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ON THE EXTENSION OF LIPSCHITZ MAPS WITH VALUES IN A FRÉCHET SPACE

BY

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1. By a pseudometric space $(X, \mathscr{A})$ we mean a set X equipped with a family of pseudometrics $\mathscr{A}$ satisfying the condition

(1) For every $a_1, a_2 \in \mathscr{A}$ there exists $a \in \mathscr{A}$ with $a \geqslant \max(a_1, a_2)$.

Examples. 1. Every metric space (X, d) is a pseudometric space.

2. Let F be a locally convex space and let $U(F)$ be a basis of absolutely convex neighbourhoods of zero in F and P_U denote the Minkowski functional generated by U and $\mathscr{A} = \{P_U\colon U \in U(F)\}$. Then $(F, \mathscr{A})$ is a pseudometric space.

Let $(X, \mathscr{A})$ and $(Y, \mathscr{B})$ be pseudometric spaces. A map $f\colon (X, \mathscr{A}) \to (Y, \mathscr{B})$ is called a *Lipschitz map* iff for every $b \in \mathscr{B}$ there exists $a \in \mathscr{A}$ and a constant $K > 0$ such that

$$b\big(f(x), f(y)\big) \leqslant Ka(x, y) \quad \text{for all} \quad x, y \in X.$$

Remark. The definition of Lipschitz maps between pseudometric spaces given here is a slight modification of one introduced by Mankiewicz ([3]).

The aim of this note is to study the extension of Lipschitz maps with values in a Fréchet space. Only the real case is considered, but the results hold true in the complex case as well.

Definition. Let Φ denote a class of pseudometric spaces. A pseudometric space Y is said to be an *$EL(\Phi)$-space* iff for every $X \in \Phi$ and for every Lipschitz map f from a subset A of X into Y there exists a Lipschitz map $\tilde{f}\colon X \to Y$ such that $\tilde{f} | A = f$.

Let $\mathfrak{M}$ be the class of all metric spaces and $\mathfrak{F}_s$ be the class of all separable Fréchet spaces. The main results of this note are the following.

THEOREM 1.1. *If E is a Fréchet $EL(\mathfrak{M})$-space then $E \hat{\otimes}_\varepsilon F$ is an $EL(\mathfrak{M})$-space for every nuclear Fréchet space F.*

THEOREM 1.2. *An infinite-dimensional Montel–Fréchet space F is an $EL(\mathfrak{F}_s)$-space if and only if $F \cong s$, where s denotes the Fréchet space of all sequences of real numbers.*

The proof of Theorem 1.1 is given in Section 2. Section 3 is devoted to the proof of Theorem 1.2. In Section 4 we apply Theorems 1.1 and 1.2 to the problem of extension of uniformly continuous maps between pseudometric spaces.

2. Proof of Theorem 1.1. Given a metric space K and a Fréchet space E. By $C_u(K, E)$ we denote the Fréchet space of all bounded uniformly continuous maps from K into E with the natural topology. By a theorem of Isbell–Lindenstrauss ([2]), $C_u(K, R^1) \in EL(\mathfrak{M})$. The following lemma is an immediate consequence of Isbell–Lindenstrauss theorem.

LEMMA 2.1. *If E is a Fréchet $EL(\mathfrak{M})$-space, then so is $C_u(K, E)$.*

Proof. Consider E as a subspace of $\prod_{j=1}^{\infty} l_\infty(S_j)$ for some S_j, where $l_\infty(S_j)$ denotes the Banach space of all bounded real functions on S_j with the supremum-norm. Since the existence of a Lipschitz projection from $\prod_{j=1}^{\infty} l_\infty(S_j)$ onto E implies the existence of a Lipschitz projection from $C_u(K, \prod_{j=1}^{\infty} l_\infty(S_j))$ onto $C_u(K, E)$ and

$$C_u(K, \prod_{j=1}^{\infty} l_\infty(S_j)) = \prod_{j=1}^{\infty} C_u(K, l_\infty(S_j))$$

it suffices to consider the case $E = l_\infty(S)$. But in this case, by the relation $C_u(K, l_\infty(S)) = C_u(K \times S, R^1)$ (S is considered as a metric space with the discrete metric), the lemma follows from the Isbell–Lindenstrauss theorem. □

LEMMA 2.2. *Let F be a nuclear Fréchet space. Then there exists an inverse system $\{l_n^1, \theta_n^m\}$ such that*

(i) $l_n^1 = l^1$ *for every* n;

(ii) $F = \operatorname{proj\,lim} \{l_n^1, \theta_n^m\}$;

(iii) θ_{n+2}^n *is nuclear for all* $n \in N$;

(iv) $\operatorname{Im} \theta_{n+2}^n$ *is dense in* $\operatorname{Im} \theta_{n+1}^n$ *for every* n;

(v) *for every* $n \in N$ *there exist linear continuous maps* q_n: $l_{n+1}^1 \to C[0, 1]$, p_{n-1}: $C[0, 1] \to l_{n-1}^1$ *such that* $\theta_{n+1}^{n-1} = p_{n-1} q_n$.

Proof. Let $U(F) = \{U_n\}$ be a basis of absolutely convex neighbourhoods of zero in F such that the canonical maps ω_{n+1}^n: $F_{n+1} \to F_n$ are nuclear, where F_n denotes the completion of $F/p_{U_n}^{-1}(0)$ with respect to the pseudonorm p_{U_n} generated by U_n. Since ω_{n+1}^n are nuclear, there exist linear continuous maps $a_{n+1}\colon F_{n+1} \to l^1$, $b_n\colon l^1 \to F_n$, $c_{n+1}\colon F_{n+1} \to l_\infty$ and $d_n\colon l_\infty \to F_n$ such that $b_n a_{n+1} = \omega^n_{n+1} = d_n c_{n+1}$. For every n take an embedding $i_n\colon F_n \to C[0, 1]$ and a linear continuous map $k_n\colon C[0, 1] \to l_\infty$ such that $k_n i_n = c_n$. Put

$$l^1_n = l^1, \quad \theta^n_{n+1} = a_n b_n, \quad q_n = i_n b_n, \quad p_{n-1} = a_{n-1} d_{n-1} k_n.$$

(i) is obvious.

To verify (ii) it is enough to observe that the diagram below commutes and $F = \operatorname{proj\,lim}\{F_n, \omega^m_n\}$

(2.1)
$$\begin{tikzcd} F_{n+1} \arrow[r, "a_{n+1}"] \arrow[d, "\omega^n_{n+1}"'] & l^1_{n+1} \arrow[d, "\theta^n_{n+1}"] \arrow[dl, "b_n"'] \\ F_n \arrow[r, "a_n"] & l^1_n \end{tikzcd}$$

(iii) follows from the fact that ω^{n-1}_n is nuclear and from the relation $\theta^{n-1}_{n+1} = a_{n-1}\omega^{n-1}_n b_n$.

(iv) Since $\operatorname{Im}\omega^n_{n+1}$ is dense in F_n, by the commutability of (2.1) we infer that $\operatorname{Im}\theta^n_{n+2}$ is dense in $\operatorname{Im}\theta^n_{n+1}$.

(v) $p_{n-1}q_n = a_{n-1}d_{n-1}k_n i_n b_n = a_{n-1}d_{n-1}c_n b_n = a_{n-1}\omega^{n-1}_n b_n = \theta^{n-1}_{n+1}$.

LEMMA 2.3. *Let* $F = \operatorname{proj\,lim}\{F_n, \omega^m_n\}$ *and* $F'' = \operatorname{proj\,lim}\{F''_n, \omega''^m_n\}$ *where* F_n *and* F''_n *are Fréchet spaces. Assume that for every* n *there exists a linear continuous map* $j_n\colon F_n \to F''_n$ *satisfying the following conditions*

(i) $\omega''^m_n j_n = j_m \omega^m_n$ *for every* $n \geqslant m$;

(ii) *for every* $z \in F''_n$ *there exists* $y \in F_{n-2}$ *such that* $j_{n-2}y = \omega''^{n-2}_n z$ *for* $n > 2$;

(iii) *for every* $n > 2$, $\omega^{n-2}_n(\operatorname{Ker} j_n)$ *is dense in* $\omega^{n-2}_{n-1}(\operatorname{Ker} j_{n-1})$.

Then the map $j = \operatorname{proj\,lim} j_n$ *is surjective.*

Proof. For every n let ϱ_n be an invariant metric inducing the topology of F_n. Without loss of generality we can assume that

$$\varrho_n(\omega^n_{n+1}x, \omega^n_{n+1}y) \leqslant \varrho_{n+1}(x, y)$$

for all $x, y \in F_{n+1}$. Let $y = \{y_n\} \in F''$ with $y_n \in F''_n$. We shall construct by induction a sequence of elements $\{x_n\}$ such that

$$x_n \in F_n, \tag{2.2}$$

$$j_n x_n = y_n, \tag{2.3}$$

$$\varrho_{n-2}(\omega^{n-2}_n x_n, \omega^{n-2}_{n-1} x_{n-1}) \leqslant 1/2^n \quad \text{for all} \quad n > 2. \tag{2.4}$$

Select $x_1 \in F_1$ such that $j_1 x_1 = \omega''^1_3 y_3 = y_1$ and assume that $x_1, \ldots, x_n$ satisfying (2.2), (2.3) and (2.4) have been defined for some n. By (ii) there exists

$z_{n+1} \in F_{n+1}$ such that $j_{n+1} z_{n+1} = \omega''^{n+1}_{n+3} y_{n+3} = y_{n+1}$. Since $\omega^n_{n+1} z_{n+1} - x_n \in \operatorname{Ker} j_n$ using (ii) twice one can take a point $x_{n+2} \in \operatorname{Ker} j_{n+2}$ such that

$$\varrho_{n-1}(\omega^{n-1}_{n+2} x_{n+2}, \omega^{n-1}_{n+1} z_{n+1} - \omega^{n-1}_n x_n) \leqslant 1/2^{n+1}.$$

Setting $x_{n+1} = z_{n+1} + \omega^{n+1}_{n+2} x_{n+2}$ we have $x_{n+1} \in F_{n+1}$, $j_{n+1} x_{n+1} = y_{n+1}$ and

$$\varrho_{n-1}(\omega^{n-1}_{n+1} x_{n+1}, \omega^{n-1}_n x_n) = \varrho_{n-1}(\omega^{n-1}_{n+2} x_{n+2}, \omega^{n-1}_{n+1} z_{n+1} - \omega^{n-1}_n x_n) \leqslant 1/2^{n+1}.$$

Thus the sequence $\{x_n\}$ satisfying (2.2), (2.3), (2.4) is constructed. Since for all $n > p+1$

$$\varrho_p(\omega^p_n x_n, \omega^p_{n-1} x_{n-1}) \leqslant \varrho_{n-2}(\omega^{n-2}_n x_n, \omega^{n-2}_{n-1} x_{n-1}) \leqslant 1/2^n,$$

we infer that $\omega^p_n x_n \to z_p$ for all p. By the continuity of ω^p_{p+1} and j_p we have $j_p z_p = y_p$ for every p. Consequently $z = \{z_p\} \in F$ and $jz = y$. □

LEMMA 2.4. *Let $F = \operatorname{proj\,lim}\{F_n, \omega^m_n\}$ and $\eta_n\colon F \to F_n$ are canonical maps, where F_n are Fréchet spaces. If $\operatorname{Im}\eta_n$ is dense in F_n, then*

$$E \hat{\otimes}_\varepsilon F = \operatorname{proj\,lim}\{E \hat{\otimes}_\varepsilon F_n, \operatorname{id}_E \hat{\otimes}_\varepsilon \omega^m_n\}$$

for every Fréchet space E.

Proof. Define a map $\theta\colon E \hat{\otimes}_\varepsilon F \to \operatorname{proj\,lim}\{E \hat{\otimes}_\varepsilon F_n, \operatorname{id}_E \hat{\otimes}_\varepsilon \omega^m_n\}$ by

$$\theta \sum_{j=1}^m v_j \otimes u_j = \Big\{\sum_{j=1}^m v_j \otimes \eta_n u_j\Big\}.$$

Since $\operatorname{Im}\eta_n$ is dense in F_n we infer that $\operatorname{Im}\theta$ is dense in $\operatorname{proj\,lim}\{E \hat{\otimes}_\varepsilon F_n, \operatorname{id}_E \hat{\otimes}_\varepsilon \omega^m_n\}$. Whence, the relation

$$\sup_{\substack{v^* \in V^0 \\ u \in \eta_n^{-1}(U)}} \Big|\sum_{j=1}^m v^*(v_j) u^*(u_j)\Big| = \sup_{\substack{v^* \in V^0 \\ \tilde{u}^* \in U^0}} \Big|\sum_{j=1}^m v^*(v_j) \tilde{u}^*(\eta_n u_j)\Big|,$$

for all $V \in U(E)$ and $U \in U(F_n)$ implies that θ is isomorphic. □

Proof of Theorem 1.1. Let E be a Fréchet $EL(\mathfrak{M})$-space and A be a subset of a metric space X. By $\operatorname{Lip}(X, E \hat{\otimes}_\varepsilon F)$ we denote the Fréchet space of Lipschitz maps from X into $E \hat{\otimes}_\varepsilon F$ endowed with the topology generated by all pseudonorms of the form

$$q_L(f) = q(fa_0) + \sup_{x \neq y} \frac{q(fx - fy)}{d(x, y)}$$

where a_0 is an arbitrary fixed point in A and q is a continuous pseudonorm on $E \hat{\otimes}_\varepsilon F$. By Lemmas 2.1 and 2.4 we have:

$$\operatorname{Lip}(X, E \hat{\otimes}_\varepsilon F) = \operatorname{proj\,lim}\{\operatorname{Lip}(X, E \hat{\otimes}_\varepsilon l^1_n), \hat{\theta}^m_n\}$$

and

$$\operatorname{Lip}(A, E \hat{\otimes}_\varepsilon F) = \operatorname{proj\,lim}\{\operatorname{Lip}(A, E \hat{\otimes}_\varepsilon l^1_n), \hat{\theta}''^m_n\}$$

where $\hat{\theta}_n^m$ and $\hat{\theta}_n''^m$ are maps induced by $\mathrm{id}_E \hat{\otimes}_\varepsilon \theta_n^m$. Let j_n: $\mathrm{Lip}(X, E \hat{\otimes}_\varepsilon l_n^1) \to \mathrm{Lip}(A, E \hat{\otimes}_\varepsilon l_n^1)$ be the restriction map. Then $j = \mathrm{proj\,lim}\, j_n$ is the restriction map from $\mathrm{Lip}(X, E \hat{\otimes}_\varepsilon F)$ into $\mathrm{Lip}(A, E \hat{\otimes}_\varepsilon F)$.

To finish the proof of Theorem 1.1 it suffices to show that j_n satisfy the conditions (i), (ii) and (iii) of Lemma 2.3.

(i) is obvious.

(ii) In the notation of Lemma 2.2, let $g \in \mathrm{Lip}(A, E \hat{\otimes}_\varepsilon l_{n+1}^1)$ and $\tilde{g} = (\mathrm{id}_E \hat{\otimes}_\varepsilon q_n) g$. By Lemma 2.1 there exists $\tilde{f} \in \mathrm{Lip}(X, E \hat{\otimes}_\varepsilon C[0, 1])$ such that $\tilde{f} | A = \tilde{g}$. Setting $f = (\mathrm{id}_E \hat{\otimes}_\varepsilon p_{n-1}) \tilde{f}$ we obtain an element $f \in \mathrm{Lip}(X, E \hat{\otimes}_\varepsilon l_{n-1}^1)$ such that

$$\begin{aligned} j_{n-1} f &= (\mathrm{id}_E \hat{\otimes}_\varepsilon p_{n-1})(\tilde{f} | A) = (\mathrm{id}_E \hat{\otimes}_\varepsilon p_{n-1} q_n) \tilde{g} \\ &= (\mathrm{id}_E \hat{\otimes}_\varepsilon \theta_{n+1}^{n-1}) g = \hat{\theta}_{n+1}^{n-1}(g) = \hat{\theta}_{n+1}''^{n-1}(g). \end{aligned}$$

(iii) Let $\{e_k\}$ be the canonical basis of l^1 and $\varepsilon > 0$. By the compactness of $\overline{\{\theta_{n+1}^n e_k\}}$ and by the relation $\overline{\mathrm{Im}\,\theta_{n+2}^n} = \overline{\mathrm{Im}\,\theta_{n+1}^n}$ it follows that there exists a bounded sequence $\{x_k\} \subset l_{n+2}^1$ such that

$$\|\theta_{n+2}^n x_k - \theta_{n+1}^n e_k\| \leqslant \varepsilon \quad \text{for all } k.$$

We define a linear continuous map h: $l_{n+1}^1 \to l_{n+2}^1$ by

$$h(\{\xi_k\}) = \sum_{k=1}^{\infty} \xi_k x_k;$$

obviously

$$\|\theta_{n+2}^n h - \theta_{n+1}^n\| \leqslant \varepsilon.$$

Suppose that $f \in \mathrm{Ker}\, j_{n+1}$. Put $g = (\mathrm{id}_E \hat{\otimes}_\varepsilon h) f$. Then $g \in \mathrm{Lip}(X, E \hat{\otimes}_\varepsilon l_{n+2}^1)$, $j_{n+2} g = 0$ and

$$\begin{aligned} q_L(\hat{\theta}_{n+2}^n g - \hat{\theta}_{n+1}^n f) &= \|(\mathrm{id}_{E_q} \hat{\otimes}_\varepsilon \theta_{n+2}^n h) f - (\mathrm{id}_{E_q} \hat{\otimes}_\varepsilon \theta_{n+1}^n) f\| \\ &= \|(\mathrm{id}_{E_q} \hat{\otimes}_\varepsilon (\theta_{n+2}^n h - \theta_{n+1}^n)) f\| \leqslant \varepsilon q_L(f) \end{aligned}$$

for every continuous pseudonorm q on E, where $E_q = \hat{E}/q^{-1}(0)$.

3. Proof of Theorem 1.2. Since the dual space of a reflexive Fréchet space is bornological, similarly as in [4] we have

LEMMA 3.1 ([4], p. 61). *If $\mathscr{S}$ is the class of all pseudometric spaces and $F \in EL(\mathscr{S})$ is a reflexive Fréchet space, then for every Fréchet space E containing F as a subspace of E, F is complemented in E.*

LEMMA 3.2. *Let F be a infinite-dimensional Montel–Fréchet space and let $U(F) = \{U_n\}$ be a basis of absolutely convex neighbourhoods of zero in F. Let I be the canonical embedding of F into $E = \prod_{n=1}^{\infty} F_n$. Then $I(F)$ is complemented in E if and only if F is isomorphic to the space s.*

Proof. If $F \cong s$, then by Hahn–Banach Theorem $I(F)$ is complemented in E.

Now let T be a linear continuous projection from E onto $I(F)$. Let

$$G_n = \prod_{i=1}^{n} U_i \times \prod_{i>n} F_i.$$

Since T is open, it follows that $\{T(G_n)/m\}$ is a basis of neighbourhoods of zero in $I(F)$ and for every n, T induces a linear continuous map T_n from $E/p_{G_n}^{-1}(0) = \prod_{i=1}^{n} F_i$ onto $I(F)/p_{TG_n}^{-1}(0)$ by the formula:

$$T_n(\{x_1, \ldots, x_n\}) = \eta_n T(\{x_1, \ldots, x_n, 0, \ldots\}).$$

Since $I(F)$ is a Montel space, $T_n(\prod_{i=1}^{n} U_i)$ is a precompact neighbourhood of zero in $I(F)/p_{TG_n}^{-1}(0)$. Hence $\dim I(F)/p_{TG_n}^{-1}(0) < \infty$ for all n. This implies that $I(F) \cong s$. □

Proof of Theorem 1.2. In notations of Lemma 3.2, let P be a Lipschitz projection from E onto $I(F)$. By Lemma 3.1 there exists a linear continuous projection of E onto $I(F)$. Thus by Lemma 3.2 F is isomorphic to s.

The converse part of Theorem 1.2 is obvious. □

4. Some applications. In this section we will apply Theorems 1.1 and 1.2 to the problem of extension of uniformly continuous maps between pseudometric spaces.

Definition. Let f be a uniformly continuous map from a metric space X into a pseudometric space Y. We say that (X, f, Y) has the *unlimited uniform extension property* iff for every metric space Z containing X isometrically there exists a uniformly continuous map $\tilde{f}$: $Z \to Y$ such that $\tilde{f}|X = f$.

Let us prove the following

THEOREM 4.1. *Let (X, d) be a metric space, F be a nuclear Fréchet space and f: $X \to F$ be a uniformly continuous map. Then (X, f, F) has the unlimited uniform extension property if and only if*

$$\limsup_{t \to \infty} t^{-1} \omega_n(f, t) < \infty \quad \textit{for every } n \tag{4.1}$$

where

$$\omega_n(f, t) = \sup\{P_n(fx - fy): d(x, y) \leqslant t\} \tag{4.2}$$

and $\{P_n\}$ is a increasing sequence of pseudonorms inducing the topology of F.

Proof. The necessity of (4.1) follows from a theorem of Aronszajn and Panitchpakdi (see [1], Theorem 2, p. 408).

Let us assume that (4.1) holds for every n and Z is a metric space containing X isometrically. By a theorem of Kuratowski and Wojdysławski we can assume that Z is a normed space. Put

$$F_n = \widehat{F/P_n^{-1}(0)} \quad \text{and} \quad U_n^0 = \{x^* \in F_n^*: \|x^*\| \leqslant 1\}.$$

By a theorem of Aronszajn and Panitchpakdi ([1]) the condition (4.1) implies that for every n, there exists a uniformly continuous map f_n: $Z \to l_\infty(U_n^0)$ such that $f_n | X = \eta_n f$, where η_n: $F \to F_n \hookrightarrow l_\infty(U_n^0)$ is the canonical embedding. From the uniform continuity of f_n it follows that

$$\lim_{t \to 0} \omega_n(f_n, t) = 0.$$

For every $n \in N$, select an $\varepsilon_n > 0$ such that $\omega_n(f_n, \varepsilon_n) \leqslant 1$. By Theorem 1 [1], the condition (4.1) implies the existence of a nondecreasing subadditive function φ_n: $[0, \infty) \to [0, \infty)$ such that

$$\lim_{t \to 0} \varphi_n(t) = 0 = \varphi_n(0)$$

and

$$\omega_n(f_n, t) \leqslant \varphi_n(t) \quad \text{for all} \quad t \in [0, \infty).$$

For every $x, y \in Z$ put

$$d_n(x, y) = \varphi_n(\min \varepsilon_n, \|x-y\|) \leqslant \varphi_n(\varepsilon_n). \tag{4.3}$$

Since φ_n is subadditive, d_n is a pseudometric on Z. Obviously d_n is uniformly continuous.

Define a metric ϱ on Z by

$$\varrho(x, y) = \|x-y\| + \sum_{n=1}^{\infty} d_n(x, y)/2^n M_n \quad \text{where} \quad M_n = \varphi_n(\varepsilon_n).$$

Obviously ϱ is uniformly equivalent to the norm $\|\cdot\|$ of Z. Thus to complete the proof of Theorem 4.1, by Theorem 1.1 remains to check that f: $(X, \varrho) \to F$ is a Lipschitz map.

Let $n \in N$, $x, y \in X$. If $\|x-y\| = t \geqslant \varepsilon_n$ then we can take $x_0, x_1, \ldots, x_k \in Z$ such that $x_0 = x$, $x_k = y$ and that:

$$\|x_i - x_{i-1}\| = \frac{\|x-y\|}{k} \leqslant \varepsilon_n \quad \text{for} \quad i = 1, \ldots, k$$

where $k = [t/\varepsilon_n] + 1$. Then we get

$$P_n(fx - fy) \leqslant \sum_{i=1}^{k} P_n(f_n x - f_n x_{i-1}) \leqslant k \leqslant t/\varepsilon_n + 1 \leqslant 2\|x-y\|/\varepsilon_n.$$

If $\|x-y\| \leqslant \varepsilon_n$, then

$$P_n(fx-fy) \leqslant \omega_n(f, \|x-y\|) \leqslant \varphi_n(\|x-y\|)$$
$$= \varphi_n(\min \varepsilon_n, d_n(x, y)) = d_n(x, y) \leqslant 2^n M_n \varrho(x, y).$$

Thus $f\colon (X, \varrho) \to F$ is a Lipschitz map and the proof of Theorem 4.1 is finished by Theorem 1.1. □

THEOREM 4.2. *Let F be a infinite dimensional Montel–Fréchet space. Then F is a uniform retract of every Fréchet space E containing F as a subspace if and only if F is isomorphic to s.*

Proof. The argument of Mankiewicz [3] shows that the existence of a uniformly continuous retraction from a separable Fréchet space E containing F as a subspace onto F implies the existence of a Lipschitz retraction from E onto F. Thus the result follows from Theorem 1.2. □

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