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CORRIGENDA TO "OPERATOR SEMI-STABLE PROBABILITY MEASURES ON R^N"

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In the formulation of Theorem 3.1 in [1] the words: "The measure μ..." should read as: "The full non-Gaussian measure μ...". The proof of the theorem on p. 298 after defining the norm

$$|||x||| = \sum_{l=1}^{N} |\beta_l(x)| \quad \text{for} \quad x = \sum_{l=1}^{N} \beta_l(x) f_l,$$

should go as follows:

Since $\int_{\|x\|>1} \|x\|^\alpha M(dx) < \infty$ if and only if $\int_{\|x\|>1} |||x|||^\alpha M(dx) < \infty$, it suffices to estimate the last integral. We have

$$\int_{\|x\|>1} |||x|||^\alpha M(dx) = \sum_{n=1}^{\infty} a^n \int_{Z_A} |||A^{-n}x|||^\alpha M(dx)$$
$$= \sum_{n=1}^{\infty} a^n \int_{Z_A} \Big|\sum_{j=1}^{k} \sum_{l=1}^{n_j-n_{j-1}} \beta_{n_{j-1}+l}(A^{-n}x)\Big|^\alpha M(dx).$$

According to (3.1), the finiteness of the last expression is equivalent to the condition

$$\sum_{j=1}^{k} \sum_{l=1}^{n_j-n_{j-1}} \sum_{n=1}^{\infty} a^n \int_{Z_A} |\beta_{n_{j-1}+l}(A^{-n}x)|^\alpha M(dx) < \infty. \tag{3.2}$$

Let us now assume that, for some j and n,

$$\int_{Z_A} |\beta_{n_{j-1}+l}(A^{-n}x)|^\alpha M(dx) = 0 \quad \text{for} \quad l = 1, \ldots, n_j - n_{j-1}. \tag{3.3}$$

Then $\beta_{n_{j-1}+l}(A^{-n}x) = 0$ for M-almost all $x \in Z_A$. But this means that $A^{-n}x \perp [\{e_{n_{j-1}+1}, \ldots, e_{n_j}\}]$ for M-almost all $x \in Z_A$. The subspace $[\{e_{n_{j-1}+1}, \ldots, e_{n_j}\}]^\perp$ is A^{-1}-invariant, and thus A-invariant; therefore $x \perp [\{e_{n_{j-1}+1}, \ldots, e_{n_j}\}]$ for M-almost all $x \in Z_A$.

Put

$$X' = \{x \in \boldsymbol{R}^N: \beta_{n_{j-1}+l}(x) = 0,\ l = 1, \ldots, n_j - n_{j-1}\}.$$

From the above considerations and the fullness of μ_X, and thus of M, on X it follows that $X \subset X'$. (The notation as in Theorem 1.1.) Let $\tilde{X}$ and $\tilde{X}'$ denote the complex extensions of X and X', respectively. From the definition of $\tilde{X}'$ it follows that $f_{n_j} \notin \tilde{X}$; thus the corresponding eigenvalue $\theta_j \notin \operatorname{Sp} \tilde{A}^{-1} | \tilde{X}$. ($\tilde{A}^{-1}$ stands for the complex extension of A^{-1}.)

Let now θ_{j_0} be the eigenvalue of A^{-1} having the greatest absolute value. μ is not Gaussian, and therefore, by virtue of Theorem 1.1, $\theta_{j_0} \in \operatorname{Sp} \tilde{A}^{-1} | \tilde{X}$, which gives

$$\int_{Z_A} |\beta_{n_{j_0-1}+l}(A^{-n}x)|^\alpha M(dx) > 0 \tag{3.4}$$

for some l between 1 and $n_{j_0} - n_{j_0-1}$ and for every n.

After some computations we obtain, for sufficiently large n,

$$\beta_{n_{j-1}+l}(A^{-n}x) = \sum_{r=0}^{l-1} \binom{n}{r} \theta_j^{n-r} \beta_{n_{j-1}+l-r}(x).$$

Thus, by (3.2), we get the following condition, equivalent to the existence of the moment of order α,

$$\sum_{j=1}^{k} \sum_{n=1}^{\infty} a^n |\theta_j|^{\alpha n} \sum_{l=1}^{n_j - n_{j-1}} \int_{Z_A} \left| \sum_{r=0}^{l-1} \binom{n}{r} \theta_j^{-r} \beta_{n_{j-1}+l-r}(x) \right|^\alpha M(dx) < \infty. \tag{3.5}$$

The left-hand side in (3.5) is finite if and only if

$$\sum_{n=1}^{\infty} a^n |\theta_{j_0}|^{\alpha n} \sum_{l=1}^{n_{j_0} - n_{j_0-1}} \int_{Z_A} \left| \sum_{r=0}^{l-1} \binom{n}{r} \theta_{j_0}^{-r} \beta_{n_{j_0-1}+l-r}(x) \right|^\alpha M(dx) < \infty \tag{3.6}$$

since $|\theta_{j_0}|$ is maximal. By virtue of (3.4),

$$\sum_{l=1}^{n_{j_0} - n_{j_0-1}} \int_{Z_A} \left| \sum_{r=0}^{l-1} \binom{n}{r} \theta_{j_0}^{-r} \beta_{n_{j_0-1}+l-r}(x) \right|^\alpha M(dx) > 0;$$

therefore the left-hand side in (3.6) is finite if and only if $a|\theta_{j_0}|^\alpha < 1$ because

$$\sum_{r=0}^{l-1} \binom{n}{r} \theta_{j_0}^{-r} \beta_{n_{j_0-1}+l-r}(x)$$

is a polynomial with respect to n.

In other words, $a/|\lambda_{j_0}|^\alpha < 1$ where λ_{j_0} is the eigenvalue of A having the smallest absolute value. The last inequality is equivalent to $c/|\lambda_{\min}|^\alpha < 1$, which proves the theorem.

In Corollary 3.1 the measure μ should be assumed to be non-Gaussian.

REFERENCE

[1] A. Łuczak, *Operator semi-stable probability measures on* $\boldsymbol{R}^N$, Colloquium Mathematicum 45 (1981), p. 287–300.

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