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# TENSOR PRODUCT OF STABLE MEASURES IN BANACH SPACES OF STABLE TYPE

BY

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**1. Introduction.** Let $M$ be a class of probability Borel measures of a fixed type in Banach spaces, e.g., the class of all Gaussian measures, $p$-stable measures, etc. Let $B_1$ and $B_2$ be real separable Banach spaces, and $B_1 \hat{\otimes}_\alpha B_2$ the Banach space being their reasonable cross-tensor product (the precise definition is given in Section 2). Further, let $\mu_1$ and $\mu_2$ be probability measures on the class of all Borel sets of $B_1$ and $B_2$, respectively, belonging to the class $M$. In a natural way, the following problem (denoted in the sequel by (P)) arises: Is it possible to construct a probability Borel measure $\nu$ on $B_1 \hat{\otimes}_\alpha B_2$ such that

(i) $\nu \in M$,

(ii) there exist linear continuous maps

$$p_1\colon B_1 \hat{\otimes}_\alpha B_2 \mapsto B_1 \quad \text{and} \quad p_2\colon B_1 \hat{\otimes}_\alpha B_2 \mapsto B_2$$

with the property

$$\mu_1(A_1) = \nu\big(p_1^{-1}(A_1)\big) \quad \text{and} \quad \mu_2(A_2) = \nu\big(p_2^{-1}(A_2)\big)$$

for all Borel sets $A_1$ from $B_1$ and $A_2$ from $B_2$?

Every such measure $\nu$ will be called an *$\alpha$-tensor product* of the measures $\mu_1$ and $\mu_2$, and denoted by $\nu = \mu_1 \otimes_\alpha \mu_2$.

For the class $M$ of all Gaussian measures in Banach spaces and for an $\varepsilon$-tensor product ($\alpha = \varepsilon$, cf. Section 2), problem (P) has a positive solution. Indeed, let $\mu_1$ and $\mu_2$ be Gaussian measures on real separable Banach spaces $B_1$ and $B_2$. It is known (cf. [1] and [3]) that there exists a unique Gaussian measure, say $\mu_1 \otimes_\varepsilon \mu_2$, on $B_1 \hat{\otimes}_\varepsilon B_2$ which satisfies

$$(1.1) \quad \int\limits_{B_1 \hat{\otimes}_\varepsilon B_2} \langle x_1^* \otimes x_2^*, u\rangle \langle y_1^* \otimes y_2^*, u\rangle d[\mu_1 \otimes_\varepsilon \mu_2](u) = \int\limits_{B_1} \langle x_1^*, u\rangle \langle y_1^*, u\rangle d\mu_1(u) \int\limits_{B_2} \langle x_2^*, u\rangle \langle y_2^*, u\rangle d\mu_2(u)$$

for all $x_1^*, y_1^* \in B_1^*$ and $x_2^*, y_2^* \in B_2^*$.

To see that (ii) holds we choose $x_1^* \in B_1^*$ and $x_2^* \in B_2^*$ such that

$$\int_{B_1} \langle x_1^*, x\rangle^2 d\mu_1(x) = 1 \quad \text{and} \quad \int_{B_2} \langle x_2^*, x\rangle^2 d\mu_2(x) = 1.$$

If we identify $B_1 \hat{\otimes}_\varepsilon \boldsymbol{R}$ with $B_1$, and $B_2 \hat{\otimes}_\varepsilon \boldsymbol{R}$ with $B_2$, and look at $p_1 = 1_{B_1} \otimes x_2^*$ and $p_2 = x_1^* \otimes 1_{B_2}$ as maps from $B_1 \hat{\otimes}_\varepsilon B_2$ into $B_1$ and $B_2$, respectively, then by (1.1) the condition (ii) in the problem (P) is an easily verifiable fact (cf. [1]).

In this paper we show that

(I) The problem (P) has the solution for the class $M$ of all $p$-stable Radon measures with $0 < p < 1$ (for all reasonable cross-tensor products of Banach spaces).

(II) The problem (P) has the solution for the class $M$ of all $p$-stable Radon symmetric measures with $1 \leqslant p < 2$, in the class of $L^s$-Banach spaces, where $p < s \leqslant 2$.

In the sequel we use the characterization of the class of all Banach spaces of type $p$ by the spectral measures of $p$-stable Radon measures given in [7]. Finally, we present some elementary properties of the reasonable cross-tensor products of $p$-stable measures, analogous to the properties of Gaussian $\varepsilon$-tensor products.

**2. Notation and preliminaries.** Let $(B_1, \|\cdot\|_1)$ and $(B_2, \|\cdot\|_2)$ be real Banach spaces. Following Schatten [8], a norm $\alpha$ on $B_1 \otimes B_2$ (the algebraic tensor product of $B_1$ and $B_2$) is called a *cross-norm* (of $\|\cdot\|_1$ and $\|\cdot\|_2$) if

$$\alpha(x_1 \otimes x_2) = \|x_1\|_1 \cdot \|x_2\|_2 \quad \text{for all } x_1 \in B_1 \text{ and } x_2 \in B_2.$$

If $\alpha$ is a cross-norm on $B_1 \otimes B_2$, then we denote by $B_1 \otimes_\alpha B_2$ the normed space $(B_1 \otimes B_2, \alpha)$, and by $B_1 \hat{\otimes}_\alpha B_2$ the completion of $B_1 \otimes_\alpha B_2$.

A cross-norm $\alpha$ on $B_1 \otimes B_2$ is said to be *reasonable* if

$$B_1^* \otimes B_2^* \subset (B_1 \otimes_\alpha B_2)^*$$

and the dual norm $\alpha^*$ of $\alpha$ is a cross-norm on $B_1^* \otimes B_2^*$.

More interesting examples of reasonable cross-norms are the $\varepsilon$- and $\pi$-norms.

The $\varepsilon$-norm $\|\cdot\|_\varepsilon$ is defined by

$$\|x\|_\varepsilon = \sup\{|\langle x_1^* \otimes x_2^*, x\rangle|: x_1^* \in U_1, x_2^* \in U_2\},$$

where $x \in B_1 \otimes B_2$, and $U_1$ and $U_2$ are the closed unit balls of $B_1^*$ and $B_2^*$, respectively.

The $\pi$-norm $\|\cdot\|_\pi$ is defined by

$$\|x\|_\pi = \inf\{\sum_i \|x_i\|_1 \|y_i\|_2: x = \sum_i x_i \otimes_i y_i, x_i \in B_1, y_i \in B_2\}.$$

Moreover, every reasonable cross-norm $\alpha$ on $B_1 \otimes B_2$ satisfies the condition

$$\|\cdot\|_\varepsilon \leqslant \alpha(\cdot) \leqslant \|\cdot\|_\pi.$$

Further discussion of tensor products of Banach spaces can be found in [10].

Let $(X_1, \mathscr{B}_1, m_1)$ and $(X_2, \mathscr{B}_2, m_2)$ be two measure spaces ($m_1$ and $m_2$ are finite or not). Consider the Lebesgue spaces $L^s(X_i, m_i)$, $s \geqslant 1$, $i = 1, 2$, and $L^s(X_1 \times X_2, m_1 \times m_2)$. Then it is obvious that the norm

$$\alpha_s(f) = \Big(\int_{X_1 \times X_2} |f(x_1, x_2)|^s (m_1 \times m_2)(dx_1, dx_2)\Big)^{1/s}$$

is a reasonable cross-norm of the norms

$$\|f\|_i = \Big(\int_{X_i} |f(x)|^s m_i(dx)\Big)^{1/s}, \quad i = 1, 2.$$

Thus

$$L^s(X_1 \times X_2, m_1 \times m_2) = L^s(X_1, m_1) \hat{\otimes}_{\alpha_s} L^s(X_2, m_2).$$

Let us recall that a real Banach space $B$ is of *stable type* $p$ if there exists a constant $c > 0$ such that for all $x_1, x_2, \ldots, x_n$ from $B$ we have

$$\Big(\mathrm{E}\Big\|\sum_{i=1}^{n} x_i \xi_i\Big\|^r\Big)^{1/r} \leqslant c\Big(\sum_{i=1}^{n} \|x_i\|^p\Big)^{1/p}$$

for some (each) $r$ with $0 < r < p$, where $\xi_1, \xi_2, \ldots, \xi_n$ is a sequence of i.i.d. random variables with characteristic function $\exp(-|t|^p)$, $t \in \boldsymbol{R}$. Here $0 < p < 2$.

PROPOSITION 2.1 (cf. [11]). (i) *Each normed space is of stable type $p$ whenever $0 < p < 1$.*

(ii) *The Lebesgue spaces $L^s$ and $l^s$ are of stable type $p$ for each $s$ with $1 \leqslant p < s \leqslant 2$.*

For a detailed discussion of Banach spaces of $p$-stable type (cotype) and related topics we refer the reader to [11].

Let $B$ be a real separable Banach space, $\mathscr{B}(B)$ the Borel $\sigma$-algebra, and let $\mu$ be a $p$-stable Radon measure on $\mathscr{B}(B)$ with $0 < p < 2$. Then it is known ([4], [5], [7], [9]) that the characteristic function (ch. f.) $\hat{\mu}$ of $\mu$ is given by

$$\hat{\mu}(x^*) = \exp\Big\{-\int_{S_B} |\langle x, x^*\rangle|^p \, dm(x)\Big\}, \quad x^* \in B^*, \tag{2.1}$$

where $S_B$ is the boundary of the unit ball of $B$, and $m$ is a finite Borel measure on $S_B$ (cf. [9]). The measure $m$ on $\mathscr{B}(S_B)$ is often called the *spectral measure* of $\mu$. If (2.1) holds, then we write $\mu = N_p(m)$.

In our definition of the tensor product of $p$-stable measures, the measure $m$ plays such a role as the covariance operator in the definition of the tensor product of Gaussian measures (cf. [1]–[3]).

THEOREM 2.1 ([5], [7]). *Let $B$ be a real separable Banach space and $S_B$ the boundary of its unit ball. Assume $0 < p < 2$. Then the following statements are equivalent*:

(i) *$B$ is of stable type $p$.*

(ii) *Every function of the form*

$$\psi(x^*) = \exp\{-\int_{S_B} |\langle x, x^*\rangle|^p dm(x)\}$$

*for a finite measure $m$ on $\mathscr{B}(S_B)$ is the ch.f. of a $p$-stable probability measure on $B$.*

**3. Tensor products of $p$-stable Radon measures in Banach spaces.** Let $(B_1, \|\cdot\|_1)$ and $(B_2, \|\cdot\|_2)$ be separable real Banach spaces and let $\alpha$ be a reasonable cross-norm of $\|\cdot\|_1$ and $\|\cdot\|_2$. Let $\mu_1$ and $\mu_2$ be two $p$-stable Radon measures on $B_1$ and $B_2$ with the spectral measures $m_1$ and $m_2$, respectively. By (2.1) we have

$$\hat{\mu}_1(x^*) = \exp\{-\int_{S_{B_1}} |\langle x, x^*\rangle|^p dm_1(x)\},$$

$$\hat{\mu}_2(y^*) = \exp\{-\int_{S_{B_2}} |\langle y, y^*\rangle|^p dm_2(y)\},$$

where $x^* \in B_1^*$ and $y^* \in B_2^*$.

Now, denote by $i$ the map from $B_1 \times B_2$ into $B_1 \hat{\otimes}_\alpha B_2$ given by

$$i(x, y) = x \otimes y, \quad x \in B_1 \text{ and } y \in B_2.$$

By the definition of a cross-norm,

$$\alpha(i(x, y)) = \alpha(x \otimes y) = \|x\|_1 \cdot \|y\|_2 \quad \text{for all } x \in B_1 \text{ and } y \in B_2.$$

Hence $i$ is continuous in the product topology on $B_1 \times B_2$, and if $x \in S_{B_1}$ and $y \in S_{B_2}$, then $i(x, y) \in S_{B_1 \hat{\otimes}_\alpha B_2}$. Let $i_s = i|(S_{B_1} \times S_{B_2})$ and let $m_1 \times m_2$ be a product measure on $\mathscr{B}(S_{B_1} \times S_{B_2})$. If we put

$$(3.1) \qquad (m_1 \otimes m_2)(A) = (m_1 \times m_2)(i_s^{-1}(A)) \quad \text{for } A \in \mathscr{B}(S_{B_1 \hat{\otimes}_\alpha B_2}),$$

then by the previous remarks formula (3.1) defines the finite measure $m_1 \otimes m_2$ on $\mathscr{B}(S_{B_1 \hat{\otimes}_\alpha B_2})$, which will be called a *spectral $\alpha$-tensor product of the measures $m_1$ and $m_2$*.

Now, consider two cases:

Case 1. *The Banach spaces $B_1$ and $B_2$ are arbitrary and $0 < p < 1$.*

By Proposition 2.1 (i) and Theorem 2.1, the function of the form

$$(B_1 \hat{\otimes}_\alpha B_2)^* \ni z^* \mapsto \exp\{-\int_{S_{B_1 \hat{\otimes}_\alpha B_2}} |\langle z^*, z\rangle|^p d(m_1 \otimes m_2)(z)\}$$

is the ch.f. of some $p$-stable Radon measure, say $\mu_1 \otimes_\alpha \mu_2$, on $\mathscr{B}(B_1 \hat{\otimes}_\alpha B_2)$.

Case 2. $1 \leqslant p < 2$.

Let $B_1 = L^s(X, v_1)$ and $B_2 = L^s(Y, v_2)$, where $v_1$ or $v_2$ are arbitrary and $p < s \leqslant 2$. Then, by Proposition 2.1 (ii) and Theorem 2.1, the function

$$L^r(X \times Y, v_1 \times v_2) \ni z^* \mapsto \exp\{-\int_{S_{L^s(X \times Y, v_1 \times v_2)}} |\langle z^*, z\rangle|^p d(m_1 \otimes m_2)(z)\}$$

is the ch.f. of some $p$-stable measure, say $\mu_1 \otimes_\alpha \mu_2$, on $L^s(X \times Y, v_1 \times v_2)$. Here $1/s + 1/r = 1$.

PROBLEM (cf. [3] for cotype 2). Assume that $B_1$ and $B_2$ are of stable type $p$ with $1 \leqslant p < 2$. Is the space $B_1 \hat{\otimes}_\alpha B_2$ of stable type $p$? (**P 1327**)

DEFINITION. The measure $\mu_1 \otimes_\alpha \mu_2$ defined in Cases 1 and 2 is called the *$\alpha$-tensor product of p-stable Radon measures $\mu_1$ and $\mu_2$*.

THEOREM 3.1. (i) *The construction given in Case* 1 *is the solution of the problem* (P) *for the class M of all p-stable Radon measures with* $0 < p < 1$, *in the class of all Banach spaces.*

(ii) *The construction given in Case* 2 *is the solution of the problem* (P) *for the class M of all p-stable Radon measures with* $1 \leqslant p < 2$, *in the class of all $L^s$-spaces,* $p < s \leqslant 2$.

Proof. The condition (i) from (P) follows immediately from the definition of $\mu_1 \otimes_\alpha \mu_2$. We show that the measure $\mu_1 \otimes_\alpha \mu_2$ satisfies the condition (ii) from the problem (P). Let $x_1^* \in B_1^*$ and $x_2^* \in B_2^*$ be such that

$$(3.2) \qquad \int_{S_{B_1}} |\langle x_1^*, u\rangle|^p dm_1(u) = 1 \quad \text{and} \quad \int_{S_{B_2}} |\langle x_2^*, u\rangle|^p dm_2(u) = 1.$$

Here $m_1$ and $m_2$ are the spectral measures of $\mu_1$ and $\mu_2$, $m_1 \neq 0$, $m_2 \neq 0$, on the boundaries $S_{B_1}$ and $S_{B_2}$ of unit balls in $B_1$ and $B_2$, respectively. If $m_1$ or $m_2$ is identically zero, then (ii) is clear. When we identify $B_1 \hat{\otimes}_\alpha \boldsymbol{R}$ with $B_1$, and $B_2 \hat{\otimes}_\alpha \boldsymbol{R}$ with $B_2$ in the usual way (cf. Introduction), we can look at $p_1 = 1_{B_1} \otimes x_2^*$ and $p_2 = x_1^* \otimes 1_{B_2}$ as maps from $B_1 \hat{\otimes}_\alpha B_2$ into $B_1$ on one hand, and from $B_1 \hat{\otimes}_\alpha B_2$ into $B_2$ on the other. We compute the ch.f.'s of the measures $p_1(\mu_1 \otimes_\alpha \mu_2)$ on $B_1$ and $p_2(\mu_1 \otimes_\alpha \mu_2)$ on $B_2$. Let $x_1^* \in B_1$. We have

$$\begin{aligned} p_1(\mu_1 \otimes_\alpha \mu_2)\hat{}\,(x_1^*) &= \int_{B_1} \exp\{i\langle x_1^*, u\rangle\} dp_1(\mu_1 \otimes_\alpha \mu_2)(u) \\ &= \int_{B_1 \hat{\otimes}_\alpha B_2} \exp\{i\langle x_1^*, p_1(u)\rangle\} d(\mu_1 \otimes_\alpha \mu_2)(u) \\ &= \int_{B_1 \hat{\otimes}_\alpha B_2} \exp\{i\langle p_1^*(x_1^*), u\rangle\} d(\mu_1 \otimes_\alpha \mu_2)(u) \\ &= (\mu_1 \otimes_\alpha \mu_2)\hat{}\,(x_1^* \otimes x_2^*), \end{aligned}$$

since it is easy to calculate that $p_1^*(u_1^*) = u_1^* \otimes x_2^*$ for all $u_1^* \in B_1^*$. But from the definition of $\mu_1 \otimes_\alpha \mu_2$ and by (3.2) we have

$$(\mu_1 \otimes_\alpha \mu_2)\hat{}(x_1^* \otimes x_2^*)$$
$$= \exp\{-\int_{S_{B_1 \hat{\otimes}_\alpha B_2}} |\langle x_1^* \otimes x_2^*, u\rangle|^p d(m_1 \otimes m_2)(u)\}$$
$$= \exp\{-\int_{S_{B_1} \times S_{B_2}} |\langle x_1^*, u_1\rangle \cdot \langle x_2^*, u_2\rangle|^p dm_1(u_1)\, dm_2(u_2)\}$$
$$= \exp\{-\int_{S_{B_1}} |\langle x_1^*, u_1\rangle|^p dm_1(u_1)\} = \hat{\mu}_1(x_1^*).$$

Thus, we show that $p_1(\mu_1 \otimes_\alpha \mu_2)\hat{}(x_1^*) = \hat{\mu}_1(x_1^*)$ for all $x_1^* \in B_1^*$, i.e., that $p_1(\mu_1 \otimes_\alpha \mu_2) = \mu_1$. Similarly we can show that $p_2(\mu_1 \otimes_\alpha \mu_2) = \mu_2$. Thus the measure $\mu_1 \otimes_\alpha \mu_2$ is an $\alpha$-tensor product of $p$-stable measures in the sense of problem (P).

**4. Some elementary properties of the tensor products of $p$-stable measures.** We end the paper with the presentation of properties of the tensor product of $p$-stable measures. From the definition we have immediately

PROPOSITION 4.1. *Let $\mu_1 = N_p(m_1)$, $\mu_2 = N_p(m_2)$ and $\mu_3 = N_p(m_3)$ be p-stable Radon measures on Banach spaces $B_1$, $B_2$ and $B_3$, respectively, $0 < p < 2$. Then*

(i) $(\mu_1 \otimes_\alpha \mu_2) \otimes_\alpha \mu_3 = \mu_1 \otimes_\alpha (\mu_2 \otimes_\alpha \mu_3)$,
(ii) $\mu_1 \otimes_\alpha \mu_2 = \mu_2 \otimes_\alpha \mu_1$,
(iii) $\mu_1 \otimes_\alpha 0 = 0$.

PROPOSITION 4.2. *Let $B_1$ and $B_2$ be real separable Banach spaces and let $\mu$ be a p-stable Radon measure on $B_1 \hat{\otimes}_\alpha B_2$. A necessary and sufficient condition for the existence of p-stable measures $\mu_1$ and $\mu_2$ on $B_1$ and $B_2$ such that $\mu = \mu_1 \otimes_\alpha \mu_2$ is that there are finite measures $m_1$ and $m_2$ on $S_{B_1}$ and $S_{B_2}$ which satisfy*

$$\int_{S_{B_1 \hat{\otimes}_\alpha B_2}} |\langle u, x^* \otimes y^*\rangle|^p dm(u) = \int_{S_{B_1}} |\langle x^*, u_1\rangle|^p dm_1(u_1) \int_{S_{B_2}} |\langle y^*, u_2\rangle|^p dm_2(u_2) \tag{4.1}$$

*for every $x^* \in B_1^*$ and $y^* \in B_2^*$, where $m$ is the spectral measure of $\mu$.*

Proof. The necessity of (4.1) is obvious, and the sufficiency is clear if $m_1$ or $m_2$ is identically zero. Otherwise, there are $x_1^* \in B_1^*$ and $x_2^* \in B_2^*$ such that (3.2) holds. Then we have

$$\mu = p_1(\mu) \otimes_\alpha p_2(\mu),$$

where $p_1$ and $p_2$ are defined in the proof of Theorem 3.1.

PROPOSITION 4.3. *For $i = 1, 2$ let $\{m_n^{(i)}: n \geqslant 1\}$ be a sequence of finite* *measures which converges weakly on the boundary $S_{B_i}$ of the unit ball in a real separable Banach space $B_i$ of stable type $p$ (with $0 < p < 2$) to a finite measure $m^{(i)}$. Let $\mu_n = N_p(m_n^{(1)})$, $\nu_n = N_p(m_n^{(2)})$ and $\mu = N_p(m^{(1)})$, $\nu = N_p(m^{(2)})$. Then*

(i) *$\{\mu_n\}$ converges weakly to $\mu$ and $\{\nu_n\}$ converges weakly to $\nu$;*

(ii) *the sequence $\{\mu_n \otimes_\alpha \nu_n: n \geqslant 1\}$ converges weakly on $B_1 \hat{\otimes}_\alpha B_2$ to the measure $\mu \otimes_\alpha \nu$ if all tensor products considered here exist.*

Proof. Under our assumptions, since $m_n^{(i)}$ converges weakly to $m^{(i)}$ for $i = 1, 2$, then also $m_n^{(1)} \times m_n^{(2)}$ (and $m_n^{(1)} \otimes m_n^{(2)}$) converges weakly to $m^{(1)} \times m^{(2)}$ (to $(m^{(1)} \otimes m^{(2)})$), respectively (a spectral tensor product of finite measures is the image of the product of measures by the continuous map $i_s$). By the theorem of Marcus and Woyczynski (see, e.g., [6], Theorem 1.1), the weak convergence of the spectral measures $m_n^{(i)}$ to $m^{(i)}$ ($i = 1, 2$) and $m_n^{(1)} \otimes m_n^{(2)}$ to $m^{(1)} \otimes m^{(2)}$ implies the weak convergence of $\{\mu_n\}$ to $\mu$ and $\{\nu_n\}$ to $\nu$, and finally $\{\mu_n \otimes_\alpha \nu_n\}$ to $\mu \otimes_\alpha \nu$, which completes the proof.

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