Gauss and Weingarten derived equations are given by

$$D_{X_c} X_b = \left\{ {a \atop bc} \right\} X_a + h_{bc} N \tag{2.2}$$

and

$$D_{X_b} N = -h_b^a X_a, \tag{2.3}$$

respectively, where D_{X_c} means the covariant derivative with respect to the Riemannian metric G in the direction of X_c, and h_{bc} are components in $\mathscr{U}$ of the second fundamental tensor of M.

The integrability condition of (2.2) is easily seen to be

$$R(X_c, X_d) X_b - f\big(K(e_c, e_d) e_b\big) = (h_{bc,d} - h_{bd,c}) N + (-h_{bc} h_d^a + h_{bd} h_c^a) X_a,$$

where R and K mean the curvature tensors of S^3 and M, respectively. As S^3 is of constant sectional curvature 1, we have

$$R(X_c, X_d) X_b = G(X_b, X_c) X_d - G(X_b, X_d) X_c. \tag{2.4}$$

So, by virtue of the assumption that M is flat, we can easily see that the equation reduces to

$$-h_{ad} h_{bc} + h_{ac} h_{bd} = g_{ad} g_{bc} - g_{ac} g_{bd}, \qquad h_{bc,d} = h_{bd,c}.$$

Since dim $M = 2$, these reduce to

$$h_{11} h_{22} - h_{12}^2 = -(g_{11} g_{22} - g_{12}^2) \tag{2.5}$$

and

$$h_{11,2} = h_{12,1}, \qquad h_{22,1} = h_{21,2}. \tag{2.6}$$

The integrability condition of (2.3) reduces to (2.6). Equations (2.5) and (2.6) are Gauss and Codazzi integrability conditions, respectively.

As $h_{11}h_{22}-h_{12}^2<0$ by (2.5), we see that through each point of $\mathscr{U}$ there pass two mutually distinct asymptotic curves. So, we may take a cubic local coordinate neighborhood $\mathscr{U}_1$ around any fixed point in $\mathscr{U}$ so that coordinate curves are asymptotic. Then, we see that

$$h_{11}=h_{22}=0, \quad h_{12}\neq 0 \tag{2.7}$$

hold good in $\mathscr{U}_1$. We may, moreover, take local coordinates so that

$$h_{12}>0 \quad \text{in } \mathscr{U}_1. \tag{2.8}$$

Now, the equations of (2.6) reduce in this case to

$$\frac{\partial \log h_{12}}{\partial u^1}+\left\{ {2 \atop 12} \right\}-\left\{ {1 \atop 11} \right\}=0$$

and an equations which is obtained by interchanging the indices 1 and 2. However, as

$$h_{12}=W\ (>0), \quad W^2=g_{11}g_{22}-g_{12}^2 \tag{2.9}$$

by (2.5), (2.7), (2.8) and by

$$\frac{\partial \log W}{\partial u^b}=\sum_{a=1}^{2}\left\{ {a \atop ab} \right\}$$

holds good, (2.6) reduces to

$$\left\{ {2 \atop 12} \right\}=0, \quad \left\{ {1 \atop 12} \right\}=0.$$

These equations are equivalent to

$$[12,\ 2]=0, \quad [21,\ 1]=0,$$

whence we see that

g_{11} does not depend upon u^2 and g_{22} does not depend upon u^1.

Hence, if we perform local coordinate transformation in $\mathscr{U}_1$ defined by

$$\bar{u}^1=\int^{u^1}\sqrt{g_{11}(u^1)}\,du^1, \quad \bar{u}^2=\int^{u^2}\sqrt{g_{22}(u^2)}\,du^2,$$

then we see that components of the metric tensor in new local coordinate system satisfy the relation $\bar{g}_{11}\equiv\bar{g}_{22}\equiv 1$. As this transformation preserves properties (2.7) and (2.8), we see that we can take local coordinate system in $\mathscr{U}_1$ of M so that

$$\begin{aligned} g_{11}&=g_{22}=1, & 1-g_{12}^2&>0, \\ h_{11}&=h_{22}=0, & h_{12}&>0 \end{aligned} \tag{2.10}$$

hold good. In such coordinate system, u^1 is the arc length for each asymptotic curve $u^2 = \text{const}$ and u^2 is the arc length for each asymptotic curve $u^1 = \text{const}$. The two families of asymptotic curves constitute a Tschebyscheff net, i.e., opposite sides of each small quadrilateral, whose sides are asymptotic curves, have equal lengths (cf. Blaschke [1]). So, we call such a system a *Tschebyscheff local coordinate system* for brevity.

Now, the Gaussian curvature of M is everywhere zero, by assumption, so we see by the first equation of (2.10) that

$$(2.11) \qquad -\frac{1}{W}\left\{\frac{\partial}{\partial u^2}\left(\frac{\partial g_{12}}{\partial u^1}\Big/ W\right)+\frac{\partial}{\partial u^1}\left(\frac{\partial g_{12}}{\partial u^2}\Big/ W\right)\right\} = 0.$$

If we put

$$(2.12) \qquad g_{12} = \cos\omega,$$

then (2.11) reduces to

$$\frac{\partial^2 \omega}{\partial u^1 \partial u^2} = 0$$

(cf. also Takagi [6]). Hence, ω is a function of the form

$$(2.13) \qquad \omega = a(u^1)+b(u^2)$$

whose value lies in an open interval $(0, \pi)$.

3. Complete flat surfaces. In this and the next section, we assume that the flat surface M in S^3 in consideration is complete with respect to the induced Riemannian metric.

THEOREM 1. *Every asymptotic curve of a complete flat surface M in S^3 can be extended indefinitely.*

Proof. As the universal covering surface $\tilde{M}$ of a 2-dimensional complete flat Riemannian manifold is the Euclidean plane E^2, M can be regarded as the image of an isometric immersion f of E^2 in S^3. So, M has neither boundary points nor singularities, and hence every asymptotic curve can be extended indefinitely, q.e.d.

Now, a local coordinate system $(\mathcal{U}, \varphi, D)$, where D and $\mathcal{U}$ are open sets in the (u^1, u^2)-plane and M, respectively, and $\varphi: D \to \mathcal{U}$ is a diffeomorphism, is said to be *cubic* if D is an open cube of the form

$$|u^1 - u_0^1| < c^1, \qquad |u^2 - u_0^2| < c^2.$$

Let us consider two cubic Tschebyscheff local coordinate systems $(\mathcal{U}, \varphi, D)$ and $(\mathcal{U}', \varphi', D')$ with local coordinates (u^1, u^2) and $('u^1, 'u^2)$, respectively, such that $\mathcal{U} \cap \mathcal{U}'$ is not empty. Then, we can easily see that there are relations of the form $'u^1 = \pm u^1 + a^1$, $'u^2 = \pm u^2 + a^2$ in $\mathcal{U} \cap \mathcal{U}'$, where a^1 and a^2 are constants. We can easily modify Tschebyscheff local coordinates in the second coordinate neighborhood so that $'u^1 = u^1$, $'u^2 = u^2$ in $\mathscr{U} \cap \mathscr{U}'$. Then, in $\mathscr{U} \cap \mathscr{U}'$ components of the first and second fundamental tensors in $\mathscr{U}$ and $\mathscr{U}'$ coincide. So, we get a Tschebyscheff local coordinate system extended over $\mathscr{U} \cup \mathscr{U}'$.

Next, let us fix a point p_0 on the surface M and let us take a cubic Tschebyscheff local coordinate system $\mathscr{U}$ around p_0 so that p_0 has the coordinates (0,0). We consider the asymptotic curve $u^2 = 0$ through p_0 in $\mathscr{U}$ and extend it indefinitely in both directions. Then, the cubic Tschebyscheff local coordinate system $\mathscr{U}$ can be extended indefinitely preserving the Tschebyscheff property along the extended asymptotic curve Γ_1. On Γ_1 there is a point p_1 with local coordinates $(u_0^1, 0)$ for arbitrary fixed number u_0^1. Through p_1 there passes another asymptotic curve which can be also extended indefinitely. We denote the curve by Γ_2. We pick up the part of cubic Tschebyscheff local coordinate system around p_1 and extend it along Γ_2. Then on Γ_2 there is a point p_2 whose local coordinates are (u_0^1, u_0^2) for arbitrary fixed value u_0^2. Thus, we have a differentiable mapping of the (u^1, u^2)-plane on M^2. It is an isometric immersion of the (u^1, u^2)-plane regarded as a flat Riemannian manifold with the metric

$$g_{11} = g_{22} = 1, \quad g_{12} = \cos\big(a(u^1) + b(u^2)\big). \tag{3.1}$$

4. The curvature and torsion of asymptotic curves. Let us calculate the geodesic curvature of asymptotic curves. First, for a curve C on M in S^3 the geodesic curvature $\varkappa$ is given by $\varkappa n = \nabla_t t$, where t means the unit tangent vector of C, ∇_t means the covariant derivative along the curve with respect to the induced metric g and $\{t, n\}$ is an orthonormal frame.

To calculate the geodesic curvature $\varkappa_1$ of the asymptotic curve $u^2 = \text{const}$, we note

$$t = (1, 0), \quad \varkappa_1 n = \left\{ {a \atop 11} \right\}. \tag{4.1}$$

Then, as

$$\left\{ {1 \atop 11} \right\} = -\frac{g_{12}}{W^2}\frac{\partial g_{12}}{\partial u^1}, \quad \left\{ {2 \atop 11} \right\} = \frac{1}{W^2}\frac{\partial g_{12}}{\partial u^1}, \tag{4.2}$$

by the first equation of (2.10), we see that

$$\varkappa_1^2 = g_{ab}\left\{ {a \atop 11} \right\}\left\{ {b \atop 11} \right\} = \left(\frac{1}{W}\frac{\partial g_{12}}{\partial u^1}\right)^2 = \left(\frac{\partial \omega}{\partial u^1}\right)^2 = \left(\frac{da}{du^1}\right)^2.$$

We define the sign of $\varkappa_1$ so that (t, n) give the same orientation of the (u^1, u^2)-plane as the natural frame. This is equivalent to take

$$n = \left(-\frac{g_{12}}{W}, \frac{1}{W}\right) \tag{4.3}$$

and

$$\varkappa_1 = -\frac{\partial \omega}{\partial u^1} = -\frac{da}{du^1}. \tag{4.4}$$

In the same way, the geodesic curvature $\varkappa_2$ of the asymptotic curve $u^1 = \text{const}$ is given by

$$t = (0, 1), \quad \varkappa_2 n = \begin{Bmatrix} a \\ 22 \end{Bmatrix}. \tag{4.5}$$

We take the sign of $\varkappa_2$ by the same convention as above. This amounts to take

$$n = \left(-\frac{1}{W}, \frac{g_{12}}{W}\right) \tag{4.6}$$

and

$$\varkappa_2 = \frac{\partial \omega}{\partial u^2} = \frac{db}{du^2}. \tag{4.7}$$

Thus, we get the following

THEOREM 2. *On a complete flat surface M in S^3, each asymptotic curve in the same family has the same geodesic curvature.*

By (4.4) and (4.7) we get also the following

THEOREM 3. *On a complete flat surface M in S^3, each curve of a family of asymptotic curves $u^2 = \text{const}$ (respectively, $u^1 = \text{const}$) is a geodesic if and only if $a(u^1)$ (respectively, $b(u^2)$) is a constant function.*

In the next place, we calculate the curvature and torsion of each asymptotic curve of M^2 as a curve in S^3. First, for any curve C on M^2, we put $T = f(t)$. Then, we have

$$D_T T = f(\nabla_t t) + h(t, t) N. \tag{4.8}$$

So, if the curve in consideration is asymptotic, we get

$$D_T T = \varkappa f(n). \tag{4.9}$$

Thus, $H = f(n)$ and $\varkappa$ can be regarded as the principal normal vector and curvature, respectively. (The convention for the direction of H and the sign of curvature are a little different from the usual one.)

For the asymptotic curve $u^2 = \text{const}$, we see that the curvature is equal to the geodesic curvature $\varkappa_1$ given by (4.4). To get the torsion of the curve, we utilize the Frénet frame (T, H, B) with

$$B = N \tag{4.10}$$

and the Frénet formula

$$D_T T = \varkappa H, \quad D_T H = -\varkappa T + \tau N, \quad D_T N = -\tau H.$$

By virtue of (2.3), we see that

$$D_T N = -h_1^a X_a = -W(g^{a2}X_a) = -H.$$

This shows that the torsion τ_1 of the asymptotic curve in consideration is equal to 1 everywhere.

In the same way, the curvature and the torsion of the asymptotic curve $u^1 = \text{const}$ are given by $\varkappa_2$ in (4.7) and $\tau_2 = -1$. Thus, we get the following

THEOREM 4. *On a complete flat surface M in S^3, the curvature of each curve of a family of asymptotic curves is equal to its geodesic curvature and the torsion is equal to a constant* 1 *or* -1.

As the corollary of Theorems 2 and 4, we get the following

THEOREM 5. *On a complete flat surface M in S^3, all curves in a family of asymptotic curves are congruent with each other.*

By (4.9) we see that an asymptotic curve on M, which is also a geodesic of M, is also a geodesic of S^3. So, by virtue of Theorem 3, we get

THEOREM 6. *On a complete flat surface M in S^3, each curve of a family of asymptotic curves $u^2 = \text{const}$ (respectively, $u^1 = \text{const}$) is a great circle of S^3 if and only if $a(u^1)$ (respectively, $b(u^2)$) is a constant function.*

As a corollary, we get the following

THEOREM 7. *Any complete flat minimal surface in S^3 is a Clifford torus with $\theta = \pi/2$.*

Proof. Since the condition for a surface to be minimal is $H = \frac{1}{2}g^{ab}h_{ab} = 0$, we see that $g_{12} = \cos(a+b) = 0$ holds identically for a surface in consideration. So, $a(u^1)$ and $b(u^2)$ are both constant and the surface is a Clifford torus with $\theta = \pi/2$.

5. Geometric construction of complete flat surfaces. The arguments in the preceeding sections suggest us a method how to construct candidates of complete flat surfaces in S^3. To explain it, we first define two functions $a(u^1)$ and $b(u^2)$ both defined on a whole real line R to be an admissible pair if

$$0 < a(u^1) + b(u^2) < \pi,$$

where u^1 and u^2 vary independently.

THEOREM 8. *Let $a(u^1)$ and $b(u^2)$ be an admissible pair of functions. We first draw a curve Γ_1 with curvature $\varkappa_1 = -da/du^1$ and torsion $\tau_1 = 1$. Using the moving Frénet frame (T, H, B) of Γ_1, we define a moving frame (X_1, X_2, N) on Γ_1 by*

$$\begin{cases} X_1(u^1, 0) = T(u^1), \\ X_2(u^1, 0) = g_{12}(u^1, 0)T(u^1) + W(u^1, 0)H(u^1), \\ N(u^1, 0) = B(u^1), \end{cases} \tag{5.1}$$

where we have put

$$(5.2)\qquad g_{12}=\cos\big(a(u^1)+b(u^2)\big),\qquad W=\sin\big(a(u^1)+b(u^2)\big).$$

Then, taking

$$(5.3)\qquad \begin{cases} \bar{T}(u^1,0)=X_2(u^1,0),\\ \bar{H}(u^1,0)=-\dfrac{1}{W(u^1,0)}X_1(u^1,0)+\dfrac{g_{12}(u^1,0)}{W(u^1,0)}X_2(u^1,0),\\ \bar{B}(u^1,0)=N(u^1,0),\end{cases}$$

as the initial Frénet frame for each fixed value u^1, we draw a curve $\Gamma_2(u^1)$ with curvature $\kappa_2=db/du^2$ and torsion $\tau_2=-1$. Then, the locus of all $\Gamma_2(u^1)$ $(u^1\in R)$ is a flat surface in S^3.

Proof. We put

$$(5.4)\qquad \begin{aligned} g_{11}&=g_{22}=1, & g_{12}&=\cos\big(a(u^1)+b(u^2)\big),\\ h_{11}&=h_{22}=0, & h_{12}&=\sin\big(a(u^1)+b(u^2)\big).\end{aligned}$$

Then, we may easily see that these sets of functions defined over $R^2=R\times R$ satisfy the Gauss and Codazzi equations (2.5) and (2.6), respectively. So, by the fundamental theorem of surfaces which also holds for surfaces in S^3 (cf. Sasaki [5]), we see that there exist surfaces in S^3 whose first and second fundamental tensors coincide with the tensors given in the first and second equations of (5.4) and any two of them are congruent under a motion of S^3.

We take any one of these surfaces and denote it by M which can be regarded as an isometric immersion f of R^2 with the Riemannian metric given by the first equation of (5.4) in S^3. At each point $f(u^1,u^2)$ of M, we have the Gaussian frame (X_1,X_2,N). We define an orthonormal frame (T,H,B) by

$$(5.5)\qquad \begin{cases} T(u^1,u^2)=X_1(u^1,u^2),\\ H(u^1,u^2)=-\dfrac{g_{12}(u^1,u^2)}{W(u^1,u^2)}X_1(u^1,u^2)+\dfrac{1}{W(u^1,u^2)}X_2(u^1,u^2),\\ B(u^1,u^2)=N(u^1,u^2).\end{cases}$$

We fix the value u^2 and consider (T,H,B) as a moving orthonormal frame along the u^1-curve (asymptotic curve). Then, by Gauss and Weingarten derived equations, we can show that

$$(5.6)\qquad D_{X_1}T=\kappa_1 H,\qquad D_{X_1}H=-\kappa_1 T+B,\qquad D_{X_1}B=-H,$$

where $\kappa_1=\kappa_1(u)=da/du^1$.

As an example, we prove only the first one:

$$D_{X_1}T = \begin{Bmatrix} a \\ 11 \end{Bmatrix} X_a = \frac{1}{W^2}\frac{\partial g_{12}}{\partial u^1}(-g_{12}X_1+X_2). \tag{5.7}$$

However, as $\partial g_{12}/\partial u^1 = W\varkappa_1$, we get the first equation of (5.6). Equations (5.6) show that (T, H, B) for each fixed value u^2 is the moving Frénet frame along the asymptotic u^1-curve. Especially, for the asymptotic curve $u^2 = 0$, the moving Frénet frame $\big(T(u^1), H(u^1), B(u^1)\big)$ relates to the Gauss frame of M on the curve $u^2 = 0$ by (5.1).

In the same way we can show that

$$\begin{cases} \bar{T}(u^1, u^2) = X_2(u^1, u^2), \\ \bar{H}(u^1, u^2) = -\dfrac{1}{W(u^1, u^2)} X_1(u^1, u^2) + \dfrac{g_{12}(u^1, u^2)}{W(u^1, u^2)} X_2(u^1, u^2), \\ \bar{B}(u^1, u^2) = N(u^1, u^2) \end{cases} \tag{5.8}$$

satisfy the differential equations

$$D_{X_2}\bar{T} = \varkappa_2\bar{H}, \quad D_{X_2}\bar{H} = -\varkappa_2\bar{T}-\bar{B}, \quad D_{X_2}\bar{B} = \bar{H}, \tag{5.9}$$

where $\varkappa_2 = \varkappa_2(u^2) = db/du^2$.

Thus $(\bar{T}, \bar{H}, \bar{B})$ for each fixed u^1 is the moving Frénet frame of the asymptotic u^2-curves. For $u^2 = 0$, it reduces to the orthonormal frame defined in (5.3). These facts show that the assertion of the theorem is true, q.e.d.

The surface M is a candidate for a complete flat surface. It is complete if and only if R^2 with the Riemannian metric

$$g_{11} = g_{22} = 1, \quad g_{12} = \cos\big(a(u')+b(u^2)\big) \tag{5.10}$$

is complete. Recently, Prof. K. Nomizu kindly wrote me that Cecil remarked the following fact: The Riemannian metric (5.10) is complete if there exist constants α, β, A and B such that

$$\begin{gathered} 0 < \alpha \leqslant a(u^1)+b(u^2) \leqslant \beta < \pi, \\ \left|\frac{da}{du^1}\right| < A, \quad \left|\frac{db}{du^2}\right| < B \quad (A, B > 0). \end{gathered} \tag{5.11}$$

We can prove it easily by showing the following facts under conditions (5.11):

(i) Let C be a smooth curve in R^2 and denote its Euclidean length by $J_E(C)$ and the Riemannian length corresponding to the metric (5.10) by $J_R(C)$. Then, we have

$$\sqrt{2}kJ_E(C) < J_R(C) < \sqrt{2}J_E(C),$$

where

$$k = \min\left\{\cos\frac{\beta}{2}, \sin\frac{a}{2}\right\}.$$

(ii) If C is a geodesic for the Riemannian metric (5.10), then the curvature of C is bounded.

From this consideration, we see that there are many complete flat surfaces immersed in S^3 which are not Clifford torus.

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