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# *ON BOHR CLUSTER SETS*

BY

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In this article* we construct an infinite set $E \subset \boldsymbol{Z}$, the integers, which cannot be partitioned into two infinite subsets with mutually disjoint closures in the Bohr compactification $\overline{\boldsymbol{Z}}$ of $\boldsymbol{Z}$. That is, the set of Bohr cluster points is connected. This question was originally posed by S. Hartman in 1970 and appeared later in [2]. It is equivalent to: given an infinite $E \subset \boldsymbol{Z}$ must there always exist a discrete measure $\mu$ on $\boldsymbol{T}$ such that the sets $E_0 = E \cap (\hat{\mu})^{-1}(0)$ and $E_1 = E \cap (\hat{\mu})^{-1}(1)$ are both infinite and partition $E$? In this form, but without the word "discrete", the question was posed a few years later by M. Bożejko and was recently solved by McGehee (see [1], p. 226). He showed that such measures always exist by constructing an appropriate continuous $\mu$ using Riesz products whenever $E$ has the property that either $E \cap S$ or $E \setminus S$ is finite for every $S$ in the coset ring of $\boldsymbol{Z}$. Sets $E$ of the form $\{j!k\colon 0 \leqslant k \leqslant N_j, 1 \leqslant j\}$ are of this type, yet at the same time they "approximate" subgroups of $\boldsymbol{Z}$. Our set is one of these. The idea is to choose $N_j \to \infty$ rapidly enough so that the joint behavior of any finite collection of characters on $E$ reflects their behavior on $\boldsymbol{Z}$ close enough to obtain the connectivity required.

The author is greatly indebted to O. C. McGehee and T. Ramsey for pointing out this problem and for the many conversations concerning it, and to S. Hartman who encouraged the author through an earlier somewhat erroneous version of this paper. This result has been independently announced jointly by Y. Katznelson and T. Ramsey [4].

We begin with notation. Let $\boldsymbol{T}^n$ denote the $n$-dimensional torus $[0, 2\pi)^n \cong \boldsymbol{R}^n/2\pi\boldsymbol{Z}^n$ under the quotient Euclidean metric

$$\|t\| = \inf\{\|t-2\pi a\|_2\colon a \in \boldsymbol{Z}^n\},$$

where $\|\cdot\|_2$ denotes the usual Euclidean metric on $\boldsymbol{R}^n$. Let $B(\varepsilon)$ denote

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the open ball of radius $\varepsilon$ in $T^n$ (the dimension of $B$ will be clear in the context). For $t \in T^n$ and $M > 0$ define

$$R(t, M) = \{kt\colon 0 \leqslant k \leqslant M, k \in Z\} \quad \text{and} \quad R(t) = R(t, \infty).$$

Given $x, y \in R^n$ and $A \subset R^n$, we write $x \cdot y$ for the usual inner product, $x \cdot A = \{x \cdot a\colon a \in A\}$, and $A^{\perp} = \{x\colon x \cdot A = \{0\}\}$. In particular, if $H \subset T^n$ is a subgroup, then $H^{\perp} \subset Z^n$ is its annihilator. If $H \subset T^n$ is a closed subgroup, then the component of $H$ which contains zero is a subgroup which will be denoted by $H_0$. Note that $H_0 \cong T^k$ for some $k \leqslant n$. Define $\dim(H) = \dim(H_0) = k$. A subset $K \subset H$ is *$\varepsilon$-dense* in $H$ if $H \subset K + [B(\varepsilon) \cap H_0]$. Note that this implies that $K$ occurs quite frequently in each component of $H$ and is considerably stronger than $H \subset K + B(\varepsilon)$ for most $H$. We use the usual characterization of basic Bohr open sets in $Z$. That is, given $x = (x_i)$, $a = (a_i) \in T^n$, and $\delta = (\delta_i) \in \Delta^n \equiv [0, 1)^n$, let

$$U(x, a, \delta) = \{k \in Z\colon \|kx_i - a_i\| < \delta_i \text{ for all } i\}.$$

The collection of all such sets forms a base for the relative Bohr topology on $Z$. This is precisely the weak topology generated by all the maps $t = (t_1, \ldots, t_l) \in T^l$ for $1 \leqslant l < \infty$ which map $Z \to T^l$ by $k \mapsto tk$.

The first lemma uses a classical tool from the theory of Rajchman sets (see [3], p. 58) and is no doubt well known.

LEMMA. *Let $\varepsilon > 0$ and let $n$ be a positive integer. Then there is an $r > 0$ such that $R(t)$ is $\varepsilon$-dense in $T^n$ whenever $t \in T^n$ and*

$$\inf\{\|a\|_2\colon a \in R(t)^{\perp}, a \neq 0\} > r.$$

Proof. Denote the algebra of absolutely converging Fourier series on $T^n$ by $A(T^n)$. Let $0 < f \in A(T^n)$ with $\operatorname{supp} f \subset B(\varepsilon)$. Choose $r > 0$ so that

$$\Big(\sum_a |\hat{f}(a)|\colon \|a\|_2 \geqslant r\Big) < \hat{f}(0) = \|f\|_1.$$

Now suppose $t \in T^n$ and that $R(t)$ is not $\varepsilon$-dense in $T^n$. Then for some $y \in T^n$ we must have $[y + B(\varepsilon)] \cap \overline{R(t)} = \emptyset$. Denote the Haar measure on $\overline{R(t)}$ by $\mu$. Then

$$0 = \int f_y d\mu = \sum \hat{f}(a) e^{ia\cdot y} \hat{\mu}(a) = \|f\|_1 + \sum_{a \neq 0} f(a) e^{-ia\cdot y} \hat{\mu}(a).$$

Since $\hat{\mu}$ is the characteristic function of $R(t)^{\perp}$, our choice of $r$ implies that $\hat{\mu}(a) = 1$ for some $a$ such that $0 < \|a\|_2 < r$.

Note that this lemma implies immediately that for each $\varepsilon > 0$ there is a finite set $C(T^n, \varepsilon)$ of $(n-1)$-dimensional closed subgroups of $T^n$ such that either $R(t)$ is $\varepsilon$-dense in $T^n$ or $t \in S$ for some $S \in C(T^n, \varepsilon)$, for all $t \in T^n$.

The lemma works equally well for any connected closed subgroup $H$ of $T^m$ with $\dim(H) = n$ and, with slight modifications, for any closed subgroup $H$ of $T^n$, as the following corollary shows:

COROLLARY. *Let $\varepsilon > 0$ and suppose $H$ is a closed subgroup of $T^m$ of dimension $k$. Then there are a finite collection $C_k(H, \varepsilon)$ of proper $k$-dimensional subgroups of $H$ and a finite collection $C_{k-1}(H, \varepsilon)$ of $(k-1)$-dimensional subgroups of $H$ such that $R(t)$ is $\varepsilon$-dense in $H$ for each $t$ belonging to the set $H\setminus\bigcup\{S\in C_k(H,\varepsilon)\cup C_{k-1}(H,\varepsilon)\}$.*

Proof. The quotient group $H/H_0$ is compact and discrete, hence finite, and $mH_0 = H_0$ for any $m\in \mathbf{Z}$. It follows that $H \cong H_0\oplus D$ for some finite group $D$ in $H$ with order, say, $m$. Fix such an isomorphism and let $C_k(H, \varepsilon)$ be the collection of those subgroups of $H$ that correspond to $H_0\oplus D'$ for some proper subgroup $D'$ of $D$. If $t\in H$ does not lie in any of these subgroups, then $t \pmod{H_0}$ must be a generator of $H/H_0$. Write $t = t_0+t_1$, where $t_0\in H_0$ and $mt_1 = 0$. If $R(t_0)$ is $(\varepsilon/m)$-dense in $H_0$, then

$$\{lt\colon\ l\equiv j \pmod m\}+[B(\varepsilon)\cap H_0] = H_0+jt_1.$$

On the other hand, if $t_0$ is not $(\varepsilon/m)$-dense in $H_0$, then $t_0$ belongs to a group in $C(H_0, \varepsilon/m)$ and $t$ must be a member of some group in $C_{k-1}(H, \varepsilon) \equiv \{S+D\colon\ S\in C(H_0, \varepsilon/m)\}$.

We are now in a position to prove our main result.

THEOREM 1. *Let $n$ be a fixed positive integer. Then there is a sequence $\{N_j\}$ such that for each $t\in T^n$ and each $\varepsilon > 0$ there are a connected set $C\subset T^n$ and a finite set $F\subset E = \{j!\,k\colon\ 0\leqslant k\leqslant N_j, 1\leqslant j<\infty\}$ such that*

$$\{lt\colon\ l\in E\setminus F\}\subset C+B(\varepsilon/2)\subset \{lt\colon\ l\in E\setminus F\}+B(\varepsilon).$$

Proof. Given a closed subgroup $H\subset T^n$ of dimension $k$, $0\leqslant k\leqslant n$, set $C(H, \varepsilon) = C_k(H, \varepsilon)\cup C_{k-1}(H, \varepsilon)$. It follows from the proof of the corollary that there is a finite, partially ordered (by set inclusion) family $\mathscr{C}(H, \varepsilon)$ of subgroups of $H$ which satisfy

(i) $H\in\mathscr{C}(H, \varepsilon)$,

(ii) $S\in\mathscr{C}(H, \varepsilon) \Rightarrow C(S, \varepsilon)\subset\mathscr{C}(H, \varepsilon)$.

For each $l$ $(0\leqslant l\leqslant \dim(H))$ let

$$\mathscr{C}_l(H, \varepsilon) = \{S\in\mathscr{C}(H, \varepsilon)\colon\ \dim(S) = l\}.$$

Let $\mathscr{O}(H, \varepsilon)$ denote the set of components of elements in $\mathscr{C}(H, \varepsilon)$ and let $\omega(H, \varepsilon) > 0$ denote the minimum distance between disjoint elements of $\mathscr{O}(H, \varepsilon)$. Since $\mathscr{O}(H, \varepsilon)$ is a finite collection of compact sets, for each $\delta > 0$ there is some $\sigma = \sigma(H, \varepsilon, \delta) > 0$ such that if $K, L\in\mathscr{O}(H, \varepsilon)$, then

$$\big(K+B(\sigma)\big)\cap\big(L+B(\sigma)\big)\subset (K\cap L)+B(\delta).$$

Now we fix our attention on $H = T^n$ and proceed to choose the $N_j$. Let $\{\varepsilon_j\}_{j\geqslant 1}$ be a positive sequence bounded by 1 such that $j!\varepsilon_j$ decreases to zero. Note that we can assume, by taking finite unions if necessary, that the $\mathscr{C}(T^n, \varepsilon_j)$ are increasing with $j$. Set

$$\omega_{j+1} = \omega(T^n, \varepsilon_{j+1}) \quad \text{and} \quad \sigma_{j+1} = \min\{\sigma(T^n, \varepsilon_{j+1}, \omega_{j+1}/2), \varepsilon_{j+1}/2\}.$$

Note that $2\sigma_{j+1} < \omega_{j+1}$. Let $\mathscr{K}_j(l) = \bigcup\{S \in \mathscr{C}_l(T^n, \varepsilon_j)\}$ for $0 \leqslant l \leqslant n$ and set $N_j(0) = \mathrm{LCM}\{\mathrm{order}(S)\colon S \in \mathscr{C}_0(T^n, \varepsilon_j)\}$. Let

$$K_j(0) = \mathscr{K}_j(0), \quad K_j(1) = \mathscr{K}_j(1)\diagdown\big[\mathscr{K}_j(0) + B\big(\sigma_{j+1}/j!2N_j(0)\big)\big].$$

Each $h \in K_j(1)$ must have $R(h)$ at least $\varepsilon_j$-dense in some $S \equiv S_j(h) \in \mathscr{C}_1(T^n, \varepsilon_j)$. Choose any particular $S_j(h)$ if more than one exists. Hence there are $N$ and a neighborhood $U$ of $h$ in $S$ such that $R(z, N)$ is $\varepsilon_j$-dense in $S$ for each $z \in U$. Since $S \cap K_j(1)$ is compact and $\mathscr{C}_1(T^n, \varepsilon_j)$ is finite, there is some $N_j(1) \geqslant N_j(0)$ such that $R\big(h, N_j(1)\big)$ is $\varepsilon_j$-dense in $S_j(h)$ for each $h \in K_j(1)$. Similarly, set

$$K_j(2) = \mathscr{K}_j(2)\diagdown\big[\mathscr{K}_j(1) + B\big(\sigma_{j+1}/j!2^2N_j(1)\big)\big]$$

and pick $N_j(2) \geqslant N_j(1)$, and continue, eventually obtaining

$$K_j(n) = \mathscr{K}_j(n)\diagdown\big[\mathscr{K}_j(n-1) + B\big(\sigma_{j+1}/j!2^nN_j(n-1)\big)\big]$$

and $N_j(n) \geqslant N_j(n-1)$ from which it follows that $R\big(h, N_j(n)\big)$ is $\varepsilon_j$-dense in $S_j(h)$ for some $S_j(h) \in \mathscr{C}_n(T^n, \varepsilon_j)$, and each $h \in K_j(n)$, realizing of course that at this $n$-th stage $S_j(h) = T^n$. Set $N_j = N_j(n)$.

We now show that $\{N_j\}$ satisfies the assertion of the theorem. There are two cases. Let $t \in T^n$. Either $t \in K_j(n)$ for infinitely many $j$'s or not. If so, set $C = T^n$ and $F = \emptyset$. Let $\varepsilon > 0$ be given and choose $j$ so that $t \in K_j(n)$ and $j!\varepsilon_j < \varepsilon$. Then $R(t, N_j)$ $\varepsilon_j$-dense in $T^n$ implies that $R(j!t, N_j)$ is $(j!\varepsilon_j)$-dense in $T^n$. Hence $T^n \subset \{lt\colon l \in E\} + B(\varepsilon)$. Thus we can assume that $t \notin K_j(n)$ for some $j_0 > 3$ and all $j \geqslant j_0$. Choose any $\varepsilon > 0$ and fix $k \geqslant j_0$ so that $2k!\varepsilon_k < \varepsilon$. For each $m \geqslant k$, our construction yields an $l_m$, $0 \leqslant l_m < n$, and a point $h_m \in K_m(l_m)$ which is the orthogonal projection of $t$ onto some $S_m \in \mathscr{C}_{l_m}(T^n, \varepsilon_m)$ with $R\big(h_m, N_m(l_m)\big)$ $\varepsilon_m$-dense in $S_m$ and with $t \in h_m + B\big(\sigma_{m+1}/m!2^{l_m}N_m(l_m)\big)$. Actually we obtain $h_m$ through a finite succession of orthogonal projections

$$t \to t_{n-1} \to t_{n-2} \to \ldots \to h_m,$$

where $t_{n-i} \in \mathscr{K}_m(n-i)$ and $t_{n-i} \in t_{n-i-1} + B\big(\sigma_{m+1}/m!2^{n-i}N_m(n-i-1)\big)$. In particular,

$$R\big(m!t, N_m(l_m)\big) \subset R\big(m!h_m, N_m(l_m)\big) + B(\sigma_{m+1}) \subset m!S_m + B(\sigma_{m+1}).$$

Moreover, $R\big(m!h_m, N_m(l_m)\big)$ is $(m!\varepsilon_m)$-dense in $m!S_m$. Recall that $S_m + B(\sigma_{m+1})$ is a disjoint union of open "bands" about the components of $S_m$. Indeed, our choice of $\sigma_{m+1}$ insures that if $K, L \in \mathcal{O}(T^n, \varepsilon_{m+1})$ and if $(K+B(\sigma_{m+1}))\cap(L+B(\sigma_{m+1})) \neq \emptyset$, then $K\cap L \neq \emptyset$. It follows that if $\tilde{S}_m$ denotes the component of $S_m$ which contains $h_m$, then $\tilde{S}_m\cap\tilde{S}_{m+1} \neq \emptyset$. Thus the set $l(\tilde{S}_m\cup\tilde{S}_{m+1})$ is connected for any integer $l$. Similarly, for any $j \geqslant m$ and for any $l$ the set $l(\tilde{S}_m\cup \ldots \cup\tilde{S}_j)$ is connected. Set $m = k$ and choose $j \geqslant k$ so that $\operatorname{order}(S_k/(S_k)_0)$ divides $j!$. Then the set

$$j!l(\tilde{S}_k\cup \ldots \cup\tilde{S}_j) = j!l[(S_k)_0\cup\tilde{S}_{k+1}\cup \ldots \cup\tilde{S}_j]$$

is connected for any $l$. Fix such a $j$ ($n, k, j$ are now fixed). It follows that

$$C' = \bigcup_{m\geqslant j}\bigcup_{1\leqslant l<\infty} m!l[(S_k)_0\cup\tilde{S}_{k+1}\cup \ldots \cup\tilde{S}_m] = j!\big((S_k)_0\cup S_{k+1}\cup \ldots \cup S_j\big)\cup\Big(\bigcup_{m>j} m!S_m\Big)$$

is a connected set. Put

$$\begin{aligned} F = {} & \{m!l\colon 1\leqslant m<k, 1\leqslant l\leqslant N_m\}\cup \\ & \cup\{m!l\colon k\leqslant m<j, 1\leqslant l\leqslant N_m(l_m), m!lS_m \not\subset j!S_m\}\cup \\ & \cup\{m!l\colon k\leqslant m\leqslant j, N_m(l_m)<l\leqslant N_m\}. \end{aligned}$$

As the notation suggests, $C'$ must be modified to obtain the required set $C$. The problem is that usually $N_m \gg N_m(l_m)$ so that $m!lt$ is not close to $m!lh_m$ throughout the segment $0 \leqslant l \leqslant N_m$. To adjust for this let $m > j$ and consider the sets

$$M_r = \big\{m!lt\colon rN_m(l_m) < l \leqslant \min\big((r+1)N_m(l_m), N_m\big)\big\}$$

for $0 \leqslant r \leqslant r_1 = [(N_m-1)/N_m(l_m)]$. Since $t = t_0+h_m$, where $t_0$ belongs to $B(\sigma_{m+1}/m!2^{l_m}N_m(l_m))$, and since $R(m!h_m, N_m(l_m))$ is $(m!\varepsilon_m)$-dense in $m!S_m$, the set $[M_r - m!rN_m(l_m)t_0 + B(\sigma_{m+1})]\cap S_m$ must be $(m!\varepsilon_m)$-dense in $m!S_m$ for $0 \leqslant r < r_1$. Hence we consider the sequence

$$m!S_m,\ m!S_m+t_1,\ \ldots,\ m!S_m+(r_1-1)t_1,$$

where $t_1 = m!N_m(l_m)t_0 \in B(\sigma_{m+1}) \subset B(\varepsilon_{m+1}/2) \subset B(\varepsilon/(m+1)!2)$. The corresponding components in adjacent elements of this sequence are within $\sigma_{m+1}$ of each other; hence we can connect them with line segments of length bounded by $\sigma_{m+1}$. Let $C_m$ be the set $m!S_m\cup \ldots \cup(m!S_m+(r_1-1)t_1)$ together with these connecting segments. Then $C_m \subset R(m!t, N_m)+B(\varepsilon)$. Moreover, since

$$m!S_m+(r_1-1)t_1+B(\varepsilon/2) \supset m!S_m+(r_1-1)t_1+B(\sigma_{m+1}) \supset M_{r_1},$$

we have $C_m+B(\varepsilon/2) \supset R(m!t, N_m)$. Put

$$C = C'\cup\Big(\bigcup_{m>j} C_m\Big).$$

Note that $C$ is connected since each component of $C_m$ contains a component of $m!S_m \subset C'$.

It remains only to show that $C$ and $F$ satisfy the final condition of the theorem. We have already shown that for $m > j$

$$R(m!t, N_m) \subset C_m + B(\varepsilon/2) \subset R(m!t, N_m) + B(\varepsilon);$$

note that $F$ contains none of the indices used here. For $k \leqslant m \leqslant j$ the set $R(m!h_m, N_m(l_m))$ is $(m!\varepsilon_m)$-dense in $m!S_m$, which, by the definition of "$\varepsilon$-dense", implies that the set

$$R(m!h_m, N_m(l_m)) \cap j!S_m = \{m!lh_m\colon\ m \text{ fixed},\ m!l \in E \setminus F,\ l \leqslant N_m\}$$

is $(m!\varepsilon_m)$-dense in $j!S_m$. Since $t \in h_m + B(\sigma_{m+1}/m!N_m(l_m))$, we conclude that

$$\{m!lt\colon\ m \text{ fixed}, m!l \in E \setminus F, l \leqslant N_m\} \subset j!S_m + B(\varepsilon/2)$$
$$\subset \{m!lt\colon\ m \text{ fixed}, m!l \in E \setminus F, l \leqslant N_m\} + B(\varepsilon).$$

Taking the union over $k \leqslant m \leqslant j$ and then over $m > j$, we obtain

$$\{lt\colon\ l \in E \setminus F\} \subset C + B(\varepsilon/2) \subset \{lt\colon\ l \in E \setminus F\} + B(\varepsilon).$$

The sequence $\{N_j\} = \{N_j(n)\}$ in Theorem 1 can be replaced by any increasing sequence $M_j \geqslant N_j$. In particular, for each $T^n$ Theorem 1 produces a sequence $\{N_j(n)\}_j$. Define $M_j = \max\{N_j(n)\colon 1 \leqslant n \leqslant j\}$ and observe that the following corollary is an immediate consequence of Theorem 1 and its proof.

COROLLARY. *There is a sequence $\{M_j\}$ such that for each $n$, each $t \in T^n$, and each $\varepsilon > 0$ there are a connected set $C \subset T^n$ and a finite set $F \subset E = \{j!k\colon 0 \leqslant k \leqslant M_j, 1 \leqslant j < \infty\}$ such that*

$$\{lt\colon\ l \in E \setminus F\} \subset C + B(\varepsilon/2) \subset \{lt\colon\ l \in E \setminus F\} + B(\varepsilon).$$

THEOREM 2. *Let $\{M_j\}$ be as in the corollary. Then the set of Bohr cluster points in $\bar{Z}$ of the set*

$$E = \{j!l\colon\ 0 \leqslant l \leqslant M_j, j = 1, 2, \ldots\}$$

*is connected.*

Proof. The Bohr group $\bar{Z}$ can be continuously and isomorphically represented as a subgroup of $T^\omega$ by taking the closure $B$ in $T^\omega$ of the range of $\varphi\colon Z \to T^\omega$ by $(\varphi(k))_t = tk$ and observing that $\varphi$ extends to a continuous isomorphism $\varphi'\colon \bar{Z} \to B$. Since $B$ is also compact, $\varphi'$ is bicontinuous. To say that the set of cluster points to $E$ in $\bar{Z}$ is not connected is to imply the existence of two infinite subsets $A$, $E \setminus A$ of $E$ with disjoint closures $\bar{A}$, $\overline{E \setminus A}$ in $\bar{Z}$. Hence $K = \varphi'(\bar{A})$ and $L = \varphi'(\overline{E \setminus A})$ are disjoint compact sets in $B$; consequently, there are open sets $U$, $V$ in $B$ with disjoint closures such that $L \subset U$, $K \subset V$, and $U$, $V$ are finite unions of basic Bohr open sets. Hence there is a finite-dimensional projection $\pi\colon T^\omega \to T^n$ such that $U = \pi^{-1}\pi(U)$, $V = \pi^{-1}\pi(V)$, and therefore $\pi(K \cup L)$ is not connected in $T^n$. Let $t = (t_1, \ldots, t_n)$ denote the coordinates of projection by $\pi$. Then

$$\pi(K) = \overline{\{lt\colon l \in A\}} \quad \text{and} \quad \pi(L) = \overline{\{lt\colon l \in E \setminus A\}}.$$

Let $3\varepsilon > 0$ denote the minimum distance between these two sets. Now apply the corollary to Theorem 1. There are a finite set $F$ and a connected set $C \subset T^n$ such that

$$\overline{\{lt\colon l \in E \setminus F\}} \subset \overline{C} + \overline{B(\varepsilon/2)} \subset \overline{\{lt\colon l \in E \setminus F\}} + \overline{B(\varepsilon)},$$

which implies that $\pi(K)$ and $\pi(L)$ can differ by at most $2\varepsilon$, a contradiction.

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