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PROBLEMS ON BANACH SPACES

COLLECTED BY

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We present the list of problems which were discussed at the Winter School on Functional Analysis held in January 1978 in Nowy Sącz, Poland. The meeting was organized by the Institute of Mathematics of the Polish Academy of Sciences and Sektion Mathematik, Friedrich-Schiller-Universität Jena, the German Democratic Republic.

1. We say that a subset A of a Banach space X has the ***Radon-Nikodým property*** if for every measure space (Ω, Σ, μ) with $\mu(\Omega) < \infty$ and every μ-continuous $G\colon \Sigma \to A$ of bounded variation there exists $g \in L_1(A, \mu)$ such that

$$G(E) = \int_E g\,d\mu \quad \text{for all } E \in \Sigma.$$

Is l_∞ generated by a set with Radon-Nikodým property? (J. Diestel; **P 1217**)

2. We say that a norm $\|\cdot\|$ in a Banach space X is ***weakly locally uniformly convex*** if, for every sequence $(y_n) \subset X$ with $\|y_n\| = 1$ and $\lim_n \|y_n + y_0\| = 2$,

$$\lim_n y_n = y_0 \text{ weakly}.$$

Given a dual space X^*, are there general conditions that make weakly locally uniformly convex norms dual norms? (J. Diestel; **P 1218**)

3. It is known that if X has the Radon-Nikodým property, then the unit ball in X has the fixed-point property for contractions, i.e. if U is the unit ball in X and $T\colon U \to U$ is a mapping such that $\|Tx - Ty\| \leqslant \|x - y\|$ for $x, y \in U$, then T has a fixed point. Is this property isomorphically invariant? (S. Kwapień; **P 1219**)

4. A Banach space X is called ***primary*** if from the equality $X = Y \oplus Z$ it follows that X is isomorphic to Y or X is isomorphic to Z. Is the disc algebra A primary? The same question for H^1 and H^∞. (P. G. Casazza; **P 1220**)

5. Let $D \subset C$ be a connected domain. Is $H^\infty(D)$ isomorphic to H^∞? (P. Wojtaszczyk, **P 1221**)

6. Let G be a compact abelian group, and Γ its dual. We say that a subset M of Γ is *Sidon* if

$$\Big\|\sum_{\gamma\in M} c_\gamma \gamma\Big\| \sim \sum_{\gamma\in M} |c_\gamma|$$

for all scalars $(c_\gamma)_{\gamma\in M}$. It is known that if E_M^∞ has cotype 2, then M is Sidon (E_M^∞ denotes the subspace of $C(G)$ generated by $\gamma \in M$). Does the last statement remain true after replacing 2 by some $q < \infty$? (A. Pełczyński; **P 1222**)

7. Let $S(R^n)$ be a set of all symmetric norms a on R^n satisfying the condition $a(e_i) = 1$ $(i = 1, \ldots, n)$. Describe the set of extreme points of $S(R^n)$. It is known that $\|\cdot\|_1$ and $\|\cdot\|_\infty$ belong to this set. (A. Pietsch; **P 1223**)

8. Let $1 \leqslant p < 2$ and let $(f_i)_{i=1}^\infty$ be a sequence of p-stable independent random variables, i.e. the Fourier transform of f_i equals $e^{-|t|^p}$. Does there exist a Banach space X such that, for some sequence $(x_i)_{i=1}^\infty \in X$, $\sum x_i f_i$ is bounded a.e. but not convergent a.e.? (W. Linde; **P 1224**; this question was originally stated by Garling in [2])

9. Let $1 \leqslant p < 2$. Let μ_p^i $(i = 1, \ldots, n)$ be a p-stable measure on R and let $\mu_p = \mu_p^1 \times \ldots \times \mu_p^n$ be a measure on R^n. Let T: $R^n \to R^n$ be a linear operator such that $\|T\colon l_{p'}^n \to l_p^n\| \leqslant 1$. Let q be any seminorm on R^n. Is it true that $\mathrm{E}q(Tx) \leqslant \mathrm{E}q(x)$? (W. Linde; **P 1225**)

10. Let $1 \leqslant p < 2$. Let X be a Banach space and let $T\colon l_{p'} \to X$ be a linear operator such that $\mathrm{E}\big\|\sum T(e_i) f_i\big\| < \infty$ ((f_i) is a sequence of p-stable independent random variables). Is T p-absolutely summing? (W. Linde; **P 1226**)

11. It is known that if for every subspace E of a Banach space X there is a continuous projection from X onto E, then X is isomorphic to a Hilbert space [4]. Now suppose that a linear operator $T: X \to Y$ (X, Y being Banach spaces) satisfies the following condition:

(*) There is a constant C such that for every finite-dimensional subspace $E \subset X$ there is a linear operator $T_E: X \to T(E)$ with $T_E e = Te$ for $e \in E$ and $\|T_E\| \leqslant C$.

Does (*) imply that T is hilbertian, i.e. there exist a Hilbert space H and linear operators $A: X \to H$ and $B: H \to Y$ with $T = BA$? (T. Figiel and A. Pełczyński; **P 1227**)

For the next two problems the reader can see also the paper by König [3].

12. Let $p < 2$ and let $S_p^{\max}$ denote the maximal operator ideal on all Banach spaces which on a Hilbert space coincides with $S_p(H)$ (the p-th trace class). For a Banach space X we define

$$\mathcal{E}_{q,\infty}(X) = \{T : X \to X \mid \{\lambda_i(T)\} \in l_{q,\infty}\},$$
$$\mathcal{E}_q(X) = \{T : X \to X \mid \{\lambda_i(T)\} \in l_q\},$$

where $\{\lambda_i(T)\}$ is the sequence of eigenvalues of the operator T. It is known that $S_p^{\max}(X) \subset \mathcal{E}_{q,\infty}(X)$, where $q^{-1} = p^{-1} - 2^{-1}$. Is this true for $\mathcal{E}_q(X)$ instead of $\mathcal{E}_{q,\infty}(X)$? (H. König; P 1192)

13. Is it true that $S_p^{\max}(L_r) \subset \mathcal{E}_q(L_r)$, where $q^{-1} = p^{-1} - 2^{-1} + r^{-1}$? (H. König; P 1193)

14. Let E, F, G be Banach spaces and let $T \in \Pi_2(E, F)$, $S \in \Pi_2(F, G)$. Does the operator ST admit a factorization

$$E \xrightarrow[u]{} L_\infty \xrightarrow[w_1]{} L_1 \xrightarrow[w_2]{} L_2 \xrightarrow[v]{} G$$

with $\|u\|\|w_1\|\|w_2\|\|v\| \leqslant C\Pi_2(T)\Pi_2(S)$, where C is a universal constant? (A. Pietsch; **P 1228**)

15. Let a Banach space of dimension n have type 2 and cotype 2 with Gaussian constants α and β, respectively. It is known that to calculate these constants it is enough to consider sums $\sum x_i \gamma_i$ of length $n(n+1)/2$ only [1]. Is this (or some other version) true for $\Pi_p = \Pi_p(\mathrm{id} : X \to X)$ instead of α and β? (T. Figiel; **P 1229**)

16. For an n-dimensional real Banach space X we define the volume ratio of X by $\mathrm{vr}(X) = (\mathrm{vol} B(X)/\mathrm{vol}\mathcal{E})^{1/n}$, where $B(X)$ is the unit ball of X and $\mathcal{E}$ is the ellipsoid of maximal volume contained in $B(X)$ (see [5]). Does there exist a function f: $R^+ \to R^+$ such that $\mathrm{vr}(X) \leqslant f(\beta(X))$ for every X ($\beta(X)$ is as in Problem 15)? (T. Figiel, S. J. Szarek and N. Tomczak; **P 1230**)

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