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ON THE REDUCIBILITY OF CONNECTIONS ON THE PROLONGATIONS OF VECTOR BUNDLES

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It is well known that a *connection* on a vector bundle (E, p, B) can be defined as a splitting of the exact sequence

$$0 \to E \otimes T^*(B) \to J^1 E \to E \to 0. \tag{1}$$

Since the r-th non-holonomic prolongation $\tilde{J}^r E$ of E is also a vector bundle, it is natural to introduce a connection on $\tilde{J}^r E$ as a splitting of the exact sequence

$$0 \to \tilde{J}^r E \otimes T^*(B) \to \tilde{J}^{r+1} E \to \tilde{J}^r E \to 0. \tag{2}$$

However, accepting this point of view, we neglect entirely the fact that $\tilde{J}^r E$ is a special vector bundle constructed by means of the successive jet prolongations of E. Let $\Phi(E)$ be the groupoid of all linear isomorphisms between the fibres of E. Then, by the general theory of prolongations of fibre bundles, [1], the r-th non-holonomic prolongation $\tilde{\Phi}^r(E)$ of $\Phi(E)$ is a groupoid of operators on $\tilde{J}^r E$. Since $\tilde{\Phi}^r(E)$ is evidently a proper subgroupoid of the groupoid $\Phi(\tilde{J}^r E)$ of all linear isomorphisms between the fibres of $\tilde{J}^r E$, there appears a natural question: under what conditions a connection on $\Phi(\tilde{J}^r E)$ given by an arbitrary splitting of (2) can be reduced to $\tilde{\Phi}^r(E)$? In the present paper, we solve this problem for $r = 1$ and $r = 2$, but we hope that one meets all essential features of the general problem already in the case $r = 2$ and we feel inconvenient to treat directly the general case because of a great number of different conditions which should appear there. After that, we investigate the reducibility of a connection on $\tilde{\Phi}^2(E)$ to some natural subgroupoids of $\tilde{\Phi}^2(E)$. We should like to underline the remarkable role played in our considerations by the so-called lateral projections of non-holonomic jets introduced recently in [6]. The standard terminology and notations of the theory of jets are used throughout the paper. In addition, j^s_r means the usual projection of r-jets into s-jets, $s < r$. Our considerations are in the category C^∞.

1. Let (E, p, B) be a vector bundle over B of fibre dimension m and let $n = \dim B$. Further, let $\Phi(E)$ be the groupoid of all linear isomorphisms between the fibres of E and let $a\colon \Phi(E) \to B$ or $b\colon \Phi(E) \to B$ be its *source* or *target projection*, respectively (i. e., if $\theta \in \Phi$ is a linear isomorphism of E_x into E_t, then $a(\theta) = x$, $b(\theta) = t$). Obviously, $\Phi(E)$ is a Lie groupoid over B [8]. A *connection* on E can be equivalently defined either as a cross-section $C\colon B \to Q^1(\Phi(E))$, where $Q^1(\Phi(E))$ means the fibre bundle of all elements of connection of the first order on $\Phi(E)$, [2], or as a splitting $\gamma\colon E \to J^1 E$ of the exact sequence (1), see [8]. If a connection C is given, then the value of the corresponding splitting γ on a vector $v \in E_x$ is determined as follows. If $C(x) = j^1_x \varrho(t)$, where ϱ is a mapping of a neighbourhood U of $x \in B$ into $\Phi(E)$ such that $a(\varrho(t)) = x$, $b(\varrho(t)) = t$, $\varrho(x) = \mathrm{id}_{E_x}$, $t \in U$, then $t \mapsto \varrho(t)(v)$ is a local cross-section of E over U. By [8], $\gamma(v)$ is the 1-jet of $\varrho(t)(v)$ at x, i. e.

$$\gamma(v) = j^1_x[\varrho(t)(v)]. \tag{3}$$

In what follows we systematically denote a cross-section of $Q^1(\Phi(E))$ by a capital Roman letter and we shall use the corresponding lower-case Greek letter for the associated splitting $E \to J^1 E$.

We use frequently the evaluations in some local coordinates. For the sake of simplicity, we assume in such a situation that E is a trivial vector bundle $R^n \times R^m$ over $R^n = B$ and that

$$x^i, y^a, \quad \begin{aligned} &i, j, \ldots = 1, \ldots, n, \\ &a, \beta, \ldots = 1, \ldots, m, \end{aligned}$$

are the natural coordinates on $R^n \times R^m$. Then $\Phi(R^n \times R^m) = R^n \times L^1_m \times R^n$. This induces the coordinates

$$\bar{x}^j, b^a_\beta, x^i,$$

$\det |b^a_\beta| \neq 0$, on $\Phi(R^n \times R^m)$. We have $a(\bar{x}^j, b^a_\beta, x^i) = (x^i)$, $b(\bar{x}^j, b^a_\beta, x^i) = (\bar{x}^j)$ and the action of $\Phi(R^n \times R^m)$ on $R^n \times R^m$ is given by

$$(\bar{x}^j, b^a_\beta, x^i)(x^i, y^\beta) = (\bar{x}^j, b^a_\beta y^\beta). \tag{4}$$

On $J^1(R^n \times R^m)$ there are further coordinates y^a_i determined by

$$y^a_i(j^1_x\sigma) = \partial_i y^a(\sigma(x)),$$

provided ∂_i denotes the partial differentiation with respect to x^i. Thus, if $C(x) = j^1_x(t^j, b^a_\beta(t), x^i)$, $b^a_\beta(x) = \delta^a_\beta$, then

$$\gamma(x^i, y^a) = (x^i, y^a, \Gamma^a_{\beta i}(x) y^\beta), \tag{5}$$

where $\Gamma^a_{\beta i}(x) = \partial_i b^a_\beta(x)$. The functions $\Gamma^a_{\beta i}$ are called the *Christoffel's symbols* of C. In short, we say that the splitting (5) is given by

$$y^a_i = \Gamma^a_{\beta i} y^\beta. \tag{6}$$

2. Consider the tensor product $E_1 \otimes E_2$ of two vector bundles over B. Let C_a be a connection on E_a, $a = 1, 2$. Then C_1 and C_2 determine a connection $C_1 \otimes C_2$ on $E_1 \otimes E_2$ in the following well known manner. If $C_a(x) = j_x^1 \varrho_a(t)$, then $\varrho_1(t) \otimes \varrho_2(t) \in \Phi(E_1 \otimes E_2)$ and we write

$$(C_1 \otimes C_2)(x) = j_x^1\big(\varrho_1(t) \otimes \varrho_2(t)\big).$$

In coordinates, if

$$\text{(7)} \quad \begin{aligned} y_i^a &= \Gamma^a_{\beta i} y^\beta, && m \text{ — the fibre dimension of } E_1, \\ y_i^\lambda &= \Gamma^\lambda_{\mu i} y^\mu, && \lambda, \mu, \ldots = m+1, \ldots, m + \text{the fibre dimension of } E_2, \end{aligned}$$

are the corresponding splittings, $\Gamma^a_{\beta i}(x) = \partial_i b^a_\beta(x)$, $\Gamma^\lambda_{\mu i}(x) = \partial_i b^\lambda_\mu(x)$, then the Christoffel's symbols of $C_1 \otimes C_2$ satisfy $\Gamma^{a\lambda}_{\beta\mu i}(x) = \partial_i(b^a_\beta b^\lambda_\mu)(x)$. This implies

$$\text{(8)} \qquad \Gamma^{a\lambda}_{\beta\mu i} = \Gamma^a_{\beta i} \delta^\lambda_\mu + \delta^a_\beta \Gamma^\lambda_{\mu i}.$$

A connection C on $E_1 \otimes E_2$ will be said to be *decomposable* if there exist connections C_1 and C_2 such that $C = C_1 \otimes C_2$. The decomposability of C is characterized by linear equations (8) with unknown $\Gamma^a_{\beta i}$, $\Gamma^\lambda_{\mu i}$. Since the general solution of the homogeneous system depends on one parameter, the factors C_1 and C_2 of a decomposable connection are not uniquely determined. We shall further say that a connection C on $E_1 \otimes E_2$ is decomposable with respect to a connection C_1 on E_1, if there exists a connection C_2 on E_2 such that $C = C_1 \otimes C_2$. By (8), we obtain

LEMMA 1. *If a connection C on $E_1 \otimes E_2$ is decomposable with respect to a connection C_1 on E_1, then the second factor C_2 on E_2 is uniquely determined.*

3. We shall state an algebraic lemma generalizing Lemma 2 of [7]. Consider a commutative diagram with exact rows and columns of vector bundles over the same base and of their base-preserving homomorphisms:

$$\text{(9)} \qquad \begin{array}{ccccccccc} & & 0 & & 0 & & 0 & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & A_1 & \xrightarrow{i_1} & B_1 & \xrightarrow{p_1} & C_1 & \to & 0 \\ & & \downarrow{\scriptstyle \varphi_1} & & \downarrow{\scriptstyle \psi_1} & & \downarrow{\scriptstyle \zeta_1} & & \\ 0 & \to & A_2 & \xrightarrow{i_2} & B_2 & \underset{p_2}{\overset{\gamma}{\rightleftarrows}} & C_2 & \to & 0 \\ & & \downarrow{\scriptstyle \varphi_2} & & \downarrow{\scriptstyle \psi_2} & & \downarrow{\scriptstyle \zeta_2} & & \\ 0 & \to & A_3 & \xrightarrow{i_3} & B_3 & \xrightarrow{p_3} & C_3 & \to & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & 0 & & 0 & & 0 & & \end{array}$$

Let $\gamma\colon C_2 \to B_2$ be a splitting in the middle row. By exactness, the values of the composition $\omega(\gamma) = \psi_2 \gamma \zeta_1$ lie in $A_3 \subset B_3$. Hence $\omega(\gamma)$ is an element of $\mathrm{Hom}(C_1, A_3)$, which will be said to be the *obstruction* associated with γ.

LEMMA 2. *A necessary and sufficient condition for the existence of a splitting* $\gamma_0 \colon C_3 \to B_3$ *or* $\gamma^0 \colon C_1 \to B_1$ *compatible with diagram* (9) *is* $\omega(\gamma) = 0 \in \mathrm{Hom}(C_1, A_3)$.

Proof. Necessity is obvious. Let $\omega(\gamma) = 0$. If $x + \zeta_1(C_1) \in C_3$, then we define $\gamma_0\big(x + \zeta_1(C_1)\big) = \psi_2\gamma(x)$. Since $\psi_2\gamma\zeta_1(C_1) = 0$, this definition is correct. Further, we have

$$p_3\gamma_0\big(x + \zeta_1(C_1)\big) = p_3\psi_2\gamma(x) = \zeta_2 p_2\gamma(x) = x + \zeta_1(C_1),$$

so that γ_0 is a splitting. Moreover, if $y \in C_1$, then $\omega(\gamma) = 0$ implies $\gamma\zeta_1(y) \in \mathrm{Im}(\psi_1)$. Hence there is a unique $\gamma^0(y) \in B_1$ with $\psi_1\gamma^0(y) = \gamma\zeta_1(y)$. Then

$$\zeta_1 p_1\gamma^0(y) = p_2\psi_1\gamma^0(y) = p_2\gamma\zeta_1(y) = \zeta_1(y).$$

4. The *first prolongation* J^1E of E is a vector bundle of fibre dimension $N = m(n+1)$. Let $\Phi(J^1E)$ be the groupoid of all linear isomorphisms between the fibres of J^1E. In the trivial case,

$$\Phi\big(J^1(R^n \times R^m)\big) = R^n \times L^1_N \times R^n.$$

Taking into account that J^1E is the prolongation of E, we shall say that an element $\theta \in \Phi(J^1E)$ with source x and target t is *projectable*, if there exists an element $\varphi \in \Phi(E)$ such that the diagram

$$\begin{array}{ccc} J^1_xE & \xrightarrow{\ \theta\ } & J^1_tE \\ \downarrow{\scriptstyle j^0_1} & & \downarrow{\scriptstyle j^0_1} \\ E_x & \xrightarrow{\ \varphi\ } & E_t \end{array} \tag{10}$$

commutes.

Denote by $\Phi_p(J^1E)$ the subgroupoid of all projectable elements of $\Phi(J^1E)$. If the coordinate expression of θ is

$$(x^i, y^a, y^a_i) \mapsto (t^j, c^a_\beta y^\beta + c^{ai}_\beta y^\beta_i, c^a_{\beta i} y^\beta + c^{aj}_{\beta i} y^\beta_j),$$

then

$$\theta \in \Phi_p(J^1E) \text{ if and only if } c^{ai}_\beta = 0. \tag{11}$$

In general, if a Lie groupoid Ψ is a subgroupoid of a Lie groupoid Φ over the same base B, then a connection $C \colon B \to Q^1(\Phi)$ is said to be *reducible to* Ψ if $C(B) \subset Q^1(\Psi)$. We shall first investigate the reducibility of a connection $\gamma \colon J^1E \to \tilde{J}^2E$ to $\Phi_p(J^1E)$.

We recall that beside the usual projection $j^1_2 \colon \tilde{J}^2E \to J^1E$ there is another canonical projection $l^1_2 \colon \tilde{J}^2E \to J^1E$ defined by $l^1_2(j^1_x\sigma) = j^1_x(j^0_1\sigma)$; this projection is said to be *lateral*, [6]. On $\tilde{J}^2(R^n \times R^m)$ there are further coordinates y^a_{0i}, y^a_{ij} given by

$$y^a_{0i}(j^1_x\sigma) = \partial_i y^a\big(\sigma(x)\big), \quad y^a_{ij}(j^1_x\sigma) = \partial_j y^a_i\big(\sigma(x)\big).$$

According to [6], if x^i, y^a, y^a_i, y^a_{0i}, y^a_{ij} are the coordinates of an element $Y \in \tilde{J}^2(R^n \times R^m)$, then

$$j^1_2(Y) = (x^i, y^a, y^a_i), \qquad l^1_2(Y) = (x^i, y^a, y^a_{0i}). \tag{12}$$

PROPOSITION 1. *A connection $\gamma\colon J^1E \to \tilde{J}^2E$ is reducible to $\Phi_p(J^1E)$ if and only if there exists a connection $\gamma_0\colon E \to J^1E$ compatible with the diagram*

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & E \otimes (\overset{2}{\otimes} T^*(B)) & \to & J^1(E \otimes T^*(B)) & \to & E \otimes T^*(B) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & J^1E \otimes T^*(B) & \longrightarrow & \tilde{J}^2E & \overset{\gamma}{\rightleftarrows} & J^1E & \to & 0 \\
 & & \downarrow & & \downarrow l^1_2 & j^1_2 & \downarrow j^0_1 & & \\
0 & \to & E \otimes T^*(B) & \longrightarrow & J^1E & \underset{j^0_1}{\overset{\gamma_0}{\rightleftarrows}} & E & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array} \tag{13}$$

Proof. In general, the coordinate form of γ is

$$\begin{aligned} y^a_{0i} &= \Gamma^a_{\beta 0i} y^\beta + \Gamma^{aj}_{\beta 0i} y^\beta_j, \\ y^a_{ij} &= \Gamma^a_{\beta ij} y^\beta + \Gamma^{ak}_{\beta ij} y^\beta_k. \end{aligned} \tag{14}$$

The obstruction $\omega(\gamma) \in \mathrm{Hom}(E \otimes T^*(B), E \otimes T^*(B))$ vanishes if and only if $\Gamma^{aj}_{\beta 0i} = 0$. Comparing with (11), we obtain our assertion.

The connection C_0 can be described geometrically in the following simple way. The mapping $\pi\colon \Phi_p(J^1E) \to \Phi(E)$, $\theta \mapsto \varphi$ is a functor. In general, if Φ and Ψ are two Lie groupoids over B and if $\lambda\colon \Phi \to \Psi$ is a base-preserving functor, then every connection $C\colon B \to Q^1(\Phi)$ is transformed into a connection $\lambda_*(C)\colon B \to Q^1(\Psi)$. In particular, in the situation of Proposition 1, we have $C_0 = \pi_*(C)$.

5. By the general theory of prolongations of fibre bundles, if Φ is a groupoid of operators on a fibred manifold (S, q, B), then the first prolongation Φ^1 of Φ is a groupoid of operators on J^1S, [1]. In particular, the first prolongation $\Phi^1(E)$ of $\Phi(E)$ is a groupoid of operators on J^1E. Obviously, $\Phi^1(E) \subset \Phi_p(J^1E)$. In the trivial case,

$$\Phi^1(R^n \times R^m) = R^n \times T^1_n(L^1_m) \overline{\times} L^1_n \times R^n,$$

where $T^1_n(L^1_m) \overline{\times} L^1_n$ means the semi-direct product of $T^1_n(L^1_m)$ and L^1_n with respect to the action $(S, A) \mapsto SA$ of L^1_n on $T^1_n(L^1_m)$, $S \in T^1_n(L^1_m)$, $A \in L^1_n$. (In the other words, the multiplication in $T^1_n(L^1_m) \overline{\times} L^1_n$ is given by

$(S_1, A_1)(S_2, A_2) = \big((S_1 A_2)\cdot S_2, A_1 A_2\big)$, where the dot denotes the multiplication in $T^1_n(L^1_m)$.) We shall study the reducibility of connection (14) to $\Phi^1(E)$. On $\Phi^1(R^n \times R^m)$ there are coordinates

$$\bar{x}^j, b^a_\beta, b^a_{\beta i}, a^i_j, x^i,$$

$\det|a^i_j| \neq 0$, and the action of $\Phi^1(R^n \times R^m)$ on $J^1(R^n \times R^m)$ is given by

$$(15) \qquad (\bar{x}^j, b^a_\beta, b^a_{\beta i}, a^i_j, x^i)(x^i, y^a, y^a_i) = \big(\bar{x}^j, b^a_\beta y^\beta, (b^a_{\beta j} y^\beta + b^a_\beta y^\beta_j) A^j_i\big),$$

provided $a^i_j A^j_k = \delta^i_k$ (cf. [4]).

Let C be a connection on $\Phi^1(E)$,

$$C(x) = j^1_x\big(t^j, b^a_\beta(t), b^a_{\beta i}(t), a^i_j(t), x^i\big).$$

Using (15) and $b^a_\beta(x) = \delta^a_\beta$, $b^a_{\beta i}(x) = 0$, $a^i_j(x) = \delta^i_j$, one finds easily that the corresponding splitting $J^1E \to \tilde{J}^2E$ is

$$(16) \qquad \begin{aligned} y^a_{0i} &= \Gamma^a_{\beta i} y^\beta, \\ y^a_{ij} &= \Gamma^a_{\beta ij} y^\beta + \Gamma^a_{\beta i} y^\beta_j - \Gamma^k_{ij} y^a_k, \end{aligned}$$

where

$$(17) \qquad \Gamma^a_{\beta i}(x) = \partial_i b^a_\beta(x), \qquad \Gamma^a_{\beta ij}(x) = \partial_j b^a_{\beta i}(x), \qquad \Gamma^i_{jk}(x) = \partial_k a^i_j(x).$$

Consider now an arbitrary connection (14) on J^1E. If (14) is reducible to $\Phi_p(J^1E)$, then there is a splitting γ^0: $E\otimes T^*(B) \to J^1\big(E\otimes T^*(B)\big)$ in the top row of (13) compatible with the diagram. By (16), if the connection (14) is further reducible to $\Phi^1(E)$, then C^0 is decomposable with respect to C_0, i. e., there is a linear connection L on B such that $C^0 = C_0 \otimes L$. One sees easily that this condition is also sufficient. Hence we obtain

PROPOSITION 2. *A connection γ: $J^1E \to \tilde{J}^2E$ is reducible to $\Phi^1(E)$ if and only if there exists a connection γ_0: $E \to J^1E$ compatible with diagram* (13) *and the induced connection C^0 on $E\otimes T^*(B)$ is decomposable to C_0.*

6. We shall now discuss the similar problem for $r = 2$. Since the lateral projections play a fundamental role even in this case, we shall first give a survey of some of their properties. According to [6] there are three canonical projections j^2_3, l^2_3, ${}^2l^2_3$ of $\tilde{J}^3E$ into $\tilde{J}^2E$. We shall state the coordinate form of these projections. On $\tilde{J}^3(R^n \times R^m)$ there are further coordinates $y^a_{00i}, y^a_{i0j}, y^a_{0ij}, y^a_{ijk}$ given by

$$(18) \qquad \begin{aligned} y^a_{00i}(j^1_x\sigma) &= \partial_i y^a\big(\sigma(x)\big), & y^a_{i0j}(j^1_x\sigma) &= \partial_j y^a_i\big(\sigma(x)\big), \\ y^a_{0ij}(j^1_x\sigma) &= \partial_j y^a_{0i}\big(\sigma(x)\big), & y^a_{ijk}(j^1_x\sigma) &= \partial_k y^a_{ij}\big(\sigma(x)\big). \end{aligned}$$

If $x^i, y^a, y^a_i, y^a_{0i}, y^a_{ij}, y^a_{00i}, y^a_{i0j}, y^a_{0ij}, y^a_{ijk}$ are the coordinates of an element $Y \in \tilde{J}^3(R^n \times R^m)$, then

$$(19) \qquad \begin{aligned} j^2_3(Y) &= (x^i, y^a, y^a_i, y^a_{0i}, y^a_{ij}), \\ l^2_3(Y) &= (x^i, y^a, y^a_i, y^a_{00i}, y^a_{i0j}), \\ {}^2l^2_3(Y) &= (x^i, y^a, y^a_{0i}, y^a_{00i}, y^a_{0ij}). \end{aligned}$$

7. Considering an arbitrary splitting $\gamma\colon \tilde{J}^2E \to \tilde{J}^3E$ of the form

$$(20)\qquad \begin{aligned} y^a_{00i} &= \Gamma^a_{\beta 00i}y^\beta + \Gamma^{aj}_{\beta 00i}y^\beta_j + \Gamma^{a0j}_{\beta 00i}y^\beta_{0j} + \Gamma^{ajk}_{\beta 00i}y^\beta_{jk},\\ y^a_{i0j} &= \Gamma^a_{\beta i0j}y^\beta + \Gamma^{ak}_{\beta i0j}y^\beta_k + \Gamma^{a0k}_{\beta i0j}y^\beta_{0k} + \Gamma^{akl}_{\beta i0j}y^\beta_{kl},\\ y^a_{0ij} &= \Gamma^a_{\beta 0ij}y^\beta + \Gamma^{ak}_{\beta 0ij}y^\beta_k + \Gamma^{a0k}_{\beta 0ij}y^\beta_{0k} + \Gamma^{akl}_{\beta 0ij}y^\beta_{kl},\\ y^a_{ijk} &= \Gamma^a_{\beta ijk}y^\beta + \Gamma^{al}_{\beta ijk}y^\beta_l + \Gamma^{a0l}_{\beta ijk}y^\beta_{0l} + \Gamma^{alm}_{\beta ijk}y^\beta_{lm}, \end{aligned}$$

we investigate a connection on $\Phi(\tilde{J}^2E)$. By [1], the *second non-holonomic prolongation* $\tilde{\Phi}^2(E)$ of $\Phi(E)$ is a groupoid of operators on $\tilde{J}^2E$ satisfying $\tilde{\Phi}^2(E) \subset \Phi(\tilde{J}^2E)$. We study the reducibility of the connection (20) to $\tilde{\Phi}^2(E)$. (Similarly to Proposition 1, some of the following conditions can be explained even separately. Nevertheless, we shall not formulate explicitly any of the corresponding assertions.) In the trivial case we have

$$\tilde{\Phi}^2(R^n\times R^m) = R^n\times \tilde{T}^2_n(L^1_m)\,\overline{\times}\,\tilde{L}^2_n\times R^n,$$

where $\tilde{T}^2_n(L^1_m)\,\overline{\times}\,\tilde{L}^2_n$ means the semi-direct product with respect to the action $(S, A)\mapsto SA$ of $\tilde{L}^2_n$ on $\tilde{T}^2_n(L^1_m)$, $S\in \tilde{T}^2_n(L^1_m)$, $A\in\tilde{L}^2_n$. On $\tilde{\Phi}^2(R^n\times R^m)$ there are coordinates

$$\bar{x}^j,\ b^a_\beta,\ b^a_{\beta i},\ b^a_{\beta 0i},\ b^a_{\beta ij},\ a^i_j,\ a^i_{0j},\ a^i_{jk},\ x^i,$$

$\det|a^i_{0j}|\neq 0$, and the action of $\tilde{\Phi}^2(R^n\times R^m)$ on $\tilde{J}^2(R^n\times R^m)$ is given by

$$(21)\qquad \begin{aligned} &(\bar{x}^j, b^a_\beta, b^a_{\beta i}, b^a_{\beta 0i}, b^a_{\beta ij}, a^i_j, a^i_{0j}, a^i_{jk}, x^i)(x^i, y^a, y^a_i, y^a_{0i}, y^a_{ij})\\ &\quad = \big(\bar{x}^j, b^a_\beta y^\beta, (b^a_{\beta j}y^\beta + b^a_\beta y^\beta_j)A^j_i, (b^a_{\beta 0j}y^\beta + b^a_\beta y^\beta_{0j})A^j_{0i},\\ &(b^a_{\beta kl}y^\beta + b^a_{\beta k}y^\beta_{0l} + b^a_{\beta 0l}y^\beta_k + b^a_\beta y^\beta_{kl})A^k_iA^l_{0j} - (b^a_{\beta k}y^k + b^a_\beta y^\beta_k)A^k_l a^l_{mn}A^m_iA^n_{0j}\big), \end{aligned}$$

provided $a^i_jA^j_k = \delta^i_k$, $a^i_{0j}A^j_{0k} = \delta^i_k$ (cf. [4]).

Let C be a connection on $\tilde{\Phi}^2(E)$,

$$C(x) = j^1_x\big(t^j, b^a_\beta(t), b^a_{\beta i}(t), b^a_{\beta 0i}(t), b^a_{\beta ij}(t), a^i_j(t), a^i_{0j}(t), a^i_{jk}(t), x^i\big),$$

where

$$b^a_\beta(x) = \delta^a_\beta,\qquad b^a_{\beta i}(x) = b^a_{\beta 0i}(x) = 0,$$
$$b^a_{\beta ij}(x) = 0,\qquad a^i_j(x) = a^i_{0j}(x) = \delta^i_j,\qquad a^i_{jk}(x) = 0.$$

Introduce the corresponding Γ's analogously to (17). Then (21) implies that C determines a splitting $\tilde{J}^2E\to\tilde{J}^3E$ of the form

$$(22)\qquad \begin{aligned} y^a_{00i} &= \Gamma^a_{\beta i}y^\beta,\\ y^a_{i0j} &= \Gamma^a_{\beta ij}y^\beta + \Gamma^a_{\beta i}y^\beta_j - \Gamma^k_{ij}y^a_k,\\ y^a_{0ij} &= \Gamma^a_{\beta 0ij}y^\beta + \Gamma^a_{\beta i}y^\beta_{0j} - \Gamma^k_{i0j}y^a_{0k},\\ y^a_{ijk} &= \Gamma^a_{\beta ijk}y^\beta + \Gamma^a_{\beta ik}y^\beta_{0j} + \Gamma^a_{\beta 0jk}y^\beta_i - \Gamma^l_{ijk}y^a_l + \Gamma^a_{\beta k}y^\beta_{ij} - \Gamma^l_{ik}y^a_{lj} - \Gamma^l_{0jk}y^a_{il}. \end{aligned}$$

Our next investigation will be based on the comparison of (20) and (22). Naturally, we shall deduce all conditions in an invariant form.

8. Consider first the lateral projection l_3^2 and a commutative exact diagram

$$
\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & J^1E\otimes(\overset{2}{\otimes} T^*(B)) & \to & J^1(J^1E\otimes T^*(B)) & \overset{\gamma_1^0}{\rightleftarrows} & J^1E\otimes T^*(B) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & \tilde J^2E\otimes T^*(B) & \to & \tilde J^3 E & \overset{\gamma}{\underset{j_3^2}{\rightleftarrows}} & \tilde J^2 E & \to & 0 \\
 & & \downarrow & & \downarrow l_3^2 & & \downarrow j_2^1 & & \\
0 & \to & J^1E\otimes T^*(B) & \to & \tilde J^2 E & \overset{\gamma_1}{\underset{j_2^1}{\rightleftarrows}} & J^1 E & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}
\tag{23}
$$

By the first two formulas in (22) there should be a splitting γ_1: $J^1E \to \tilde J^2 E$ compatible with the diagram. By Lemma 2, the obstruction is an element of $\mathrm{Hom}(J^1E\otimes T^*(B), J^1E\otimes T^*(B))$, which will be denoted by $\omega_1(\gamma)$. In coordinates, $\omega_1(\gamma) = 0$ means

$$
\Gamma^{\alpha 0j}_{\beta 00i} = 0, \quad \Gamma^{\alpha jk}_{\beta 00i} = 0, \quad \Gamma^{\alpha 0k}_{\beta i0j} = 0, \quad \Gamma^{\alpha kl}_{\beta i0j} = 0. \tag{24}
$$

Taking into account the second lateral projection ${}^2l_3^2$, we obtain a commutative exact diagram:

$$
\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & J^1(E\otimes T^*(B))\otimes T^*(B) & \to & \tilde J^2(E\otimes T^*(B)) & \overset{\gamma_2^0}{\rightleftarrows} & J^1(E\otimes T^*(B)) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & \tilde J^2E\otimes T^*(B) & \to & \tilde J^3 E & \overset{\gamma}{\underset{j_3^2}{\rightleftarrows}} & \tilde J^2 E & \to & 0 \\
 & & \downarrow & & \downarrow {}^2l_3^2 & & \downarrow l_2^1 & & \\
0 & \to & J^1E\otimes T^*(B) & \to & \tilde J^2 E & \overset{\gamma_2}{\underset{j_2^1}{\rightleftarrows}} & \tilde J^1 E & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}
\tag{25}
$$

By the first and third formulas in (22) there should be a splitting γ_2: $J^1E \to \tilde J^2 E$ compatible with the diagram. The obstruction $\omega_2(\gamma)$ is an element of $\mathrm{Hom}(J^1(E\otimes T^*(B)), J^1E\otimes T^*(B))$. In coordinates, $\omega_2(\gamma) = 0$ is equivalent to

$$
\Gamma^{\alpha j}_{\beta 00i} = 0, \quad \Gamma^{\alpha jk}_{\beta 00i} = 0, \quad \Gamma^{\alpha k}_{\beta 0ij} = 0, \quad \Gamma^{\alpha kl}_{\beta 0ij} = 0. \tag{26}
$$

In what follows, we assume $\omega_1(\gamma) = 0$, $\omega_2(\gamma) = 0$. Then (24) and (26) imply that there is a splitting γ_0: $E \to J^1 E$ compatible with the following diagram ($a = 1, 2$):

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & E \otimes (\overset{2}{\otimes} T^*(B)) & \to & J^1(E \otimes T^*(B)) & \underset{}{\overset{\gamma_0 \otimes \lambda_a}{\leftrightarrows}} & E \otimes T^*(B) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & J^1 E \otimes T^*(B) & \longrightarrow & \tilde{J}^2 E & \underset{j_2^1}{\overset{\gamma_a}{\leftrightarrows}} & J^1 E & \to & 0 \\
 & & \downarrow & & \downarrow{\scriptstyle l_2^1} & & \downarrow{\scriptstyle j_1^0} & & \\
0 & \to & E \otimes T^*(B) & \longrightarrow & J^1 E & \underset{j_1^0}{\overset{\gamma_0}{\leftrightarrows}} & E & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array} \tag{27}$$

By the first three formulas in (22), the induced connection on $E \otimes T^*(B)$ should be reducible to the connection C_0 on E, i. e., there should exist a linear connection L_a on B such that the induced connection in the top row of (27) is of the form $C_0 \otimes L_a$. Further, by the third and fourth formulas in (22), we conclude that the connection C_1^0 of the top row of (23) should satisfy

$$C_1^0 = C_1 \otimes L_2. \tag{28}$$

(In particular, the connection induced on the intersection $E \otimes \left(\overset{2}{\otimes} T^*(B)\right)$ of the kernels of both projections j_2^1 and l_2^1 is $C_0 \otimes L_1 \otimes L_2$.)

It remains to deduce an analogous condition for C_2^0, but this requires an auxiliary consideration.

9. Let E_1 and E_2 be two vector bundles over B, let C_a be a connection on $J^1 E_a$ reducible to $\Phi^1(E_a)$, $a = 1, 2$, and let both C_1 and C_2 have a common underlying linear connection L. Then the splitting γ_1 or γ_2 is of the form

$$y_{0i}^\alpha = \Gamma_{\beta i}^\alpha y^\beta, \qquad y_{ij}^\alpha = \Gamma_{\beta ij}^\alpha y^\beta + \Gamma_{\beta i}^\alpha y_j^\beta - \Gamma_{ij}^k y_k^\alpha \tag{29}$$

or

$$y_{0i}^\lambda = \Gamma_{\mu i}^\lambda y^\mu, \qquad y_{ij}^\lambda = \Gamma_{\mu ij}^\lambda y^\mu + \Gamma_{\mu i}^\lambda y_j^\mu - \Gamma_{ij}^k y_k^\lambda, \tag{30}$$

respectively. Hence we may write

$$C_1(x) = j_x^1\left(t^j, b_\beta^\alpha(t), b_{\beta i}^\alpha(t), a_j^i(t), x^i\right) = j_x^1 \varrho_1(t),$$
$$C_2(x) = j_x^1\left(t^j, b_\mu^\lambda(t), b_{\mu i}^\lambda(t), a_j^i(t), x^i\right) = j_x^1 \varrho_2(t).$$

Prolonging the action of the product $\Phi(E_1) \times \Phi(E_2)$ on $E_1 \otimes E_2$, we obtain an action of the first prolongation $\left(\Phi(E_1) \times \Phi(E_2)\right)^1$ on $J^1(E_1 \otimes E_2)$. Since the sections ϱ_1 and ϱ_2 determine a section (ϱ_1, ϱ_2) of $\left(\Phi(E_1) \times \Phi(E_2)\right)^1$,

$C(x) = j^1_x(\varrho_1(t), \varrho_2(t))$ is an element of connection on $(\Phi(E_1)\times\Phi(E_2))^1$. Thus we obtain a connection on $J^1(E_1\otimes E_2)$, which will be said to be the *tensor product* of C_1 and C_2 over L and will be denoted by

$$C = C_1 \underset{L}{\otimes} C_2.$$

By the definition, the coordinate form of the above-given action of $(\Phi(E_1)\times\Phi(E_2))^1$ on $J^1(E_1\otimes E_2)$ is

$$(31)\qquad (\bar{x}^j, b^a_\beta, b^a_{\beta i}, b^\lambda_\mu, b^\lambda_{\mu i}, a^i_j, x^i)(x^i, y^{a\lambda}, y^{a\lambda}_i) = \left(\bar{x}^j, b^a_\beta b^\lambda_\mu y^{\beta\mu}, (b^a_{\beta j} b^\lambda_\mu y^{\beta\mu} + b^a_\beta b^\lambda_{\mu j} y^{\beta\mu} + b^a_\beta b^\lambda_\mu y^{\beta\mu}_j) A^j_i\right).$$

Using (31), we deduce that $C_1 \underset{L}{\otimes} C_2$ determines a splitting

$$J^1(E_1\otimes E_2) \to \tilde{J}^2(E_1\otimes E_2)$$

of the form

$$(32)\qquad \begin{aligned} y^{a\lambda}_{0i} &= \Gamma^a_{\beta i} y^{\beta\lambda} + \Gamma^\lambda_{\mu i} y^{a\mu}, \\ y^{a\lambda}_{ij} &= \Gamma^a_{\beta ij} y^{\beta\lambda} + \Gamma^\lambda_{\mu ij} y^{a\mu} + \Gamma^a_{\beta i} y^{\beta\lambda}_j + \Gamma^\lambda_{\mu i} y^{a\mu}_j - \Gamma^k_{ij} y^{a\lambda}_k. \end{aligned}$$

Further, let C be a connection on $J^1(E_1\otimes E_2)$ and let C_1 be a connection on $\Phi^1(E_1)$ with an underlying linear connection L. We say that C is *decomposable with respect to* C_1, if there exists a connection C_2 on $\Phi^1(E_2)$ with the same underlying linear connection L such that

$$C = C_1 \underset{L}{\otimes} C_2.$$

By (32), if C is decomposable with respect to C_1, then the second factor C_2 is uniquely determined.

10. By the second and fourth formulas in (22), the connection C^0_2 on $J^1(E\otimes T^*(B))$ should be decomposable with respect to C_2, i. e., there should exist a reducible connection M on $J^1(T^*(B))$ with the underlying linear connection L_2 such that

$$(33)\qquad C^0_2 = C_2 \underset{L_2}{\otimes} M.$$

Comparing now (20) and (22), we see that we have found all relations characterizing (22) with respect to (20). Hence we have proved

PROPOSITION 3. *A connection γ: $\tilde{J}^2E \to \tilde{J}^3E$ is reducible to $\tilde{\Phi}^2(E)$ if and only if all the following conditions are satisfied*:

(a) $\omega_1(\gamma) = 0$, $\omega_2(\gamma) = 0$;

(b) *the connection of the top row of* (27) *is of the form $C_0\otimes L_a$, where C_0 is the connection of the bottom row of* (27) *and L_a is a convenient linear connection on B, $a = 1, 2$;*

(c) *the connection* C_1^0 *of the top row of* (23) *is of the form* $C_1 \otimes L_2$, *where* C_1 *is the connection of the bottom row of* (23);

(d) *the connection* C_2^0 *of the top row of* (25) *is of the form* $C_2 \underset{L_2}{\otimes} M$, *where* C_2 *is the connection of the bottom row of* (25) *and* M *is a convenient connection on* $J^1(T^*(B))$.

11. Assume in the sequel that C is reducible to $\tilde{\Phi}^2(E)$. Beside $\bar{\Phi}^2(E)$ and $\Phi^2(E)$, [1], one can naturally introduce also the following subgroupoids of $\tilde{\Phi}^2(E)$:

$$\tilde{\Phi}_s^2(E) = \{\theta \in \tilde{\Phi}^2(E);\ b\theta \in \bar{\Pi}^2(B)\},$$

$$\tilde{\Phi}_h^2(E) = \{\theta \in \tilde{\Phi}^2(E);\ b\theta \in \Pi^2(B)\},$$

$$\bar{\Phi}_h^2(E) = \{\theta \in \bar{\Phi}^2(E);\ b\theta \in \Pi^2(B)\};$$

$\bar{\Pi}^2(B)$ or $\Pi^2(B)$ means the groupoid of all invertible semi-holonomic or holonomic 2-jets of B into B, respectively.

In the trivial case we have

$$\tilde{\Phi}_s^2(R^n \times R^m) = R^n \times \tilde{T}_n^2(L_m^1) \bar{\times} \bar{L}_n^2 \times R^n,$$

$$\tilde{\Phi}_h^2(R^n \times R^m) = R^n \times \tilde{T}_n^2(L_m^1) \bar{\times} L_n^2 \times R^n,$$

$$\bar{\Phi}^2(R^n \times R^m) = R^n \times \bar{T}_n^2(L_m^1) \bar{\times} \bar{L}_n^2 \times R^n, \tag{34}$$

$$\bar{\Phi}_h^2(R^n \times R^m) = R^n \times \bar{T}_n^2(L_m^1) \bar{\times} L_n^2 \times R^n.$$

$$\Phi^2(R^n \times R^m) = R^n \times T_n^2(L_m^1) \bar{\times} L_n^2 \times R^n.$$

By (22), we obtain immediately

PROPOSITION 4. *A connection* C *on* $\tilde{\Phi}^2(E)$ *is reducible to*

(a) $\tilde{\Phi}_s^2(E)$ *if and only if the connections* L_1 *and* L_2 *coincide*,

(b) $\bar{\Phi}^2(E)$ *if and only if the connections* C_1 *and* C_2 *coincide*.

12. Assume $L_1 = L_2$. Then (22) implies that C is reducible to $\bar{\Phi}_h^2(E)$ if and only if $\Gamma^i_{[jk]l} = 0$, provided the square brackets denote antisymmetrization. We shall give an invariant explanation of this condition.

Libermann ([5], p. 159) has established an identification

$$J^1 T^*(B) \approx \bar{T}^{2*}(B), \tag{35}$$

There $\bar{T}^{2*}(B)$ is the vector bundle of all semi-holonomic 2-jets of B into R with target 0. Consider the injection $i: T^{2*}(B) \to \bar{T}^{2*}(B)$ of the subspace $\mathrm{w}^{2*}(B) \subset \bar{T}^{2*}(B)$ of all holonomic jets. According to [3], we have the exact sequence

$$0 \to T^{2*}(B) \xrightarrow{i} \bar{T}^{2*}(B) \xrightarrow{\Delta} \Lambda^2 T^*(B) \to 0, \tag{36}$$

where $\varDelta$ is a special case of the difference tensor map. Taking account of (35), the above-mentioned connection M determines a splitting $\mu\colon \bar{T}^{2*}(B) \to J^1\bar{T}^{2*}(B)$. We say that M is *symmetric* if $\mu\big(T^{2*}(B)\big) \subset J^1T^{2*}(B)$. We have a commutative exact diagram

$$
(37)\qquad
\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & T^{2*}(B)\otimes T^*(B) & \to & J^1T^{2*}(B) & \leftrightarrows & T^{2*}(B) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow{\scriptstyle i} & & \\
0 & \to & \bar{T}^{2*}(B)\otimes T^*(B) & \to & J^1\bar{T}^{2}(B) & \overset{\mu}{\underset{j^0_1}{\leftrightarrows}} & \bar{T}^{2*}(B) & \to & 0 \\
 & & \downarrow & & \downarrow{\scriptstyle j^1\varDelta} & & \downarrow{\scriptstyle \varDelta} & & \\
0 & \to & \varLambda^2T^*(B)\otimes T^*(B) & \to & J^1\big(\varLambda^2T^*(B)\big) & \xrightarrow{\ j^0_1\ } & \varLambda^2T^*(B) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}
$$

where $j^1\varDelta$ means the first jet prolongation of the morphism $\varDelta\colon \bar{T}^{2*}(B) \to \varLambda^2T^*(B)$. By Lemma 2, M is symmetric if and only if

$$(j^1\varDelta)\mu i = 0 \in \operatorname{Hom}\big(T^{2*}(B), \varLambda^2T^*(B)\otimes T^*(B)\big).$$

Applying the relation $L_1 = L_2$, we find the following coordinate form of μ:

$$
(38)\qquad
\begin{aligned}
y_{i0j} &= -\Gamma^k_{ij}y_k,\\
y_{ijk} &= -\Gamma^l_{ijk}y_l - \Gamma^l_{ik}y_{lj} - \Gamma^l_{jk}y_{il}.
\end{aligned}
$$

Using the coordinate expression of the difference tensor map (see [3], p. 139), we infer that M is symmetric if and only if

$$(39)\qquad 0 = y_{[ij]k} = -\Gamma^l_{[ij]k}y_l.$$

Hence we have

PROPOSITION 5. *A connection C on $\tilde{\varPhi}^2(E)$ is reducible to*

(a) *$\tilde{\varPhi}^2_h(E)$ if and only if $L_1 = L_2$ and M is symmetric,*

(b) *$\bar{\varPhi}^2_h(E)$ if and only if $C_1 = C_2$ and M is symmetric.*

13. It remains to treat the reducibility of C to $\varPhi^2(E)$. Assume that C is reducible to $\bar{\varPhi}^2(E)$. By (22), C is further reducible to $\varPhi^2(E)$ if and only if $\Gamma^a_{\beta[ij]k} = 0$ and $\Gamma^l_{[ij]k} = 0$. Denote by $\bar{\gamma}$ the restriction of γ to $\bar{J}^2E$. Since C is reducible to $\bar{\varPhi}^2(E)$, the values of $\bar{\gamma}$ lie in $J^1\bar{J}^2E$. Consider the injection $i\colon J^2E \to \bar{J}^2E$. By [3], we have the exact sequence

$$(40)\qquad 0 \to J^2E \xrightarrow{i} \bar{J}^2E \xrightarrow{\varDelta} E\otimes\varLambda^2T^*(B) \to 0,$$

where $\varDelta$ is the difference tensor map. The morphism $\varDelta$ is prolonged to a morphism $j^1\varDelta\colon J^1\bar{J}^2E \to J^1\big(E\otimes\varLambda^2T^*(B)\big)$ and we get a commutative exact diagram

$$(41)\qquad \begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & J^2E\otimes T^*(B) & \longrightarrow & J^1J^2E & \leftrightarrows & J^2E & \to & 0\\
 & & \downarrow & & \downarrow & \bar{\gamma} & \downarrow{\scriptstyle i} & & \\
0 & \to & \bar{J}^2E\otimes T^*(B) & \longrightarrow & J^1\bar{J}^2E & \leftrightarrows & \bar{J}^2E & \to & 0\\
 & & \downarrow & & \downarrow{\scriptstyle j^1\Delta} & j_3^2 & \downarrow{\scriptstyle \Delta} & & \\
0 & \to & E\otimes\Lambda^2T^*(B)\otimes T^*(B) & \to & J^1(E\otimes\Lambda^2T^*(B)) & \longrightarrow & E\otimes\Lambda^2T^*(B) & \to & 0\\
 & & \downarrow & & \downarrow & j_1^0 & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

If C is reducible to $\Phi^2(E)$, then there exists a splitting in the top row compatible with the diagram. By Lemma 2, the obstruction is an element $\omega(\bar{\gamma})$ of $\mathrm{Hom}\big(J^2E, E\otimes\Lambda^2T^*(B)\otimes T^*(B)\big)$. Using $C_1 = C_2$ and $y_i^a = y_{0i}^a$, we simplify the last row of (22) to the form

$$y_{ijk}^a = \Gamma_{\beta ijk}^a y^\beta + \Gamma_{\beta ik}^a y_j^\beta + \Gamma_{\beta jk}^a y_i^\beta - \Gamma_{ijk}^l y_l^a + \Gamma_{\beta k}^a y_{ij}^\beta - \Gamma_{ik}^l y_{lj}^a - \Gamma_{jk}^l y_{il}^a.$$

By direct evaluation, we infer that $\omega(\bar{\gamma})$ vanishes if and only if

$$0 = y_{[ij]k}^a = \Gamma_{\beta[ij]k}^a y^\beta - \Gamma_{[ij]k}^l y_l^a.$$

Thus, finally, we obtain

PROPOSITION 6. *A connection C on $\tilde{\Phi}^2(E)$ is reducible to $\Phi^2(E)$ if and only if $C_1 = C_2$ and $\omega(\bar{\gamma})$ vanishes.*

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