

*ON THE COEFFICIENTS IN FOURIER-STIELTJES SERIES
WITH NORM-BOUNDED PARTIAL SUMS*

BY

JOHN J. F. FOURNIER (VANCOUVER)

1. Introduction. Given a measure μ on the unit circle \mathcal{T} denote its Fourier-Stieltjes coefficients by $(\hat{\mu}(n))_{n=-\infty}^{\infty}$, and the partial sums of the corresponding Fourier-Stieltjes series by $(S_N(\mu))_{N=0}^{\infty}$; given a subset E of the set \mathcal{Z} of all integers, denote the cardinality of E by $|E|$. See [10] for further conventions about notation.

Our goal here is to exhibit a function L , on the interval $(0, 1)$ and with values in \mathcal{Z} , with the property that if μ is a measure for which $\|S_N(\mu)\|_1 \leq 1$ for all N , then

$$|\{n \in \mathcal{Z} : |\hat{\mu}(n)| \geq b\}| \leq L(b)$$

for all b in $(0, 1)$. Pigno and Smith [15] showed that such functions L exist, by exhibiting one. Our method is a modification of theirs; it yields significantly smaller, but not optimal values for $L(b)$.

In a well-known paper [9], Helson showed that if

$$\sup_N \|S_N(\mu)\|_1 < \infty,$$

then $\hat{\mu}(n) \rightarrow 0$ as $|n| \rightarrow \infty$; the present paper* and [15] provide information, in terms of $\sup_N \|S_N(\mu)\|_1$, about the size of sets on which $\hat{\mu}(n)$ is bounded away from 0. It is known [17], [11] that there are such measures μ that are *not* absolutely continuous with respect to the Lebesgue measure $d\theta/2\pi$.

In Section 2, we state three lemmas and deduce our main result from them. In Section 3, we prove the lemmas, exhibit a specific function L as above, and comment on related matters.

2. Three lemmas. One of our main tools in Section 3 will be the classical theorem of Paley (see [2], p. 104) to the effect that if $(h_k)_{k=1}^{\infty}$ is a sequence of positive integers with $h_{k+1} \geq 2h_k$ for all k and if f is an integrable function on

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T with $\hat{f}(n) = 0$ for all $n < 0$, then

$$(|\hat{f}(0)|^2 + \sum_{k=1}^{\infty} |\hat{f}(h_k)|^2)^{1/2} \leq 2\|f\|_1.$$

With this in mind, we introduce the following terminology.

Definitions. Call a finite sequence $(h_k)_{k=1}^K$ of integers a *standard Hadamard sequence* if it is either strictly increasing or strictly decreasing and if $|h_{k+1} - h_k| \geq 2|h_k - h_{k-1}|$ for all k ; this property is unaffected by translation. Call the upper index K the *length* of the sequence $(h_k)_{k=1}^K$. Given a set M , let $u(M)$ be the supremum of the lengths of *increasing* standard Hadamard sequences in M , and let $d(M)$ be the supremum of the lengths of *decreasing* standard Hadamard sequences in M .

Example. If $M = \{2^n: 1 \leq n \leq N\}$, then $u(M) = N$, but $d(M) = 2$.

LEMMA 1. For each integer $j > 0$ there is an integer J such that every set M with more than J elements satisfies $u(M) + d(M) \geq j$.

Given a measure μ , let

$$s(\mu) = \sup_N \|S_N(\mu)\|_1.$$

Given a positive number b , let

$$D(\mu, b) = \{m > 0: |\hat{\mu}(m)| \geq b\};$$

finally, given a positive integer n , let

$$D_n(\mu, b) = D(\mu, b) \cap [n, 2n).$$

LEMMA 2. Let μ be a measure with $s(\mu) \leq 1$ and let $b > 0$. Then

- (i) $d(D(\mu, b)) \leq 4/b^2$;
- (ii) $u(D_n(\mu, b)) \leq 64/b^2$ for all n ;
- (iii) there is a number J depending only on b so that $|D_n(\mu, b)| \leq J$ for all n .

Finally, we need to deduce estimates on $D(\mu, b)$ from estimates on $D_n(\mu, b)$.

LEMMA 3. Every integer-valued function J on the interval $(0, 1)$ determines another such function I with the following property: if μ is a measure with $\|\mu\| \leq 1$ for which $|D_n(\mu, b)| \leq J(b)$ for all numbers b in $(0, 1)$ and all integers n , then $|D(\mu, b)| \leq I(b)$ for all such b .

We now restate and prove our main result.

THEOREM. Let μ be a nontrivial measure with $s(\mu) < \infty$ and let $0 < b < s(\mu)$. Then there is an integer L depending only on $b/s(\mu)$ so that

$$|\{n: |\hat{\mu}(n)| \geq b\}| \leq L.$$

Proof. Replacing μ by $\mu/s(\mu)$, we can reduce matters to the case where $s(\mu) = 1$. This done, we can apply part (iii) of Lemma 2 and Lemma 3 to obtain a function I for which $|D(\mu, b)| \leq I(b)$.

Let $\tilde{\mu}$ be the measure given by $\tilde{\mu}(E) = \mu(-E)$ for all E , and recall that $(\tilde{\mu})^\wedge(n) = \hat{\mu}(-n)$ for all n ; thus $s(\tilde{\mu}) = 1$, and also $|D(\tilde{\mu}, b)| \leq I(b)$. Since

$$\{n: |\hat{\mu}(n)| \geq b\} \subset D(\mu, b) \cup -D(\tilde{\mu}, b) \cup \{0\},$$

we get the desired conclusion with $L(b) = 2I(b) + 1$.

We will discuss the relation between b and L further, at the end of the next section.

3. Proofs and remarks. We first offer proofs of the lemmas using the most accessible forms of auxiliary results, and then we explain how to obtain better values for L . To save writing, we use the phrase "Hadamard sequence" when we really mean "standard Hadamard sequence".

Presumably, Lemma 1 is known, but it is easier to find proofs of it than references for it. We proceed by induction on j , observing first that the assertion of the lemma clearly holds when $j = 2$ with $J = 0$, and when $j = 4$ with $J = 1$. Suppose that it holds for all $j < j_0$, and for each such j let $J(j)$ be the largest integer for which there is a set M with J elements and with $u(M) + d(M) < j$.

Now let M be a set with $u(M) + d(M) < j_0$. Then M must be bounded from above and below, because, otherwise, at least one of $u(M)$ and $d(M)$ would be equal to ∞ . Assume, without loss of generality, that the smallest element of M is 0, and denote its largest element by r . Split M into halves by letting $M_1 = M \cap [0, r/2]$ and $M_2 = M \cap (r/2, r]$. Observe that $u(M_1) < u(M)$, because if $(h_k)_{k=1}^K$ is an increasing Hadamard sequence with values in M_1 , then the longer sequence obtained by letting $h_{k+1} = r$ is also an increasing Hadamard sequence with values in M . Similarly, $d(M_2) < d(M)$, and it is clear that $d(M_1) \leq d(M)$ and $u(M_2) \leq u(M)$. Thus $|M_1|$ and $|M_2|$ are both at most $J(j_0 - 1)$, and the assertion of the lemma holds when $j = j_0$ with

$$(3.1) \quad J(j_0) \leq 2J(j_0 - 1).$$

To prove Lemma 2, we observe by Paley's theorem that if $(h_k)_{k=1}^K$ is an increasing Hadamard sequence and f is a trigonometric polynomial with $\hat{f}(n) = 0$ for all $n < h_1$, then

$$(3.2) \quad \left(\sum_{k=1}^K |\hat{f}(h_k)|^2 \right)^{1/2} \leq 2 \|f\|_1,$$

and that this conclusion also holds if $(h_k)_{k=1}^K$ is a decreasing Hadamard sequence and $\hat{f}(n) = 0$ for all $n > h_1$. To obtain part (i) of the lemma, let

$(h_k)_{k=1}^K$ be a decreasing Hadamard sequence in the set $D(\mu, b)$, let $N = h_1$, and $f = S_N(\mu)$; then (3.2) yields $\sqrt{K} b \leq 2$, as required.

To deal with part (ii), let n be a positive integer and let $(h_k)_{k=1}^K$ be an increasing standard Hadamard sequence with values in $D_n(\mu, b)$. Replace n by h_1 ; then still $h_k \in D_n(\mu, b)$ for all k . Let P_n be the trigonometric polynomial for which $\hat{P}_n(m) = 0$ for all m outside the interval $(-n, 3n)$, while $\hat{P}_n(n) = 1$ and \hat{P}_n is linear on each of the intervals $[-n, n]$ and $[n, 3n]$. Since \hat{P}_n is a translate of the transform of a Fejér kernel, $\|P_n\|_1 = 1$. Let

$$f = (\mu - S_{n-1}(\mu)) * P_n;$$

then

$$\|f\|_1 \leq \|\mu\| + \|S_{n-1}(\mu)\|_1 \leq 2$$

because

$$\|\mu\| \leq \limsup_{N \rightarrow \infty} \|S_N(\mu)\|_1.$$

Now $\hat{f}(m) = 0$ for all $m < n = h_1$; also, $|\hat{f}(h_k)| > b/2$ for all k because $\hat{P}_n > 1/2$ on the interval $[n, 2n)$. Inequality (3.2) therefore yields $\sqrt{K} b/2 \leq 4$, as required.

Finally, the third part of Lemma 2 follows easily from the first two parts and Lemma 1.

In the proof of Lemma 3, we use the fact that if $(h_k)_{k=1}^K$ is a Hadamard sequence of positive integers and f is an integrable function with $\hat{f}(m) = 0$ for all positive integers m outside the set $\{h_k: k = 1, \dots, K\}$, then

$$(3.3) \quad \left(\sum_{k=1}^K |\hat{f}(h_k)|^2 \right)^{1/2} \leq C \|f\|_1,$$

where C is an absolute constant. This assertion can be proved by Kolmogorov's inequality, much as in [8] or [16], p. 226, and it follows also from M. Riesz's inequality as in [13]; we comment further on it in Remark 1 below.

We really only need a simple consequence of inequality (3.3), concerning measures whose coefficients are small at certain positive integers outside the range of the sequence h . By the proof of Proposition 1.3.12 of [7], there is a "Riesz" polynomial F , with $\|F\|_1 = 1$, such that $\hat{F}(h_k) \geq 1/2$ for all k , while $\hat{F}(m) = 0$ for all integers m outside the set

$$R(h) = \left\{ m = \sum_{k=1}^K \varepsilon_k h_k: \varepsilon_k \in \{-1, 0, 1\} \text{ for all } k \right\}.$$

Let $R'(h)$ be the set of positive integers in $R(h)$ that do not belong to the set $\{h_k\}_{k=1}^K$. Let μ be a measure with $\|\mu\| \leq 1$, let b and β be positive numbers, and suppose that $|\hat{\mu}(h_k)| \geq b$ for all k , while $|\hat{\mu}(m)| < \beta$ for all integers m in $R'(h)$. Then

$$(3.4) \quad \sqrt{K} b/2 < C(1 + \beta \cdot 3^{K/2}).$$

To verify this, just apply inequality (3.3) with

$$f(\theta) = \mu * F(\theta) - \sum_{m \in R'(h)} \hat{\mu}(m) \hat{F}(m) e^{im\theta}$$

for all θ , noting that the part that is subtracted above has L^2 -norm less than $\beta |R'(h)|^{1/2}$.

Now let J and μ be as in the hypotheses of Lemma 3 and let $b > 0$. Choose K so that $\sqrt{K} b \geq 4C$ and let $\beta = 3^{-K/2}$. Then inequality (3.4) fails, so that there is no Hadamard sequence $(h_k)_{k=1}^K$ with the properties listed before inequality (3.4). We complete the proof of the lemma by showing that if $|D(\mu, b)|$ were sufficiently large, then there would be such a sequence $(h_k)_{k=1}^K$.

First we construct a sequence $(h'_k)_{k=1}^{K'}$ as follows. Let h'_1 be the smallest element of $D(\mu, b)$; given $(h'_i)_{i=1}^{k-1}$ we declare an integer m to be ineligible at this stage if $m \in [h'_i, 2h'_i)$ for some $i < k$, and we then let h'_k be the smallest eligible element, if any, of $D(\mu, b)$. The hypotheses of the lemma guarantee that at most $(k-1)J(b)$ elements of $D(\mu, b)$ are ineligible at the k -th stage; thus the construction yields a sequence $(h'_k)_{k=1}^{K'}$ with $K' \geq |D(\mu, b)|/J(b)$. We stop at the first such integer K' . The resulting sequence has the property that $h'_{k+1} \geq 2h'_k$ for all k .

Next we attempt to extract a subsequence $(h_k)_{k=1}^K$ of $(h'_k)_{k=1}^{K'}$ by the following procedure. We let $h_k = h'_{k'}$. Then, given $(h_j)_{j=k+1}^K$, we consider the terms $h'_{k'}$ that satisfy $h'_{k'} < h_{k+1}$, and we declare that such a term is disqualified at the k -th stage if there is a sequence $(\varepsilon_j)_{j=k}^{K-k}$ with $\varepsilon_j \in \{-1, 0, 1\}$ for all j , with $\varepsilon_k = \pm 1$, and with $\varepsilon_j \neq 0$ for some $j > k$, so that the integer m given by

$$(3.5) \quad m = \varepsilon_k h'_{k'} + \sum_{j=k+1}^K \varepsilon_j h_j$$

is positive, not in the set $\{h'_j\}_{j=1}^{K'}$, and satisfies $|\hat{\mu}(m)| \geq \beta$. We then let h_k be the largest such term $h'_{k'}$, if any, that is not disqualified at the k -th stage. The point of this procedure is that, if it works, then it yields a sequence $(h_k)_{k=1}^K$ with the properties listed just before equation (3.4).

We have already concluded that no such subsequence exists; hence, for some $k \geq 1$, all terms $h'_{k'} < h_{k+1}$ are disqualified at the k -th stage. Let us say that a term $h'_{k'}$ is disqualified by a sequence $(\varepsilon_j)_{j=k}^{K-k}$ if the latter sequence has all the properties listed just before and after equation (3.5). In particular, $\varepsilon_k = \pm 1$, while $\varepsilon_j \in \{-1, 0, 1\}$ for all $j > k$, and there is at least one such index j for which $\varepsilon_j \neq 0$. Furthermore, since the selection process guarantees that $h_j < h_n$ whenever $k < j < n$, and hence that $h_n \geq 2h_j$ whenever $n > j > k$, the requirement that m be positive forces the last such non-zero term ε_j to be equal to 1. We note that there are fewer than 3^{K-k} sequences $(\varepsilon_j)_{j=k}^{K-k}$ with the properties we have just listed.

Fix one such sequence $(\varepsilon_j)_{j=k}^K$ and let

$$n = \sum_{j=k+1}^K \varepsilon_j h_j;$$

then $n > 0$ because the last non-zero ε_j is equal to 1. Suppose that also $\varepsilon_k = 1$. Then the integer m given by (3.5) is equal to $n + h'_k$. Therefore, if h'_k is disqualified by $(\varepsilon_j)_{j=k}^K$, then $m \in D_n(\mu, \beta)$. By hypothesis, there are at most $J(\beta)$ such integers m , and hence at most $J(\beta)$ terms h'_k are disqualified by the sequence $(\varepsilon_j)_{j=k}^K$. Similarly, each sequence $(\varepsilon_j)_{j=1}^K$ with $\varepsilon_k = -1$ can disqualify at most $J(\beta)$ terms h'_k .

Therefore, fewer than $3^{K-k} J(\beta)$ terms are disqualified at the k -th stage. Suppose that

$$|D(\mu, b)| > 3^K J(b) J(\beta).$$

Then the first part of our construction yields a sequence $(h'_k)_{k'=1}^{K'}$ with $K' \geq 3^K J(\beta)$, and, by the analysis above, the second part of our construction yields a subsequence $(h_k)_{k=1}^K$ with the properties listed just before and after equation (3.5). This contradiction shows that the conclusion of the lemma holds with

$$(3.6) \quad I(b) = 3^K J(b) J(\beta).$$

Remark 1. We now comment on the relation between b and $L(b)$ in the Theorem. We first note that the proof of Lemma 1 shows that its conclusion holds with $J = 2^{j-4}$ when $j \geq 4$. This means that in part (iii) of Lemma 2 we can put

$$(3.7) \quad J(b) = 2^{68/b^2 - 4}.$$

As in the proof of Lemma 3, let K be the smallest integer for which $K \geq (4C/b)^2$ and let $\beta = 3^{-K/2}$; then the conclusion of the Theorem holds with $L(b) = 2I(b) + 1$, where $I(b)$ is given by (3.6), and J is the function specified in (3.7). In particular, the Theorem holds with

$$L(b) = 5^{5^K}, \quad \text{where } K = (4C/b)^2.$$

On the other hand, the methods of [15] yield the theorem with

$$L(b) \simeq \alpha^{\alpha^{2\alpha^2}}, \quad \text{where } \alpha = 16/b^2;$$

so, our methods yield better values for $L(b)$ whenever b is small.

We can improve our estimate significantly by reconsidering the proof of Lemma 3 in the case where the measure μ satisfies the added condition that $s(\mu) \leq 1$. It turns out that, in this case, the factor $J(\beta)$ in (3.6) can be replaced by the quantity $64/\beta^2$. To verify this, consider the k -th stage in the construction of the subsequence $(h_k)_{k=1}^K$ of the sequence $(h'_k)_{k'=1}^K$. Fix an auxiliary sequence $(\varepsilon_j)_{j=k}^K$, with $\varepsilon_k = 1$ say, and consider the integers m given by (3.5) for various

terms h'_k that are disqualified by the sequence $(\varepsilon_j)_{j=k}^K$; these integers m then form an increasing Hadamard sequence in the set $D_n(\mu, \beta)$. By Lemma 2, there are at most $64/\beta^2$ such integers m , and hence at most $64/\beta^2$ such terms h'_k can be disqualified by the sequence $(\varepsilon_j)_{j=1}^K$; a similar argument works if $\varepsilon_k = -1$. This improvement yields the Theorem with

$$L(b) = 3^{(32C^2 + 68)/b^2}.$$

There are many proofs of inequality (3.3) and generalizations of it (see [3]-[5], [12], and [14], inequality (3.3)). The best value of the constant C appearing in the inequality is not known; the best value to date [3] seems to be $C = e^{1/2}$. Hence the Theorem holds with

$$L(b) = 3^{155/b^2}.$$

By refining our methods further, we can prove our Theorem with even smaller values of $L(b)$, but still of the form a^{d/b^2} . We omit these refinements, because the cases where μ is a Dirichlet kernel suggest that the optimal form for $L(b)$ might be $a^{d/b}$.

Remark 2. A glance at the proof of Lemma 2 shows that its three conclusions still hold if the hypothesis that $s(\mu) \leq 1$ is replaced by the weaker assumption that there is a sequence $(\mu_N)_{N=1}^\infty$ of measures, with $\|\mu_N\| \leq 1$ for all N , such that $\hat{\mu}_N(n) = \hat{\mu}(n)$ for all n in the interval $[1, N]$, and $\hat{\mu}_N(n) = 0$ for all $n > N$. In this case, there is no loss of generality in assuming that μ is a weak-star limit point of the sequence $(\mu_N)_{N=1}^\infty$ such that $\|\mu\| \leq 1$. Therefore, there is a version of the Theorem in which the hypothesis on $s(\mu)$ is weakened in this manner, and the conclusion is an estimate on $|D(\mu, b)|$. The methods of [15] also yield such a version of the Theorem. See [6], Remark 1, for an application of this observation.

Similarly, it is not actually necessary to assume a priori that the trigonometric series in Theorem 1 is a Fourier-Stieltjes series, because this follows from the hypothesis that its partial sums form a bounded sequence in $L^1(T)$.

Remark 3. Next we mention another possible application of Lemma 3. Recall that [1], given indices p and q in the interval $[1, \infty]$, a subset E of Z is said to be of type $S_{p,q}$ if there is a constant C such that

$$(3.8) \quad \|\hat{f}\|_q \leq C \|f\|_p$$

for all E -polynomials f , that is for all trigonometric polynomials f for which \hat{f} vanishes off E . Suppose that E is a subset of the set Z^+ of all positive integers. Motivated by inequality (3.3), we can ask whether the assumption that E is of type $S_{1,q}$, where $q < \infty$, implies that

$$(3.9) \quad \|\hat{f}|_E\|_q \leq C' \|f\|_1$$

for all $(E \cup \mathbb{Z}^-)$ -polynomials f ; here $\hat{f}|_E$ denotes the restriction of the function \hat{f} to the set E . Such conclusions do hold for all known examples of sets of type $S_{1,q}$ for the following reasons. First [1], if $q < 2$, then all such sets are finite; second, if $2 \leq q < \infty$, then all known examples of sets of type $S_{1,q}$ are actually $\Lambda(q')$ -sets where $q' = q/(q-1)$, and inequality (3.9) holds for such sets (see [6], inequality (2.8)).

While Lemma 3 does not lead to inequalities of the form (3.9), it does yield estimates for $|D(f, b)|$ in terms of $\|f\|_1$ for $(E \cup \mathbb{Z}^-)$ -polynomials f . Indeed, given $n > 0$, let Q_n be the trigonometric polynomial for which \hat{Q}_n vanishes outside the interval $(0, 4n)$, while $\hat{Q}_n(2n) = 1$, and \hat{Q}_n is linear on each of the intervals $[0, 2n]$ and $[2n, 4n]$. Given an $(E \cup \mathbb{Z}^-)$ -polynomial f , let $f_n = f * Q_n$. Then f_n is an E -polynomial, and $|\hat{f}_n(m)| \geq |\hat{f}(m)/2|$ for all integers m in the interval $[n, 2n)$. It follows from (3.8) that

$$|\{m \in [n, 2n): |\hat{f}(m)| \geq b\}| \leq (2C\|f\|_1/b)^q$$

for all $b > 0$. Apply Lemma 3 to the measure μ given by letting

$$d\mu(\theta) = f(\theta) d\theta / (2\pi\|f\|_1);$$

the outcome is

$$|\{m > 0: |\hat{f}(m)| \geq b\}| \leq I(b/\|f\|_1),$$

where I is the function associated with the initial function J given by $J(b) = (2C\|f\|_1/b)^q$.

Remark 4. Helson's original proof [9] of his theorem used his translation lemma and the theorem of F. and M. Riesz. Louis Pigno has observed, however, that Helson's argument can be modified so that Paley's theorem is used in place of the theorem of F. and M. Riesz. In our proof of the Theorem we used two analytic facts, namely Paley's theorem and inequality (3.3). Pigno's observation suggests that the Theorem should follow from Paley's theorem alone. It turns out that there is indeed such a proof of the Theorem; the proof proceeds by Lemmas 1 and 2 and the weaker version of Lemma 3 in which the conclusion is the same, but there is the added hypothesis that $|D_n(\tilde{\mu}, b)| \leq J(b)$ for all b and n . We do not see how to deduce Lemma 3 itself from Paley's theorem alone, but Lemma 3 does follow from Theorem 2 of [5], which is a common generalization of both Paley's theorem and inequality (3.3).

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DEPARTMENT OF MATHEMATICS
UNIVERSITY OF BRITISH COLUMBIA
VANCOUVER

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