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MINIMAL AND DISTAL FUNCTIONS ON SEMIDIRECT PRODUCTS OF GROUPS

BY

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In a recent paper we answered a question of A. W. Knapp by showing the existence, on the Euclidean group $G = T \otimes_\sigma C$ of the plane, of functions that are (right) distal but not left distal on G. The basic example of such a function was an extension of a character on the plane C to G. In the present paper*, we use topological-semigroup-theoretic techniques to extend functions that are minimal, point-distal or distal on a group G_2 to (functions of similar type on) a semidirect product $G_1 \otimes_\sigma G_2$. (A consequence of this work is that one can see how to construct large classes of minimal, point-distal and distal functions on some groups, e.g., $T \otimes_\sigma C$.)

For example, we show that if G_1 is compact, then every distal function on G_2 extends in a canonical way to a distal function on G. This canonical way fails dramatically to extend the character f: $x \to e^{ix}$ on $\boldsymbol{R}$ even to a minimal function on the discrete version $(\boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R})_d$ of the affine group of the line; but, amazingly, the canonical extension is left minimal on $(\boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R})_d$. This last conclusion is shown to hold in a general setting, as is another extension method that does extend f on $\boldsymbol{R}$ to a minimal function on $\boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R}$. The question remains: can f be extended to a point-distal function on $\boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R}$?

Preliminaries. Let G be a topological group. A bounded complex-valued function F on G is called *right uniformly continuous* (r.u.c., for short) if for any $\varepsilon > 0$ there is a neighbourhood V of the identity e of G such that $|F(s) - F(t)| < \varepsilon$ whenever $st^{-1} \in V$. Let $U(G)$ be the class of such functions. $U(G)$ is a C^*-subalgebra of the C^*-algebra $C(G)$ of all continuous bounded complex-valued functions on G. The *right translate* $R_t F$ of $F \in C(G)$ is defined by

$$R_t F(s) = F(st), \qquad t \in G,$$

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and, if $F \in U(G)$, then the closure $R_G F^-$ of the orbit

$$R_G F = \{R_t F | \ t \in G\}$$

in the topology of pointwise convergence on G is compact in $C(G)$ for that topology. If G is locally compact, then this compactness property characterizes $U(G)$. (See [2] for all this, noting that the space we have called $U(G)$ here is called, for a good reason, LUC(G) in [2] and [7]. Our terminology is like that in [6].) The spectrum UG of $U(G)$ contains a canonical dense image of G from which multiplication can be extended uniquely by continuity to all of UG so that UG becomes a compact semigroup with the property that the maps $\mu \to s\mu$ and $\mu \to \mu\nu$ from UG into UG are continuous for all $s \in G$ and $\nu \in UG$. (We are identifying G with its image in UG.) In particular, UG is a compact right topological semigroup [2]. Further, the multiplication is jointly continuous on $G \times UG$ and UG has a universal mapping property ([2], Theorem III.5.5), which implies that each continuous automorphism of G extends uniquely by continuity to a continuous automorphism of UG.

If $F \in U(G)$, the translation operators R_t, $t \in G$, leave $R_G F^-$ invariant and $(R_G, R_G F^-)$ is a flow. F is called *minimal*, *point-distal*, or *distal* if that flow is minimal, point-distal with F as a special point, or distal, respectively. Specifically, an F in $U(G)$ is:

minimal if, whenever

$$H_1 = \lim_\alpha R_{t_\alpha} F \text{ (pointwise on } G),$$

there is a net $\{t_\beta\} \subset G$ such that

$$F = \lim_\beta R_{t_\beta} H_1;$$

point-distal if, whenever

$$H_1 = \lim_\alpha R_{t_\alpha} F \quad \text{and} \quad \lim_\gamma R_{t_\gamma} H_1 = H' = \lim_\gamma R_{t_\gamma} F,$$

it follows necessarily that $H_1 = F$;

distal if, whenever

$$H_1 = \lim_\alpha R_{t_\alpha} F, \quad H_2 = \lim_\beta R_{t_\beta} F \quad \text{and} \quad \lim_\gamma R_{t_\gamma} H_1 = H' = \lim_\gamma R_{t_\gamma} H_2,$$

it follows necessarily that $H_1 = H_2$.

Clearly, distal functions are point-distal, and point-distal functions are minimal [5]. Also, the limit function H_1 of the definition will be minimal or distal if F is minimal or distal, respectively, but can fail to be point-distal if F is point-distal. Thus, if F is point-distal, if H' is some minimal function, and if

$$\lim_\beta R_{t_\beta} F = \lim_\beta R_{t_\beta} H',$$

then $H' \in R_G F^-$ and $H' = F$. An analogous assertion holds for distal functions. The set $D(G)$ of distal functions on G forms a C^*-subalgebra of $C(G)$ whose spectrum DG contains a canonical dense image of G from which multiplication can be extended to DG making it a compact right topological group with continuity properties like those for UG. These assertions follow from Sections 2 and 3 in [7]. Also, DG has the following universal mapping property, which follows from Theorem 4.6 in [1] or can be proved using methods as in [2], Chapter III: if φ is a continuous homomorphism of G into a compact right topological group K such that the map

$$\mu \to \varphi(s)\mu, \qquad K \to K$$

is continuous for all $s \in G$, then there is a unique continuous homomorphism ψ of DG into K such that $\varphi(s) = \psi(s)$ for all $s \in G$. (Here, for an $s \in G$, we are using the same symbol s for the canonical image of s in DG.) Consequently, any continuous automorphism of G extends uniquely by continuity to a continuous automorphism of DG.

By a beautiful result of Ellis [4] (Proposition 5.3) or [2] (Theorem II.3.6), the distal functions can be characterized as follows. If $F \in U(G)$, put $X = R_G F^-$ and regard the translations R_G (restricted to X) as a subset of X^X, which is compact. Then F is distal if and only if R_G^- (closure in X^X) is a group of transformations of X. In [10] we displayed a number of "bad" topological properties R_G^- must have if F is distal and not almost periodic. Also, in [13] Namioka gives a concrete representation of a group that arises essentially as above, and gives some example of compact Hausdorff right topological groups that are not topological groups. Further, a question to contemplate is the following: is there a characterization of distal functions on $\boldsymbol{Z}$ analogous to Bohr's result that an almost periodic function on $\boldsymbol{Z}$ is the uniform limit of linear combinations of characters on $\boldsymbol{Z}$? For example, is each distal function on $\boldsymbol{Z}$ a uniform limit of members of the algebra generated by functions $\{n \to \exp(ian^k) \mid k = 0, 1, 2, \ldots, a \in \boldsymbol{R}\}$? **(P 1346)**(1)

Note. Our terminology would have been more accurate if we had called the functions defined above right minimal, right point-distal, etc., since the definitions involved right uniform continuity and right translation. We wish to assert here that, in this paper, a function will be called *left minimal, left point-distal*, etc., if it satisfies the appropriate analogous condition involving left uniform continuity and left translation. The phrase, left uniformly continuous, will be abbreviated to l.u.c.

Let G be a topological group. We say that G is the *semidirect product* of topological groups G_1 and G_2, denoted by $G_1 \otimes_\sigma G_2$, if $G = G_1 \times G_2$ as a topological space, if σ is a homomorphism (suitably continuous) of G_1 into the automorphism group of G_2 and, finally, if multiplication in $G = G_1 \otimes_\sigma G_2$

(1) Added in proof. The problem has been solved (see *Problèmes*, p. 387).

is defined by

$$(s, t)(s', t') = (ss', t\sigma(s)t').$$

If f is a (complex-valued) function on G_2, a way of extending f to G that will concern us for most of this paper is to define F on G by

$$F(s, t) = f(\sigma(s^{-1})t).$$

First we must know when the assumption that f is r.u.c. (or l.u.c.) implies that F is r.u.c. (or l.u.c.).

PROPOSITION 1. *Suppose that for each neighbourhood W of the identity $e_2 \in G_2$ there is another neighbourhood W' of e_2 such that $\sigma(G_1)W' \subset W$. Then F is r.u.c. if f is r.u.c.*

Proof. Let $\varepsilon > 0$ be given. To get the required neighbourhood of $e \in G$, we first take a neighbourhood W of $e_2 \in G_2$ such that $|f(t_1) - f(t_2)| < \varepsilon$ whenever $t_1 t_2^{-1} \in W$ and note that

$$(s', t')(s, t)^{-1} = (s's^{-1}, \sigma(s')[(\sigma(s')^{-1}t')(\sigma(s^{-1})t)^{-1}]).$$

It follows that if W' is a neighbourhood of $e_2 \in G_2$ such that

$$\sigma(G_1)W' = \{\sigma(s)t \mid s \in G_1, \ t \in W'\} \subset W,$$

then any neighbourhood $V \times W'$ of $e \in G$ will do; for, if

$$(s', t')(s, t)^{-1} \in V \times W',$$

then $(\sigma(s')^{-1}t')(\sigma(s^{-1})t)^{-1} \in W$.

Remarks. 1. The hypothesis of the proposition will be satisfied if the group G_1 is compact or if G_2 has a system of $\sigma(G_1)$-invariant neighbourhoods. If $G = G_1 \otimes_\sigma G_2 = T \otimes_\sigma C$ is the Euclidean group of the plane, then G satisfies both these conditions.

2. The function F of the proposition need not be l.u.c. even if f is. For example, if f is any non-trivial continuous character on the subgroup (isomorphic to) C of $T \otimes_\sigma C$ (characters being u.c.), then the corresponding function F is not l.u.c.

3. If $G = \boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R}$ is the affine group of the line and $f(x) = e^{ix}$, $x \in \boldsymbol{R}$, then F, although continuous, is neither l.u.c. nor r.u.c. Here $G_1 = \boldsymbol{R}^+$ is not compact, of course.

THEOREM 1. *Suppose $G = G_1 \otimes_\sigma G_2$ with G_1 compact. If f is a (minimal) {point-distal} [distal] function on G_2, then F is a (minimal) {point-distal} [distal] function on G.*

Proof. Suppose f is minimal and suppose H_1 is the pointwise limit of a net $\{R_{(s_\alpha, t_\alpha)}F\}$, i.e., for $(s, t) \in G$,

$$R_{(s_\alpha, t_\alpha)}F(s, t) = f(\sigma(ss_\alpha)^{-1}(t\sigma(s)t_\alpha)) = f(\sigma(ss_\alpha)^{-1}t\sigma(s_\alpha^{-1})t_\alpha) \to H_1(s, t).$$

Without loss of generality we can assume that

$$s_\alpha \to s_1 \in G_1 \quad \text{and} \quad \sigma(s_\alpha^{-1})t_\alpha \to x_1 \in UG_2.$$

Then, for $t \in G_2$,

$$\lim_\alpha f(t\sigma(s_\alpha^{-1})t_\alpha) = \hat{f}(tx_1)$$

(where, for $t \in G_2$, we are using the same symbol to denote the canonical image of t in UG_2, and also $\hat{f}$ denotes the continuous extension of f to UG_2). Continuing, we write $h_1(t) = \hat{f}(tx_1)$ and have

$$H_1(s, t) = h_1(\sigma(ss_1)^{-1}t),$$

using the joint continuity of multiplication on $G_2 \times UG_2$ (see [2], Theorem III.5.5). If $\{t_\beta\} \subset G_2$ is a net for which $\lim_\beta R_{t_\beta} h_1 = f$ (pointwise on G_2), then

$$R_{(s_1^{-1}, t_\beta)} H_1(s, t) = h_1(\sigma(s)^{-1} tt_\beta) \to f(\sigma(s^{-1})t) = F(s, t)$$

for all $(s, t) \in G$. Hence F is minimal.

Now suppose f is distal. Suppose that H_1 arises from the net $\{(s_\alpha, t_\alpha)\}$ as above, that H_2 arises analogously from the net $\{(s_v, t_v)\}$ and that $\{(s_\gamma, t_\gamma)\}$ is a net such that, for i equal to 1 or 2 and $(s, t) \in G$,

$$\begin{aligned} R_{(s_\gamma, t_\gamma)} H_i(s, t) &= h_i(\sigma(ss_\gamma s_i)^{-1}(t\sigma(s)t_\gamma)) \\ &= h_i(\sigma(ss_\gamma s_i)^{-1} t\sigma(s_\gamma s_i)^{-1} t_\gamma) \to H_3(s, t) = h_{i+2}(\sigma(ss_3 s_i)^{-1} t), \end{aligned}$$

where $h_i(t\sigma(s_\gamma s_i)^{-1} t_\gamma) \to h_{i+2}(t)$ for all $t \in G_2$ and $s_\gamma \to s_3$. We must show that $H_1 = H_2$. Since

$$h_3(\sigma(ss_3 s_1)^{-1} t) = h_4(\sigma(ss_3 s_2)^{-1} t)$$

for all $(s, t) \in G$, we have

$$h_3(\sigma(ss_1)^{-1} t) = h_4(\sigma(ss_2)^{-1} t)$$

for all $(s, t) \in G$. Now, fix $s \in G_1$. For i equal to 1 or 2, the function $t \to h_i(\sigma(ss_i)^{-1} t)$ is in $D(G_2)$ and

$$\begin{aligned} h_{i+2}(\sigma(ss_i)^{-1} t) &= \lim_\gamma h_i(\sigma(ss_i)^{-1} t\sigma(s_\gamma s_i)^{-1} t_\gamma) \\ &= \lim_\gamma h_i(\sigma(ss_i)^{-1}(t\sigma(s_\gamma s^{-1})^{-1} t_\gamma)). \end{aligned}$$

Thus, for i equal to 1 or 2, the function $t \to h_i(\sigma(ss_i)^{-1} t)$ on G_2 right translates via the net $\{\sigma(s_\gamma s^{-1})^{-1} t_\gamma\} \subset G_2$ to the same function

$$t \to h_3(\sigma(ss_1)^{-1} t) = h_4(\sigma(ss_2)^{-1} t).$$

Hence

$$H_1(s, t) = h_1(\sigma(ss_1)^{-1} t) = h_2(\sigma(ss_2)^{-1} t) = H_2(s, t) \quad \text{for all } (s, t) \in G,$$

as required.

The proof that F is point-distal, if f is, is similar to the proof in the previous paragraph.

Remarks. 1. If $G = T \otimes_\sigma C$ is the Euclidean group of the plane and f is a non-trivial continuous character on C, then the function F on G is not left distal, as it is not l.u.c. However, F is left minimal on G_d (see the next theorem).

2. If $G = \boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R}$ is the affine group of the line, where $\boldsymbol{R}^+$ is not compact, and if $f(x) = e^{ix}$, $x \in \boldsymbol{R}$, then there is a net of right translates of F converging to the constant function 1: F is not even minimal on G_d. However, F is left minimal on G_d (see the next theorem).

3. The distal functions on a group can separate the points of the group even though the almost periodic functions do not; this is the case on $T \otimes_\sigma C$ (see [11] and [12]). At this point, it is appropriate to mention a result of Ellis [3], which asserts that for discrete groups G the minimal functions always separate the points; Veech [14] proved this result for locally compact groups. Since an almost automorphic function cannot separate points not separated by the almost periodic functions (among other reasons), our method cannot extend an almost automorphic f on G_2 to an almost automorphic F on $G = G_1 \otimes_\sigma G_2$ in general; for example, when $G = T \otimes_\sigma C$. (See [9] for definitions and basic facts about almost automorphic functions.)

In view of the remoteness from being minimal of the extension F of Remark 2, the following theorem comes as a bit of surprise.

THEOREM 2. *Suppose G is a semidirect product of groups, $G = G_1 \otimes_\sigma G_2$. If f is left point-distal on G_2, then F is left minimal on the discrete group G_d.*

Proof. Let LG_2 be the left topological compactification of G_2 arising from the bounded l.u.c. functions on G_2. LG_2 is a left topological semigroup with properties analogous to those of the right topological compactification UG_2 coming from $U(G_2)$. (See [2] for all necessary details.) In particular, the characterization of point-distal functions in [8] (Proposition 1.1) implies that if ν is any minimal idempotent in LG_2, then

$$\hat{f}(\nu t) = \hat{f}(t) = f(t)$$

for all $t \in G_2$. (Here we are identifying G_2 with its canonical image in LG_2, and $\hat{f}$ is the continuous extension of f to LG_2.) Since $\lambda_\delta \to \lambda$ in LG_2 implies $\nu\lambda_\delta \to \nu\lambda$, we have, in fact, $\hat{f}(\nu\lambda) = \hat{f}(\lambda)$ for all $\lambda \in LG_2$. Also, any continuous automorphism of G_2, e.g., $\sigma(s)$, extends uniquely by continuity to an automorphism of LG_2 (by a comment in the Preliminaries) and must map minimal idempotents onto minimal idempotents. Thus

$$\hat{f}(\nu\sigma(s)\lambda) = \hat{f}(\sigma(s)(\nu\lambda))$$

for any $s \in G_1$, $\lambda \in LG_2$ and minimal idempotent $\nu \in LG_2$.

Suppose now that

$$L_{(s_\alpha, t_\alpha)} F(s, t) = f\big(\sigma(s^{-1})(\sigma(s_\alpha^{-1}) t_\alpha t)\big) \to \hat{f}\big(\sigma(s^{-1})(\mu t)\big) = H(s, t),$$

where $\sigma(s_\alpha^{-1}) t_\alpha \to \mu \in LG_2$. If ν is any minimal idempotent in LG_2, we need a net $\{t_\beta\} \subset G_2$ such that $\nu\mu t_\beta \to \nu$; such a net exists because $\nu\mu G_2$ is dense in the minimal right ideal νLG_2. It follows that

$$L_{(e, t_\beta)} H(s, t) = \hat{f}\big(\sigma(s^{-1})(\nu\mu t_\beta t)\big) \to \hat{f}\big(\sigma(s^{-1})(\nu t)\big)$$
$$= f\big(\sigma(s^{-1}) t\big) = F(s, t),$$

as required.

Remarks. 1. The examples in the remarks following Theorem 1 show that F (as in Theorem 2) need not be l.u.c.

2. We suspect that one ought to be able to find a left minimal (but not left point-distal) f on some G_2 such that F is not left minimal on $G_1 \otimes_\sigma G_2$. One ought to be able to do this on $T \otimes_\sigma C$ or $\boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R}$. **(P 1347)**

The abysmal failure of the extension method used above to extend characters on $\boldsymbol{R}$ to even minimal functions on $\boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R}$ (see Remark 3 following Proposition 1 and Remark 2 following Theorem 2) prompted us to devise the following

THEOREM 3. *Let $G = G_1 \otimes_\sigma G_2$ and suppose G_1 has a subgroup G_1' with a cross-section C for right cosets such that the identity of G_1 is in the interior of C and $\bar{C}$ is compact. Then each distal f on G_2 extends to a minimal F' on G.*

Proof. The properties of C ensure that we can get a u.c. function k on G_1 such that $0 \leqslant k \leqslant 1$, $k = 1$ at the identity of G_1, and $k = 0$ off C. For $(s, t) \in G$, write

$$H(s, t) = k(s) f\big(\sigma(s^{-1}) t\big).$$

Then H is r.u.c. on G and it follows that F', defined for $(s, t) \in G$ by

$$F'(s, t) = \sum_{s' \in G_1'} R_{(s', e)} H(s, t),$$

is also r.u.c. on G. (The hypothesis that C is a cross-section for right cosets implies that at most one of the functions $R_{s'} k$, $s' \in G_1'$, can be non-zero at any given point in G_1.)

Suppose now that $\{(s_\alpha, t_\alpha)\} \subset G$, that, for each α, $s_\alpha = r_\alpha s_\alpha'$ with $r_\alpha \in C$ and $s_\alpha' \in G_1'$, and that

$$R_{(s_\alpha, t_\alpha)} F'(s, t) = \sum_{s'} H\big(s s_\alpha s', t\sigma(s) t_\alpha\big)$$
$$= \sum_{s'} k(s r_\alpha s') f\big(\sigma(s r_\alpha s')^{-1} t \sigma(r_\alpha s')^{-1} t_\alpha\big)$$
$$\to \sum_{s'} k(s r_1 s') h_1^{s'}\big(\sigma(s r_1 s')^{-1} t\big) = H_1(s, t),$$

where $r_\alpha \to r_1 \in \bar{C}$, $\sigma(r_\alpha)^{-1} t_\alpha \to \mu_1 \in DG_2$, and

$$h_1^{s'}(t) = \lim_\alpha f\left(t\sigma(s')^{-1}\sigma(r_\alpha)^{-1} t_\alpha\right) = \hat{f}\left(t\sigma(s')^{-1}\mu_1\right) \quad \text{for all } t \in G_2.$$

(Again, for $t \in G_2$ we use the same symbol for its canonical image in DG_2; $\hat{f}$ denotes the continuous extension of f to DG_2.) Let $\{t_\beta\}$ be a net in G_2 such that $t_\beta \to \mu_1^{-1} \in DG_2$. Then $\sigma(s)(t_\beta \mu_1) \to e$ for all $s \in G_1$ and

$$\begin{aligned} R_{(r_1^{-1}, t_\beta)} H_1(s, t) &= \sum_{s'} k(ss') h_1^{s'}\left(\sigma(ss')^{-1}\left(t\sigma(s) t_\beta\right)\right) \\ &= \sum_{s'} k(ss')\hat{f}\left(\sigma(ss')^{-1} t\sigma(s')^{-1} t_\beta \sigma(s')^{-1}\mu_1\right) \\ &\to \sum_{s'} k(ss')\hat{f}\left(\sigma(ss')^{-1} t\right) = F'(s, t) \end{aligned}$$

as required, and we are done.

Remarks. The method of the last theorem extends the character f: $y \to e^{iy}$ on $\boldsymbol{R}$ at least to a minimal function on $\boldsymbol{R}^+ \otimes_\sigma \boldsymbol{R} = G_1 \otimes_\sigma G_2$ via the subgroup $\{2^n | \, n \in \boldsymbol{Z}\} \subset \boldsymbol{R}^+$ and a function on $\boldsymbol{R}^+$ with support in the cross-section $(1/\sqrt{2}, \sqrt{2}]$. The extension F', however, is not point-distal. (At this stage of the paper, one verifies readily that the extension F' of f satisfies $R_{(2^m, 0)} F' = F'$ for all $m \in \boldsymbol{Z}$ and that

$$R_{(2^m, 0)} R_{(1, \pi)} F' \to F'$$

as $m \to \infty$.)

As in the remarks preceding Theorem 3, we suspect minimal, even point-distal, functions will exist on many G_2's for which the extension F' will not be minimal on $G_1 \otimes_\sigma G_2$.

In a letter Professor T.-S. Wu has pointed out that if G_2 is a syndetic normal subgroup of G, then every (minimal) {point-distal} [distal] flow (G_2, X) can be embedded in a (minimal) {point-distal} [distal] flow (G, Y); see F. Hahn, *Some embeddings, recurrence properties, and the Birkhoff–Markov theorem for transformation groups*, Duke Math. J. 27 (1960), pp. 513–525. (The relevance of this to Theorem 1 is obvious.) He also pointed out the relevance to some of this work of a construction of Furstenberg (II.5.5 in *Proximal Flows*, by S. Glasner, Springer Lecture Notes in Math. 517, New York 1976). Further, Professor Wu indicated that the non-trivial characters on $\boldsymbol{R}$ cannot be extended to distal functions on $\boldsymbol{R}^+ \otimes \boldsymbol{R}$; this conclusion also follows from H. Abel's paper, *Which groups act distally?*, Ergodic Theory and Dynamical Systems 3 (1983), pp. 167–185. The question remains: can the characters on $\boldsymbol{R}$ be extended to point-distal functions on $\boldsymbol{R}^+ \otimes \boldsymbol{R}$? **(P 1348)**

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