COLLOQUIUM MATHEMATICUM

VOL. XXXIII 1975 FASC. 1

REDUCING INVERSE SYSTEMS OF MONOMORPHISMS

BY

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1. Introduction. Let $\mathscr{D}$ denote the full subcategory of the topological category whose objects are all finite discrete spaces. Then, as is well known, every totally disconnected compact Hausdorff space can be written as the limit of an inverse system in $\mathscr{D}$. In particular, spaces of arbitrarily large weight are limits of inverse systems in $\mathscr{D}$. In contrast, consider the category $\mathscr{L}_n$ whose objects are all compact, connected n-dimensional Lie groups, and whose morphisms are all continuous, surjective homomorphisms. The limit of any inverse system in $\mathscr{L}_n$ is *metrizable* [6].

In this paper* it is shown that the second phenomenon is a special case of a general theorem. If $\mathscr{M}$ is any category such that each morphism is a monomorphism ($\mathscr{L}_n$ is such a category), then to each inverse system $\boldsymbol{X}$ in $\mathscr{M}$ there corresponds a reduced inverse system $\hat{\boldsymbol{X}}$ in $\mathscr{M}$ which is isomorphic to $\boldsymbol{X}$. In the presence of mild assumptions about $\mathscr{M}$, the cardinality of the index set $\hat{A}$ of the reduced system $\hat{\boldsymbol{X}}$ is bounded by a fixed cardinal number λ dependent only on the category $\mathscr{M}$.

Of special interest is the case where $\mathscr{M}$ is an appropriate subcategory of a category of pointed connected topological spaces whose morphisms are base-point preserving covering projections. If $\mathscr{M}$ is such a category, then λ can be expressed in terms of properties of the fundamental groups of the spaces in $\mathscr{M}$. For example, in the case of the category $\mathscr{L}_n$, a simple calculation shows that $\lambda = \aleph_0$. An immediate consequence is the previously mentioned result that the limit of any inverse system in $\mathscr{L}_n$ is metrizable.

The appropriate setting for this work involving inverse systems is that of pro-categories. The basic facts about pro-categories which are used in this paper are contained in Section 2.

We are grateful to Carl Eberhart and John Mack for numerous helpful comments and questions concerning this research. In particular, a to-

* During this research the first author was visiting the University of Zagreb on an exchange program sponsored jointly by the National Academy of Sciences (U.S.A.) and the Yugoslav Academy of Sciences and Arts.

pological proof (unpublished) of Corollary 3 due to John Mack provided the motivation for Theorem 1.

2. Pro-categories. Pro-categories were first introduced in [4], and a general discussion of such categories is contained in [1]. For this paper the less general definition (see [2]) presented in the sequel will be sufficient.

In this paper a *directed set* A is a directed quasi-ordered set with a minimal element a_0. Since every inverse system contains a cofinal subsystem with a minimal element, there is no loss of generality in this assumption.

For any category $\mathscr{K}$ let pro$(\mathscr{K})$ denote the category whose objects are inverse systems $\boldsymbol{X}$ in $\mathscr{K}$ over arbitrary directed sets, and whose morphisms are equivalence classes of maps $\boldsymbol{f}$: $\boldsymbol{X} \to \boldsymbol{Y}$ defined as follows. Let

$$\boldsymbol{X} = \{X_\alpha;\ p_{\alpha\alpha'};\ A\} \quad \text{and} \quad \boldsymbol{Y} = \{Y_\beta;\ q_{\beta\beta'};\ B\}$$

be objects in pro$(\mathscr{K})$. A *map* $\boldsymbol{f}$: $\boldsymbol{X} \to \boldsymbol{Y}$ (not a morphism) consists of a function f: $B \to A$ (not necessarily order-preserving) and of a collection of morphisms $\{f_\beta: X_{f(\beta)} \to Y_\beta \mid \beta \in B\}$ in $\mathscr{K}$ such that, for $\beta \leqslant \beta'$ in B, there exists an $\alpha \in A$ with $f(\beta)$, $f(\beta') \leqslant \alpha$ and the diagram

$$\begin{array}{ccccc} X_{f(\beta)} & \xleftarrow{p} & X_\alpha & \xrightarrow{p} & X_{f(\beta')} \\ {\scriptstyle f_\beta}\downarrow & & & & \downarrow{\scriptstyle f_{\beta'}} \\ Y_\beta & & \xleftarrow{\quad q \quad} & & Y_{\beta'} \end{array}$$

commutes in $\mathscr{K}$. Two such maps $\boldsymbol{f}$, $\boldsymbol{g}$: $\boldsymbol{X} \to \boldsymbol{Y}$ are *equivalent* if, for each $\beta \in B$, there exists an $\alpha \in A$ such that $f(\beta)$, $g(\beta) \leqslant \alpha$ and the diagram

$$\begin{array}{ccc} X_{f(\beta)} & \xleftarrow{p} & X_\alpha \\ {\scriptstyle f_\beta}\downarrow & & \downarrow{\scriptstyle p} \\ Y_\beta & \xleftarrow{g_\beta} & X_{g(\beta)} \end{array}$$

commutes in $\mathscr{K}$. The *morphisms* of pro$(\mathscr{K})$ are the equivalence classes $[\boldsymbol{f}]$ of such maps. The composition of morphisms

$$[\boldsymbol{f}]\colon \boldsymbol{X} \to \boldsymbol{Y}, \quad [\boldsymbol{g}]\colon \boldsymbol{Y} \to \boldsymbol{Z} = \{Z_\gamma;\ r_{\gamma\gamma'};\ C\}$$

is the morphism

$$[\boldsymbol{g}][\boldsymbol{f}] = [\boldsymbol{gf}],$$

where $[\boldsymbol{gf}]$ consists of the function fg: $C \to A$ and of the morphisms $\{g_\gamma f_{g(\gamma)}\colon X_{fg(\gamma)} \to Z_\gamma \mid \gamma \in C\}$.

For each cardinal number λ, we define pro$(\mathscr{K};\lambda)$ to be the full subcategory of pro$(\mathscr{K})$ having as its objects inverse systems over directed sets of cardinality λ or less.

We shall need the following facts about pro$(\mathscr{K})$:

PROPOSITION 1. *If* $\boldsymbol{X} = \{X_a;\ p_{aa'};\ A\}$ *is an object in* $\mathrm{pro}(\mathscr{K})$, *and* $B \subset A$ *is cofinal, then* $\{X_\beta;\ p_{\beta\beta'};\ B\}$ *is isomorphic to* $\boldsymbol{X}$ *in* $\mathrm{pro}(\mathscr{K})$.

PROPOSITION 2. *If* $\boldsymbol{X}$ *and* $\boldsymbol{Y}$ *are isomorphic objects in* $\mathrm{pro}(\mathscr{K})$, *and* $X_\infty = \mathrm{invlim}\,\boldsymbol{X}$, $Y_\infty = \mathrm{invlim}\,\boldsymbol{Y}$, *then* X_∞ *is isomorphic to* Y_∞ *in* $\mathscr{K}$.

3. Construction of the reduced system. Throughout this paper $\mathscr{M}$ will denote a *category of monomorphisms*, that is, a category such that if f, g and h are morphisms and $fg = fh$, then $g = h$.

Example 1. Let S be a left-cancellative semigroup with identity. Then S can be interpreted as a category $\mathscr{M}$ of monomorphisms whose only object is an "ideal object" X, and whose morphisms are the elements of S. Composition of morphisms is given by multiplication in S. In particular, any identity containing subsemigroup of a group is such a category.

In this section we show that each object $\boldsymbol{X}$ in $\mathrm{pro}(\mathscr{M})$ determines an essentially unique *reduced inverse system* $\hat{\boldsymbol{X}}$ in $\mathscr{M}$ such that $\boldsymbol{X}$ and $\hat{\boldsymbol{X}}$ are isomorphic in $\mathrm{pro}(\mathscr{M})$. Given an inverse system $\boldsymbol{X}$ in $\mathscr{M}$, the construction of the reduced system $\hat{\boldsymbol{X}}$ will be accomplished in two steps. Intuitively, the first step is to "extend" the system $\boldsymbol{X}$ to a system $\boldsymbol{X}^*$ by "adding all possible arrows". The second step is to define $\hat{\boldsymbol{X}}$ as a certain cofinal subsystem of $\boldsymbol{X}^*$.

Let $\boldsymbol{X} = \{X_a;\ p_{aa'};\ A\}$ denote an object in $\mathrm{pro}(\mathscr{M})$, where A is a directed set with quasi-order $\leqslant$ and a minimal element a_0.

Step 1. We begin by defining a directed set A^* with quasi-order $\leqslant^*$. Let $A^* = A$ as sets, and define $a \leqslant^* a'$ in A^* if there exists a morphism f in $\mathscr{M}$ such that $p_{a_0a'} = p_{a_0a}f$. By the monomorphism property, f is unique (whenever it exists). Consequently, for $a \leqslant^* a'$ we can define $p^*_{aa'}$ to be the morphism f. It is easy to verify that $\boldsymbol{X}^* = \{X_a;\ p^*_{aa'};\ A^*\}$ is a well-defined inverse system. Notice that if $a \leqslant a'$, then $a \leqslant^* a'$ and $p_{aa'} = p^*_{aa'}$.

We wish to show that $\boldsymbol{X}^*$ is isomorphic to $\boldsymbol{X}$ in $\mathrm{pro}(\mathscr{M})$. Let $\boldsymbol{i}\colon \boldsymbol{X} \to \boldsymbol{X}^*$ be the map which consists of the identity function $i\colon A^* \to A$ and the collection of identity morphisms

$$\{i_a\colon X_{i(a)} \to X_a \,|\, a \in A^*\}.$$

If $a \leqslant^* a'$, then choose any $a'' \in A$ such that $a, a' \leqslant a''$. The diagram

$$\begin{array}{ccccc} X_{i(a)} & \xleftarrow{p} & X_{a''} & \xrightarrow{p} & X_{i(a')} \\ {\scriptstyle i_a}\downarrow & & & & \downarrow{\scriptstyle i_{a'}} \\ X_a & & \xleftarrow{\quad p^* \quad} & & X_{a'} \end{array}$$

commutes and, consequently, $[\boldsymbol{i}]$ is a morphism in $\mathrm{pro}(\mathscr{M})$. The morphism $[\boldsymbol{i}]$ has an inverse which is defined analogously. Thus $[\boldsymbol{i}]$ is the required isomorphism. This completes Step 1.

Step 2. Let $\sim$ be the equivalence relation on the index set A^* defined as follows: $a \sim a'$ if $a \leqslant^* a'$ and $a' \leqslant^* a$. An equivalence class of the relation $\sim$ is called a *level set* of the quasi-order $\leqslant^*$. Let $\hat{A}$ be any subset of A^* which contains exactly one element from each level set. Thus $\hat{A}$ is a cofinal directed subset of A^* which is order-isomorphic to the directed partially ordered quotient set $A^*/\sim$. Now the inverse system $\hat{\boldsymbol{X}} = \{X_a; p^*_{aa'}; \hat{A}\}$, obtained by restricting indices to $\hat{A}$, is the desired reduced system. Proposition 1 implies that $\hat{\boldsymbol{X}}$ is isomorphic to $\boldsymbol{X}^*$, and hence to $\boldsymbol{X}$.

We now introduce a definition which plays an important role throughout the remainder of the paper, and we record a fact about our construction.

Definition. Morphisms $f\colon X \to Z$ and $g\colon Y \to Z$ of an arbitrary category are *equivalent* if there exists an isomorphism $h\colon X \to Y$ such that $f = gh$.

PROPOSITION 3. *Let $a, a' \in \hat{A}$. If p_{a_0a} and $p_{a_0a'}$ are equivalent morphisms in $\mathscr{M}$, then $a = a'$.*

Proof. Let $h\colon X_{a'} \to X_a$ be an isomorphism in $\mathscr{M}$ such that $p_{a_0a}h = p_{a_0a'}$. Then $p_{a_0a'}h^{-1} = p_{a_0a}$, so, by Step 1, $a \leqslant^* a'$ and $a' \leqslant^* a$. Thus $a \sim a'$ and, by Step 2, $a = a'$.

Remark. If $a \sim a'$ in A^*, then the monomorphism property implies that $p^*_{aa'}$ is an isomorphism with inverse $p^*_{a'a}$. This justifies our calling $\hat{\boldsymbol{X}}$ *the reduced system* associated with $\boldsymbol{X}$.

4. The reduction theorem. We now prove our main result, the reduction theorem, and give an application involving metrization of compact spaces.

THEOREM 1. *Let $\mathscr{M}$ be a category of monomorphisms. Suppose that, for each object Y in $\mathscr{M}$, the equivalence classes of morphisms in* $\mathrm{Hom}_{\mathscr{M}}(-, Y)$ *form a set whose cardinality is bounded by a fixed cardinal λ. If $\boldsymbol{X}$ is an inverse system in $\mathscr{M}$, then the reduced system $\hat{\boldsymbol{X}}$ is an object of* $\mathrm{pro}(\mathscr{M}; \lambda)$.

Proof. Let $\boldsymbol{X} = \{X_a; p_{aa'}; A\}$ be an object of $\mathrm{pro}(\mathscr{M})$, and let $\hat{\boldsymbol{X}} = \{X_a; p^*_{aa'}; \hat{A}\}$ denote the reduced system associated with $\boldsymbol{X}$. According to Proposition 3, the cardinality of $\hat{A}$ is bounded by the cardinality of the set of equivalence classes of morphisms in $\mathrm{Hom}_{\mathscr{M}}(-, X_{a_0})$. Thus $\hat{A}$ is of cardinality λ or less, and $\boldsymbol{X}$ belongs to $\mathrm{pro}(\mathscr{M}; \lambda)$.

COROLLARY 1. *Suppose $\mathscr{M}$ is a small category of monomorphisms. There exists a cardinal λ such that $\hat{\boldsymbol{X}}$ belongs to* $\mathrm{pro}(\mathscr{M}; \lambda)$ *for every $\boldsymbol{X}$ in* $\mathrm{pro}(\mathscr{M})$.

Let $\mathscr{K}'$ denote a subcategory of an arbitrary category $\mathscr{K}$. We call the subcategory $\mathscr{K}'$ *admissible* (with respect to $\mathscr{K}$) if, for each commutative diagram

$$\begin{tikzcd} X \arrow[rr, "h"] \arrow[dr, "f"'] & & Y \arrow[dl, "g"] \\ & Z & \end{tikzcd}$$

in $\mathscr{K}$, the condition that f and g belong to $\mathscr{K}'$ implies that h also belongs to $\mathscr{K}'$. Notice that every full subcategory of $\mathscr{K}$ is admissible.

COROLLARY 2. *Let $\mathscr{M}'$ be an admissible subcategory of a category $\mathscr{M}$ of monomorphisms. Suppose that there exists a cardinal λ such that, for each object $\boldsymbol{X}$ in* $\operatorname{pro}(\mathscr{M})$, *the reduced system $\hat{\boldsymbol{X}}$ belongs to* $\operatorname{pro}(\mathscr{M}; \lambda)$. *Then, for each $\boldsymbol{X}'$ in* $\operatorname{pro}(\mathscr{M}')$, *the reduced system $\hat{\boldsymbol{X}}'$ with respect to $\mathscr{M}'$ belongs to* $\operatorname{pro}(\mathscr{M}'; \lambda)$.

Proof. Let $\boldsymbol{X}'$ be an object in $\operatorname{pro}(\mathscr{M}')$. Since $\mathscr{M}'$ is admissible, it is trivial to check that the reduced system $\hat{\boldsymbol{X}}'$ with respect to $\mathscr{M}'$ is the reduced system with respect to $\mathscr{M}$. But the latter system belongs to $\operatorname{pro}(\mathscr{M}; \lambda)$, and hence to $\operatorname{pro}(\mathscr{M}'; \lambda)$.

COROLLARY 3 (John Mack). *Let X be a metric compactum and let $\mathscr{F}$ be a countable left-cancellative semigroup of surjective self-maps on X. If $\boldsymbol{X} = \{X_a;\ p_{aa'};\ A\}$, where $X_a = X$ for each a and $p_{aa'} \in \mathscr{F}$, then $X_\infty =$ $\operatorname{inv\,lim} \boldsymbol{X}$ is a metric compactum.*

Proof. Let $\mathscr{M}$ denote the category whose only object is X and whose morphisms are the mappings in $\mathscr{F}$ together with the identity map. Then $\mathscr{M}$ is a category of monomorphisms, and the number of equivalence classes of morphisms in $\operatorname{Hom}_{\mathscr{M}}(X, X)$ is bounded by $\aleph_0$. Thus, by the reduction theorem, $\hat{\boldsymbol{X}}$ is a countable system; and, by Proposition 2, X_∞ is homeomorphic to the metric compactum $\operatorname{inv\,lim} \hat{\boldsymbol{X}}$.

The next example shows that Corollary 3 does not hold if $\mathscr{F}$ is assumed to be a countable right-cancellative semigroup of self-maps, even if the space X is a countable compactum.

Example 2. Let X denote the metric compactum consisting of the set of positive integers P and the point at infinity ∞. Let $\mathscr{F}$ be the semigroup of all self-maps $f\colon X \to X$ with the following properties:

(i) there exist m and n in P such that $f([1, m]) = [1, n]$;

(ii) $f(m+r) = n+r$ for $r \in P$; and

(iii) $f(\infty) = \infty$.

Then $\mathscr{F}$ is a countable right-cancellative semigroup, since the mappings in $\mathscr{F}$ are surjections.

Let D denote any totally disconnected compact Hausdorff space. It is well known that

$$D = \operatorname{inv\,lim}\{D_a;\ p_{aa'};\ A\},$$

where each D_a is a finite discrete space and the bonding maps are surjections. Each space D_a can be interpreted as an initial segment of the space X; and each mapping $p_{aa'}$: $D_{a'} \to D_a$ can be extended uniquely to a mapping $p^*_{aa'}$: $X \to X$ in $\mathscr{F}$ whose restriction to $X - D_{a'}$ is an order-preserving bijection onto $X - D_a$. Consequently,

$$\boldsymbol{X} = \{X_a;\ p^*_{aa'};\ A\} \qquad \text{with } X_a = X$$

is a well-defined inverse system, and $D \subset X_\infty = \operatorname{invlim} \boldsymbol{X}$. This proves that the category $\mathscr{X}$ whose only object is X and whose morphisms are the mappings in $\mathscr{F}$ admits no cardinal λ such that each object in $\operatorname{pro}(\mathscr{X})$ is isomorphic to an object in $\operatorname{pro}(\mathscr{X};\ \lambda)$.

5. Categories of covering projections. Throughout this section $\mathscr{C}$ will denote the category whose objects are all pointed Hausdorff spaces which are connected, locally pathwise connected, and semilocally 1-connected, and whose morphisms are all base point-preserving covering projections. Since covering projections on objects of $\mathscr{C}$ are fibrations with unique path lifting, composition in $\mathscr{C}$ is well defined (see [7], Theorem 6, p. 69, and Theorem 10, p. 78). According to the unique lifting property of covering projections on connected spaces ([7], Theorem 2, p. 67), $\mathscr{C}$ is a category of monomorphisms. We call $\mathscr{C}$ the *category of covering projections.*

The next theorem is our main result concerning the reduction of inverse systems in subcategories of $\mathscr{C}$.

THEOREM 2. *Let $\mathscr{C}'$ be an admissible subcategory of the category $\mathscr{C}$ of covering projections. Suppose that, for each object (Y, y_0) in $\mathscr{C}'$, the number of distinct conjugacy classes of subgroups of $\pi_1(Y, y_0)$ is bounded by a fixed cardinal λ. If $\boldsymbol{X}$ is an inverse system in $\mathscr{C}'$, then the reduced system $\hat{\boldsymbol{X}}$ (with respect to $\mathscr{C}'$) belongs to* $\operatorname{pro}(\mathscr{C}';\ \lambda)$.

Proof. According to Theorem 1, it suffices to show that, for each object (Y, y_0) of $\mathscr{C}'$, the number of equivalence classes of morphisms in $\operatorname{Hom}_{\mathscr{C}'}\big(-,\ (Y, y_0)\big)$ is bounded by λ. Suppose that

$$f_i\colon (X_i, x_i) \to (Y, y_0) \qquad (i = 1, 2)$$

are morphisms in $\mathscr{C}'$, and that there exists a homeomorphism h: $X_1 \to X_2$ (not necessarily base-point preserving) such that $f_1 = f_2 h$. Using standard covering space techniques, it is easy to verify that there exists a base--point preserving homeomorphism h': $(X_1, x_1) \to (X_2, x_2)$ such that $f_1 = f_2 h'$. Since $\mathscr{C}'$ is admissible, h' is a morphism in $\mathscr{C}'$. This observation enables us to apply the classification theory for covering projections (see [7], Corollary 3, p. 80). We conclude that the collection of equivalence classes of morphisms in $\operatorname{Hom}_{\mathscr{C}'}\big(-,\ (Y, y_0)\big)$ is a set of cardinality λ or less.

COROLLARY 4. *Let $\mathscr{C}'$ be a small admissible subcategory of the category $\mathscr{C}$ of covering projections. There exists a cardinal λ such that if $\mathbf{X}$ is in* pro$(\mathscr{C}')$, *then $\hat{\mathbf{X}}$ is in* pro$(\mathscr{C}'; \lambda)$.

COROLLARY 5. *Let $\mathscr{C}'$ be an admissible subcategory of the category $\mathscr{C}$ such that each object of $\mathscr{C}'$ is a compact metric ANR with abelian fundamental group. If $\mathbf{X}$ is an inverse system in $\mathscr{C}'$, then the reduced system $\hat{\mathbf{X}}$ is an object in* pro$(\mathscr{C}'; \aleph_0)$. *Consequently, the limit of any inverse system in $\mathscr{C}'$ is metrizable.*

Proof. If (Y, y_0) is an object in $\mathscr{C}'$, then $\pi_1(Y, y_0) = H_1(Y)$ and, consequently, $\pi_1(Y, y_0)$ is a finitely generated abelian group (see [5], Corollary 7.2, p. 141). Thus $\pi_1(Y, y_0)$ has only countably many distinct subgroups.

6. Categories of n-dimensional Lie groups. Let $\mathscr{L}_n$ denote the category of compact, connected n-dimensional Lie groups with continuous surjective homomorphisms. Standard facts about the relationship of Lie algebras to Lie groups (see, e.g., [3], Theorem 6.6.3, p. 130) imply that homomorphisms in $\mathscr{L}_n$ have discrete kernels. Thus $\mathscr{L}_n$ is a subcategory of the category $\mathscr{C}$ of covering projections. The following lemma shows that $\mathscr{L}_n$ is an admissible subcategory of $\mathscr{C}$:

LEMMA 1. *Let (G_i, e_i) $(i = 0, 1, 2)$ be Lie groups in $\mathscr{L}_n$. Consider the commutative diagram*

$$\begin{tikzcd} (G_1, e_1) \arrow[rr, "h"] \arrow[dr, "f_1"'] & & (G_2, e_2) \arrow[dl, "f_2"] \\ & (G_0, e_0) & \end{tikzcd}$$

in the category $\mathscr{C}$ of covering projections. If f_1 and f_2 are morphisms in $\mathscr{L}_n$, then so is h.

Proof. We must show that h is a homomorphism. For each $y \in G_1$, define mappings h_y and h'_y from G_1 into G_2 as follows:

$$h_y(x) = h(xy) \quad \text{and} \quad h'_y(x) = h(x)h(y), \quad x \in G_1.$$

Simple calculations show that $h_y(e_1) = h'_y(e_1)$ and $f_2 h_y = f_2 h'_y$. Consequently, h_y and h'_y coincide by the unique lifting theorem ([7], Theorem 2, p. 67), and thus $h(xy) = h(x)h(y)$.

THEOREM 3. *If $\mathbf{X}$ is an inverse system in $\mathscr{L}_n$, then the reduced system $\hat{\mathbf{X}}$ is an object in* pro$(\mathscr{L}_n; \aleph_0)$.

Proof. Let G be any group in $\mathscr{L}_n$. Then $\pi_1(G)$ is abelian ([7], Corollary 10, p. 44). Since G is a manifold, G is a compact metric ANR ([5], Corollary 8.3, p. 98). According to Lemma 1, $\mathscr{L}_n$ is an admissible subcategory of the category $\mathscr{C}$ of covering projections. The theorem now follows from Corollary 5.

COROLLARY 6 (Newburgh [6]). *If* $G_\infty = \text{invlim}\{G_a;\ p_{aa'};\ A\}$, *where each factor space is a compact, connected n-dimensional Lie group and each bonding map is a continuous surjective homomorphism, then* G_∞ *is metrizable.*

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Reçu par la Rédaction le 1. 3. 1974