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OPERATORS WHICH COMMUTE WITH CONVOLUTIONS ON SUBSPACES OF $L_\infty(G)$

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1. Introduction. Let G be a locally compact group. Let T be a bounded linear operator from $L_\infty(G)$ into $L_\infty(G)$ which commutes with left (respectively, right) translations. Then, as known (see [2], p. 300), there exists a regular Borel measure μ on G such that

$$T(f) = f*\mu \quad \text{(respectively, } T(f) = \mu*f\text{)}$$

for each continuous complex-valued function f on G vanishing at infinity. Furthermore, if T is in addition weak*-weak* continuous, then the equation holds for all f in $L_\infty(G)$.

A linear operator $T\colon L_\infty(G)\to L_\infty(G)$ is said to *commute with convolution from the left* (respectively, *right*) if

$$T(\varphi*f) = \varphi*T(f) \quad \text{(respectively, } T(f*\tilde{\varphi}) = T(f)*\tilde{\varphi}\text{)}$$

for each $\varphi\in L_1(G)$ and each $f\in L_\infty(G)$. It is not hard to see (Lemma 2) that any bounded linear operator $T\colon L_\infty(G)\to L_\infty(G)$ which commutes with convolution from the left (respectively, right) also commutes with the left (respectively, right) translations on $L_\infty(G)$. However, the converse is false unless G is discrete or T is weak*-weak* continuous even when the group G is assumed to be compact and Abelian (see [5], Theorem 2, and [10], Theorem 4.1).

Let $\mathrm{UBC}_r(G)$ (respectively, $\mathrm{UBC}_l(G)$) denote the complex-valued right (respectively, left) uniformly continuous bounded functions on G as defined in [7], p. 275. Curtis and Figà-Talamanca have proved in [3], p. 169-185, Theorem 3.3, that the dual Banach space $\mathrm{UBC}_r(G)^*$ is isometric and isomorphic to the space of bounded linear operators from $L_\infty(G)$ into $L_\infty(G)$ commuting with convolution from the left.

One of the purposes of this paper* is to extend Curtis and Figà-Talamanca's result to certain subspaces of $L_\infty(G)$. More precisely, let X be

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a topologically left (respectively, right) invariant and topologically left (respectively, right) introverted closed subspace of $L_\infty(G)$ as defined in [12], p. 356. In this paper we show that there exists a closed subspace Y of $\mathrm{UBC}_r(G)$ (respectively, $\mathrm{UBC}_l(G)$) which is also topologically left (respectively, right) introverted, and the Banach algebra Y^* with the Arens product is isometric and algebra isomorphic to the algebra of bounded linear operators from X into X commuting with convolution from the left (respectively, right).

We also show that the space of bounded linear operators from $\mathrm{UBC}_r(G)$ into $\mathrm{UBC}_r(G)$ commuting with convolution from the left coincides with the space of bounded linear operators from $\mathrm{UBC}_r(G)$ into $\mathrm{UBC}_r(G)$ commuting with left translations. Furthermore, it is also isometric and isomorphic to $\mathrm{UBC}_r(G)^*$. A similar assertion is true also for $\mathrm{UBC}_l(G)$ for operators commuting with convolution from the right.

Hively [9], Theorem 5.4, has proved, using the existence of lifting commuting with translations on a locally compact group, that if G is a locally compact non-compact Abelian group, then there exists a bounded linear operator T from $L_\infty(G)$ into $L_\infty(G)$ which commutes with left translations, and T is not weak*-weak* continuous. Actually, Hively's results remain true even for a compact Abelian group G (or, more generally, amenable as discrete) which is non-discrete. Indeed, for any such group G there exists a left invariant mean m on $L_\infty(G)$ which is not a topological left invariant mean (see [5], Theorem 2, or [10], Theorem 4.1) and the operator $T(f) = m(f)\cdot 1$ commutes with left translations but T is not weak*-weak* continuous.

We prove in this paper that if G is any locally compact non-compact group, then there exists a bounded linear operator T from $L_\infty(G)$ into $L_\infty(G)$ which commutes with convolutions from the left (hence with left translations) and T is not weak*-weak* continuous. Conversely, if G is compact, then any bounded linear operator T from $L_\infty(G)$ into $L_\infty(G)$ commuting with convolution from the left is weak*-weak* continuous. Our proof is elementary, and does not depend on the lifting theorem.

It is our pleasure to thank Professor M. Rieffel for bringing our attention to the work of Hively [9]. It is from this source that we learn of the result of Curtis and Figà-Talamanca [3], p. 169-185, which we were unaware of when preparing the first version of this paper.

2. Some notation. Let G be a locally compact group with a fixed left Haar measure dx. The modular function Δ on G and spaces $L_1(G)$ and $L_\infty(G)$ are defined exactly as in [7].

Given $\varphi \in L_1(G)$ and $f \in L_\infty(G)$, the functions $\varphi * f$ and $f * \tilde{\varphi}$ on G defined by

$$(\varphi * f)(x) = \int \varphi(y) f(y^{-1}x)\,dy \quad \text{and} \quad (f * \tilde{\varphi})(x) = \int f(y)\tilde{\varphi}(y^{-1}x)\,dy$$

(where $\tilde{\varphi}(x) = \varphi(x^{-1})$) are in $C(G)$, the space of bounded continuous complex-valued functions on G. Also, $\varphi * f \in \mathrm{UBC_r}(G)$ and $f * \tilde{\varphi} \in \mathrm{UBC_l}(G)$ (see [7], p. 295).

Finally, if $x \in G$, we define the left and right translation operators on $L_\infty(G)$ at x by $(l_x f)(y) = f(xy)$ and $(r_x f)(y) = f(yx)$ for each $y \in G$ and each $f \in L_\infty(G)$.

3. Technical lemmas. Let X be a closed topologically left (respectively, right) invariant subspace of $L_\infty(G)$, i.e. $\varphi * X \subseteq X$ (respectively, $X * \tilde{\varphi} \subseteq X$) for each $\varphi \in L_1(G)$. Following Wong [12], p. 356, we say that X is *topologically left* (respectively, *right*) *introverted* if, for each $m \in L_\infty(G)^*$ and $f \in X$, the functional on $L_1(G)$ defined by

$$\varphi \to \left\langle \frac{1}{\Delta}\tilde{\varphi} * f, m \right\rangle \quad (\text{respectively, } \varphi \to \langle f * \tilde{\varphi}, m \rangle)$$

is also an element in X.

LEMMA 1. *Let X be a topologically left invariant and left introverted subspace of $L_\infty(G)$. Let $Y = L_1(G) * X$. Then*

(a) *Y is a closed subspace of $\mathrm{UBC_r}(G)$ which is also topologically left invariant and introverted.*

(b) *For each $m \in Y^*$, $\|m_{\mathrm{L}}\| = \|m\|$, where $m_{\mathrm{L}}\colon X \to X$ is defined by*

$$\langle m_{\mathrm{L}}(f), \varphi \rangle = \left\langle \frac{1}{\Delta}\tilde{\varphi} * f, m \right\rangle.$$

(c) *$(m \odot n)_{\mathrm{L}} = m_{\mathrm{L}} \circ n_{\mathrm{L}}$, where $(m \odot n)(h) = m(n_{\mathrm{L}}(h))$ for each $h \in Y$.*

Proof. (a) It follows from [7], p. 295, and the Cohen Factorization Theorem (see [8], p. 268) that Y is a closed subspace of $\mathrm{UBC_r}(G)$. Clearly, Y is topologically left invariant. To see that Y is introverted, let $m \in L^\infty(G)$, $\psi \in L_1(G)$, and $f \in X$. Then

$$\left\langle \frac{1}{\Delta}\tilde{\varphi} * (\psi * f), m \right\rangle = \langle \psi * h, \varphi \rangle,$$

where h is the functional on $L_1(G)$ defined by the map

$$\varphi \to \left\langle \frac{1}{\Delta}\tilde{\varphi} * f, m \right\rangle.$$

Since X is introverted, $h \in X$. It follows that Y is also introverted.

(b) Clearly, $\|m_L\| \leqslant \|m\|$. To see the reverse inequality, let $\{\varphi_a\}$ be an approximate identity of $L_1(G)$ and $\|\varphi_a\| \leqslant 1$. Then $(1/\Delta)\varphi_a$ is also an approximate identity of $L_1(G)$ (see [12], Lemma 3.3 (a)). Let $f \in Y$; then

$$\left\|\frac{1}{\Delta}\varphi_a * f - f\right\|_\infty \to 0$$

(see [8], p. 286). Hence

$$\|m_L(f)\|_\infty \geqslant |m_L(f)(\varphi_a)| = \left|m\left(\frac{1}{\Delta}\varphi_a * f\right)\right| \quad \text{for each } a.$$

Consequently, $\|m_L(f)\|_\infty \geqslant |m(f)|$. Since f is arbitrary, we have $\|m_L\| \geqslant \|m\|$.

(c) Let $f \in X$ and $\varphi \in L_1(G)$; then

$$\langle (m\odot n)_L(f), \varphi\rangle = \left\langle \frac{1}{\Delta}\tilde{\varphi}*f, m\odot n\right\rangle = \left\langle n_L\left(\frac{1}{\Delta}\tilde{\varphi}*f\right), m\right\rangle$$
$$= \left\langle \frac{1}{\Delta}\tilde{\varphi}*n_L(f), m\right\rangle = \langle m_L(n_L(f)), \varphi\rangle$$

using [12], Lemma 4.3 (b).

Remark 1. It is easy to see that an analogous statement also holds for a topologically right invariant and right introverted subspace X of $L_\infty(G)$ with $Y = X*L_1(G)^{\sim}$, and m_R: $X \to X$ defined by

$$\langle m_R(f), \varphi\rangle = \langle f*\tilde{\varphi}, m\rangle.$$

If $X = L_\infty(G)$, then

$$L_1(G)*X = \mathrm{UBC}_r(G) \quad \text{and} \quad X*L_1(G)^{\sim} = \mathrm{UBC}_l(G)$$

as known from [8], p. 283.

LEMMA 2. *Let T be a bounded linear operator from $L_\infty(G)$ into $L_\infty(G)$. Let $f \in L_\infty(G)$.*

(a) *If $T(\varphi*f) = \varphi*T(f)$ for each $\varphi \in L_1(G)$, then $T(l_x f) = l_x T(f)$ for each $x \in G$.*

(b) *If $T(f*\tilde{\varphi}) = T(f)*\tilde{\varphi}$ for each $\varphi \in L_1(G)$, then $T(r_x f) = r_x T(f)$ for each $x \in G$.*

Proof. (a) Let $\{\varphi_a\}$ be an approximate identity of $L_1(G)$. Then, for each h in $L_\infty(G)$, the net $\{(1/\Delta)\tilde{\varphi}_a * h\}$ converges to h in the weak* topology of $L_\infty(G)$ (see [12], Lemma 3.3 (b)). Hence, for each $x \in G$,

$$T(l_x f) = \mathrm{w}^*\text{-}\lim_a T\left(\frac{1}{\Delta}\tilde{\varphi}_a * l_x f\right)$$

$$= \mathrm{w}^*\text{-}\lim_a \frac{1}{\Delta}(l_{x^{-1}}\varphi_a)^{\sim} * T(f) = \mathrm{w}^*\text{-}\lim_a \frac{1}{\Delta}\tilde{\varphi}_a * l_x T(f) = l_x T(f).$$

Part (b) can be proved similarly.

Let $f \in \mathrm{UBC_r}(G)$, and $m \in \mathrm{UBC_r}(G)^*$. Define a function $m_1(f)$ on G by

$$m_1(f)(x) = m(l_x f) \quad \text{for each } x \in G.$$

Then, as known (see [7], p. 275), $m_1(f)$ is also a function in $\mathrm{UBC_r}(G)$. Similarly, we define for each $f \in \mathrm{UBC_l}(G)$ and each $m \in \mathrm{UBC_l}(G)^*$ a function $m_r(f)$ in $\mathrm{UBC_r}(G)^*$ by

$$m_r(f)(x) = m(r_x f) \quad \text{for each } x \in G.$$

The following lemma relates m_L with m_1, and m_R with m_r, where m_L and m_R are defined as in Lemma 1 and Remark 1 with $X = L_\infty(G)$.

LEMMA 3. *If $f \in \mathrm{UBC_r}(G)$, then $m_L(f) = m_1(f)$. Also, if $f \in \mathrm{UBC_l}(G)$, then $m_R(f) = m_r(f)$.*

Proof. It suffices to show that $\langle m_L(f), \varphi\rangle = \langle m_1(f), \varphi\rangle$ for each $\varphi \in L_1(G)$ with compact support. Let φ be such a fixed element in $L_1(G)$. If m is the point evaluation at some point $a \in G$, then

$$\langle m_L(f), \varphi\rangle = \left(\frac{1}{\Delta}\tilde{\varphi} * f\right)(a) = \int \frac{1}{\Delta}\tilde{\varphi}(x) f(x^{-1}a)\,dx$$

$$= \int \varphi(x) m(l_x f)\,dx = \langle m_1(f), \varphi\rangle.$$

If $m \in \mathrm{UBC_r}(G)^*$, $m \geqslant 0$, and $\|m\| = 1$, then there exists a net m_a of convex combinations of point evaluations such that $m_a(h)$ converges to $m(h)$ for each $h \in \mathrm{UBC_r}(G)$. Then

$$\langle m_{a_L}(f), \varphi\rangle = \left\langle \frac{1}{\Delta}\tilde{\varphi} * f, m_a\right\rangle$$

converges to

$$\left\langle \frac{1}{\Delta}\tilde{\varphi} * f, m\right\rangle = \langle m_L(f), \varphi\rangle.$$

On the other hand, if N is the support of φ, then the set $\{l_x f;\, x \in N\}$ is a norm compact subset of $\mathrm{UBC_r}(G)$. It follows from the Mackey-Arens theorem that the net $m_a(l_x f)$ converges to $m(l_x f)$ uniformly on N. Hence

$$\langle m_{a_1}(f), \varphi\rangle = \int_N \varphi(x) m_a(l_x f)\, dx$$

converges to

$$\int_N \varphi(x) m(l_x f)\, dx = \langle m_1(f), \varphi\rangle.$$

Since every linear functional in $\mathrm{UBC_r}(G)^*$ is the linear combination of positive ones with norm one, our assertion is proved.

The proof for functions in $\mathrm{UBC_l}(G)$ is similar.

Remark 2. Lemma 3 implies immediately the following well-known result (see [4], Theorem 4, and [6], Lemma 2.2.2): If $f \in \mathrm{UBC_r}(G)$ and $m \in \mathrm{UBC_r}(G)^*$ are such that $m(l_x f) = m(f)$ for each $x \in G$, then $m(\varphi * f) = m(f)$ for each positive φ in $L_1(G)$ with norm one.

4. The main theorems. If Y is a topologically left (respectively, right) introverted closed subspace of $L_\infty(G)$, then Y^* becomes a Banach algebra with the Arens product $\odot$ as defined in Lemma 1 and Remark 1 (see [12], p. 354).

THEOREM 1. *Let X be a topologically left (respectively, right) invariant and topologically left (respectively, right) introverted closed subspace of $L_\infty(G)$. Let T be a bounded linear operator from X into X. Then the following statements are equivalent:*

(a) *T commutes with convolution from the left (respectively, right).*

(b) *There exists a bounded linear functional m on $L_1(G)*X$ (respectively, $X*L_1(G)^{\sim}$) such that $T = m_{\mathrm{L}}$ (respectively, $T = m_{\mathrm{R}}$).*

Consequently, the algebra of bounded linear operators from X into X commuting with convolution from the left (respectively, right) is isometric and algebra isomorphic to the Banach algebra $\big(L_1(G)*X\big)^*$ (respectively, $\big(X*L_1(G)\big)^*$) with the Arens product.

Proof. We prove the theorem for operators commuting with convolution from the left. The proof for operators commuting with convolution from the right is similar.

That (b) implies (a) follows from [12], Lemma 4.3 (B).

To prove that (a) implies (b), let $\{\varphi_a\}$ be a bounded net of approximate identity in $L_1(G)$. Then the net $\{T^*(\varphi_a)\}$, restricted to $Y = L_1(G)*X$, is bounded in Y^*. Let m be a weak*-cluster point of $\{T^*(\varphi_a)\}$. By passing to a subnet, if necessary, we may even assume that $T^*(\varphi_a)$ converges to m in the weak* topology of Y^*. Let $f \in L_\infty(G)$. Then for each $\varphi \in L_1(G)$

we have

$$m_L(f)(\varphi) = \lim_a \left\langle \frac{1}{\Delta}\varphi * f, T^*(\varphi_a)\right\rangle = \lim_a \left\langle T\left(\frac{1}{\Delta}\varphi * f\right), \varphi_a\right\rangle$$
$$= \lim_a \left\langle \frac{1}{\Delta}\varphi * T(f), \varphi_a\right\rangle = \lim_a \langle T(f) * \varphi_a, \varphi\rangle = \langle T(f), \varphi\rangle.$$

Hence $m_L(f) = T(f)$.

The final assertion follows immediately from Lemma 1.

Remark 3. The linear isometry between $(L_1(G) * X)^*$ when $X = L_\infty(G)$ and the space of all bounded linear operators from X into X commuting with convolution from the left is due to Curtis and Figà-Talamanca [3], p. 169-185, Theorem 3.3.

THEOREM 2. *Let G be any locally compact group. If G is non-compact, then there exists a bounded linear operator from $L_\infty(G)$ into $L_\infty(G)$ commuting with convolution from the left which is not weak*-weak* continuous.*

Proof. Assume that G is non-compact and σ is a compact subset of G, and let a_σ be an element in G but not in σ. Let m be a weak*-cluster point $\mathrm{UBC_r}(G)^*$ of the net $\{\delta_{a_\sigma}\}$ of point evaluation functionals at a_σ. Let $T = m_L$. Then T commutes with convolution from the left by Theorem 1. But T is not weak*-weak* continuous. For otherwise, if $\varphi \in C_c(G)$, where $C_c(G)$ denotes all complex continuous functions on G with compact support, then $T^*(\varphi) \in L_1(G)$. Hence, if $g \in C_c(G)$, then (by [7], Theorem 20.16)

$$\langle T^*(\varphi), g\rangle = \langle \varphi, m_L(g)\rangle = \left\langle \frac{1}{\Delta}\tilde{\varphi} * g, m\right\rangle = 0.$$

Consequently, $T^*(\varphi) = 0$ for each $\varphi \in C_c(G)$. It follows that $T^* = 0$ by density of $C_c(G)$ in $L_1(G)$. This is impossible, since $\|m\| = \|T\| = 1$ by Theorem 1.

The converse follows from Theorem 1 with the observation that if G is compact, then, for each m in $\mathrm{UBC_r}(G)^* = C(G)^* = M(G)$, and $f \in L_\infty(G)$,

$$\langle m_L(f), \varphi\rangle = \left\langle \frac{1}{\Delta}\tilde{\varphi} * f, m\right\rangle = \langle f, \varphi * m\rangle \quad \text{for each } \varphi \in L_1(G).$$

Remark 4. Theorem 2 and Lemma 2 together imply Hively's result (see [9], Theorem 5.4).

THEOREM 3. *Let G be any locally compact group and let T be any bounded linear operator from $\mathrm{UBC_r}(G)$ into $\mathrm{UBC_r}(G)$ (respectively, from $\mathrm{UBC_l}(G)$ into $\mathrm{UBC_l}(G)$). Then the following statements are equivalent:*

(a) *T commutes with convolution from the left (respectively, right).*

(b) *T commutes with left (respectively, right) translations.*

(c) *There exists a bounded linear functional m on* $\mathrm{UBC}_r(G)$ (*respectively,* $\mathrm{UBC}_l(G)$) *such that* $T = m_L = m_l$ (*respectively,* $T = m_R = m_r$).

Consequently, the space of bounded linear operators on $\mathrm{UBC}_r(G)$ (*respectively,* $\mathrm{UBC}_l(G)$) *commuting with left* (*respectively, right*) *translations is isometric and isomorphic to* $\mathrm{UBC}_r(G)^*$ (*respectively,* $\mathrm{UBC}_l(G)^*$).

Proof. That (a) implies (b) follows from Lemma 2.

If (b) holds, let δ_e denote the functional on $\mathrm{UBC}_r(G)$ defined by $\delta_e(f) = f(e)$ for each $f \in \mathrm{UBC}_r(G)$, where e is the identity of G. Let $m = T^*(\delta_e)$. Then for each $f \in \mathrm{UBC}_r(G)$ we have

$$(m_l f)(x) = \langle l_x f, T^*(\delta_e)\rangle = \langle T(l_x f), \delta_e\rangle = \langle l_x T(f), \delta_e\rangle = T(f)(x).$$

Hence $m_l = T$. By Lemma 3, $T = m_L$. Then (c) implies (a), and the lest of the assertion follows from Theorem 2, since $\mathrm{UBC}_r(G)$ is topologically reft introverted (see [12], Lemma 6.2).

Let X be a topologically left invariant closed subspace of $L_\infty(G)$ containing the constant functions. A linear functional m on X is called a *topological left invariant mean* if $m(\varphi * f) = m(f)$ for each positive element φ on $L_1(G)$ of norm one (see [6], p. 24).

THEOREM 4. *Let X be a topological left invariant closed subspace of $L_\infty(G)$ containing constants. Then the following statements are equivalent*:

(a) *X has a topological left invariant mean.*

(b) *There exists a weakly compact positive operator T from X into X of norm one commuting with convolution from the left.*

Proof. If (a) holds, and m is a topological left invariant mean on X, write $(Tf)(x) = m(f)$. Then T satisfies (b).

Conversely, if (b) holds, and $f \in X$, then $l_x T(f) = T(l_x f)$ for each $x \in G$ by Lemma 2. Hence $\{l_x T(f) = T(l_x f); x \in G\}$ is relatively compact in the weak topology of X. It follows that $T(f)$ is a weakly almost periodic function on G. Let $\mathrm{WAP}(G)$ denote the space of weakly almost periodic functions on G, and let γ be the unique left invariant mean on $\mathrm{WAP}(G)$ (see [6], p. 38). The space $\mathrm{WAP}(G)$ is topologically left invariant. By [6], the proof of Lemma 2.2.2, γ is also a topological left invariant mean on $\mathrm{WAP}(G)$. Write $m(f) = \gamma(Tf)$. Then m is a topological left invariant mean on X.

Remark 5. It is well known that a locally compact group G is compact if and only if there exists a non-zero weakly compact operator from $L_1(G)$ into $L_1(G)$ commuting with right (or left) translations (see the proof of Theorem 1 in [11], and [1], Theorem 5). Theorem 4 implies that a locally compact group G is amenable (i.e. $L_\infty(G)$ has a topological left invariant mean) if and only if there exists a non-zero positive weakly compact linear operator from $L_\infty(G)$ into $L_\infty(G)$ commuting with convolution from the left. In fact, using [6], Theorem 2.2.1, and a proof similar to that of Theorem 4, we can show that in order for G to be amenable it is sufficient that there exist a non-zero positive weakly compact linear operator T from $\mathrm{UBC}_r(G)$ into $\mathrm{UBC}_r(G)$ commuting with left translation. However, we do not know whether the condition that T be positive can be dropped.

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