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A GAME OF TIMING WITH k NOISY AND n SILENT ACTIONS VERSUS ONE NOISY ACTION

0. Introduction. The game considered in this paper is a generalization of that presented by Styszyński in [1]. We describe the structure of the game in the following way:

Player A has k noisy and n silent actions ($k \geqslant 0$, $n \geqslant 1$) and player B has one noisy action. The noisy actions of player A are undertaken before the silent ones.

The success functions $P(t)$ and $Q(t)$ for players A and B, respectively, denote the probability of achieving success by the given player at time t, $t \in [0, 1]$. We assume that $P(t)$ and $Q(t)$ are differentiable in the open interval $(0, 1)$ and, moreover, $P'(t) > 0$, $Q'(t) > 0$, and $P(0) = Q(0) = 0$, $P(1) = Q(1) = 1$.

If the first player achieves success, then the game is finished and he receives the pay-off $+1$ from his opponent. If both players achieve success at the same time or if neither does it, then the pay-off is 0. The players tend to maximize their expected pay-offs. The numbers, the types of actions, the order of their taking and the success functions are fixed and known beforehand to both players.

We show that the game has a value and we give optimal strategies for the players.

In section 1 we describe the strategy spaces, section 2 contains the definitions of the optimal strategies, and in section 3 the proof of the optimality of the formulated strategies can be found.

1. Strategy spaces. Let $\{z_i\}_{i=1}^{r}$ and $\{x_j\}_{j=1}^{s}$ denote the sets of the moments of taking by player A his noisy and silent actions, respectively. Clearly,

$$0 \leqslant z_1 \leqslant \ldots \leqslant z_r \leqslant x_1 \leqslant \ldots \leqslant x_s \leqslant 1.$$

Let y denote the moment of taking by player B his noisy action. Put $Y = \{y\colon 0 \leqslant y \leqslant 1\}$.

Let us set

$$\bar{z}_r = (z_1, \ldots, z_r), \quad \bar{x}_s = (x_1, \ldots, x_s), \quad (\bar{z}_r \bar{x}_s) = (z_1, \ldots, z_r, x_1, \ldots, x_s).$$

The vector obtained from $(\bar{z}_r, \bar{x}_s)$ by omitting the first t components, $0 \leqslant t \leqslant r$, is denoted by $(\bar{z}_{r,t}, \bar{x}_s)$, where $(\bar{z}_{r,0}, \bar{x}_s) = (\bar{z}_r, \bar{x}_s)$ and $(\bar{z}_{r,r}, \bar{x}_s) = \bar{x}_s$. We put

$$\overline{Z_i X_n} = \{(\bar{z}_i, \bar{x}_n)\colon 0 \leqslant z_1 \leqslant \ldots \leqslant z_i \leqslant x_1 \leqslant \ldots \leqslant x_n \leqslant 1\}.$$

Let $\Gamma_{ij}(s)$ denote a game of timing in which player A has i noisy and j silent actions and all of these actions are to be taken in the interval $(s, 1]$ $(0 \leqslant s < 1)$.

A strategy of player A in the game $\Gamma_{in}(0)$ means any probability distribution F_i over the space $\overline{Z_i X_n}$. The set of the strategies is denoted by A_i.

Let $B_i(s)$ denote the set of strategies $\eta_i(s)$ of player B in the game $\Gamma_{in}(s)$ defined as follows:

1° $\eta_0(s) = G_0$, where G_0 is a fixed probability distribution on $(s, 1]$.

2° Let us assume that the sets $B_p(s)$ of strategies of player B in $\Gamma_{pn}(s)$ have been defined for some p $(0 \leqslant p < i)$ and for s $(0 \leqslant s < 1)$.

3° The strategy $\eta_i(s)$ $(0 \leqslant s < 1)$ is defined in the following manner: Let $\overline{\eta_i(y)} = [y, \{\eta_{i-w}(v)\}]$, where $\eta_{i-w}(v) \in B_{i-w}(v)$ for $w = 1, \ldots, i$ and $0 \leqslant v < y$. Using the strategy $\overline{\eta_i(y)}$, player B will take his action at the moment y if player A takes no action before y. If player A takes w of his actions up to the moment v, $v < y$, and achieves no success, then player B will follow the strategy $\eta_{i-w}(v) \in B_{i-w}(v)$.

Let $G_i(y)$ be a fixed probability distribution on $[s, 1]$. We define η_i by

$$\eta_i(s) = [G_i(y), \overline{\eta_i(y)}] \in B_i(s).$$

We say that player B *adopts the strategy* $\eta_i(s) \in B_i(s)$ in the game $\Gamma_{in}(s)$ if he chooses the moment y according to the probability distribution $G_i(y)$ and if he adopts $\overline{\eta_i(y)}$ afterwards. We write $\Gamma_{in}(0) = \Gamma_{in}$, $\eta_i(0) = \eta_i$, and $B_i(0) = B_i$ by definition.

Now we define the pay-off function $K[F_i; \eta_i]$ (the expected pay-off to player A if players A and B use the strategies $F_i \in A_i$ and $\eta_i \in B_i$, respectively) as follows:

$$K[F_i; \eta_i] = \int_{\overline{Z_i X_n}} \int_Y K[(\bar{z}_i, \bar{x}_n); \overline{\eta_i(y)}]\, dG_i(y)\, dF_i[(\bar{z}_i, \bar{x}_n)]; \tag{1}$$

for $j = 1, 2, \ldots,$

$$K[(\bar{z}_j, \bar{x}_n); \overline{\eta_j(y)}] \tag{2}$$

$$
= \begin{cases}
1-2Q(y) \quad \text{if } y < z_1, \\
1-[1-P(z_1)]^s + [1-P(z_1)]^s K[(\bar{z}_{j,s}; \bar{x}_n); \eta_{j-s}(z_1)] \\
\quad \text{if } z_1 = \ldots = z_s < y, z_s < z_{s+1}, 1 \leqslant s \leqslant j, z_{j+1} = x_1; \\
1-Q(z_1)-[1-P(z_1)]^s Q(z_1) \\
\quad \text{if } z_1 = \ldots = z_s = y < z_{s+1}, 1 \leqslant s \leqslant j, \\
1-[1-P(z_1)]^{j+s} + [1-P(z_1)]^{j+s} K[\bar{x}_{n,s}; \eta_0(z_1)] \\
\quad \text{if } z_1 = \ldots = z_j = x_1 = \ldots = x_s < y, x_s < x_{s+1}, 1 \leqslant s \leqslant n, x_{n+1} = 1, \\
1-Q(z_1)-[1-P(z_1)]^{j+s} Q(z_1) \\
\quad \text{if } z_1 = \ldots = z_j = x_1 = \ldots = x_s = y < x_{s+1}, 1 \leqslant s < n, \\
\{1-[1-P(z_1)]^{j+n}\}[1-Q(z_1)]-[1-P(z_1)]^{j+n} Q(z_1) \\
\quad \text{if } z_1 = \ldots = z_j = x_1 = \ldots = x_n = y;
\end{cases}
$$

(3)
$$
K[x_j; \eta_0(s)] = \int_Y K[\bar{x}_j; y] dG_0(y);
$$

(4)
$$
K[\bar{x}_j; y] = \begin{cases}
1-[1-P(x_1)]^s + [1-P(x_1)]^s K[\bar{x}_{j,s}; y] \\
\quad \text{if } x_1 = \ldots = x_s < y, x_s < x_{s+1}, 1 \leqslant s \leqslant j, \\
1-Q(x_1)-[1-P(x_1)]^s Q(x_1) \\
\quad \text{if } x_1 = \ldots = x_s = y, x_s < x_{s+1}, 1 \leqslant s < j, \\
\{1-[1-P(x_1)]^j\}[1-Q(x_1)]-[1-P(x_1)]^j Q(x_1) \\
\quad \text{if } x_1 = \ldots = x_j = y, \\
1-2Q(y) \quad \text{if } y < x_1;
\end{cases}
$$

(5)
$$
K[\bar{x}_{j,j}; y] = -1 \quad \text{if } 0 \leqslant y \leqslant 1.
$$

This system of equations determines in a unique way the function $K[F_i; \eta_i]$. Assume that player A adopts the strategy $(\bar{z}_j; \bar{x}_n)$ and player B uses $\eta_j(y)$ in the game Γ_{jn} and let $z_1 = \ldots = z_s < y$. Then we can interpret formula (2) in the following intuitive manner: If player A achieves success with one of his first s noisy actions taken at z_1, then he will win $+1$ with probability $1-[1-P(z_1)]^s$. Otherwise, he will win $K[(\bar{z}_{j,s}, \bar{x}_n); \eta_{j-s}(z_1)]$ with probability $[1-P(z_1)]^s$. The other cases of system (1)-(4) can be explained in a similar way.

Now we can consider our game as

$$
\Gamma_{kn} = \langle A_k, B_k, K \rangle,
$$

where A_k and B_k are the sets of strategies of the players and K is the pay-off function defined on $A_k \times B_k$ by formulas (1)-(5).

Definition. The strategy $F_i \in A_i$ is said to be *optimal* and the strategy $\eta_i^\varepsilon \in B_i$ is said to be *ε-optimal* if for fixed $\varepsilon > 0$ there exists a constant v such that

$$K[F_i; \eta_i] \geqslant v \quad \text{for every } \eta_i \in B_i,$$

$$K[F; \eta_i^\varepsilon] \leqslant v+\varepsilon \quad \text{for every } F \in A_i.$$

The number v is called the *value of the game.*

2. Definitions of the optimal strategies. Let us define the function W by

$$W(t) = P(t)Q(t)+P(t)+Q(t)-1, \qquad 0 \leqslant t \leqslant 1,$$

and consider the following system of relations:

$$\int_{a_n}^{1} \frac{Q'(u)[1+P(u)]du}{W(u)} + \ln\frac{Q(z)}{2} = 0,$$

$$\int_{a_i}^{a_{i+1}} \frac{Q'(u)du}{P(u)Q^2(u)} - \frac{1}{Q(a_i)} = 0, \qquad i = n-1, \ldots, 1,\ n \geqslant 2,$$

$$f_n(x_n) = \frac{2Q'(x_n)}{W(x_n)} \exp\left[-\int_{a_n}^{x_n} \frac{Q'(u)[1+P(u)]du}{W(u)}\right], \qquad x_n \in [a_n, 1],$$

$$f_i(x_i) = \frac{Q'(x_i)P(a_i)}{P(x_i)Q^2(x_i)}, \qquad x_i \in [a_i, a_{i+1}),\ i = 1, \ldots, n-1,\ n \geqslant 2,$$

$$l_0 = P(a_1)Q(a_1), \qquad l_{i+1} = \frac{l_i}{1-P(a_{i+1})}, \qquad i = 0, \ldots, n-2,\ n \geqslant 2,$$

$$\beta = l_{n-1}\frac{W(a_n)}{[1-P(a_n)]P(a_n)Q(a_n)},$$

$$g(y) = \begin{cases} \dfrac{2\beta P'(y)}{W(y)} \exp\left[\displaystyle\int_{y}^{1} \frac{P'(u)[1+Q(u)]du}{W(u)}\right], & y \in [a_n, 1], \\[2ex] \dfrac{l_i P'(y)}{Q(y)P^2(y)}, & y \in [a_i, a_{i+1}),\ i = 1, \ldots, n-1,\ n \geqslant 2, \end{cases}$$

$$c_{k+1} = a_1, \tag{6}$$

$$Q(c_{i+1}) = \frac{Q(c_i)}{1-P(c_i)}, \qquad i = 1, \ldots, k. \tag{7}$$

Fix $\varepsilon > 0$ so that

$$\varepsilon_j = \min[\delta_j, (c_{j+1}-c_j)], \qquad j = 1, \ldots, k,$$

where $\delta_1, \ldots, \delta_k$ are obtained by solving the equations

$$(8) \qquad P(c_j+\delta_j) = \min\left[P(c_j)+\frac{\varepsilon}{2^{j+1}}, 1\right], \quad j = 1, \ldots, k.$$

Using the assumptions for $P(t)$ and $Q(t)$ and the results of [1] it is easy to show that the constants $a_1, \ldots, a_n, c_1, \ldots, c_k, l_0, \ldots, l_n, \varepsilon_1, \ldots, \varepsilon_k$, β and the functions $f_1(x_1), \ldots, f_n(x_n), g(y)$ are unique and

$$0 < c_1 < \ldots < c_k < c_{k+1} = a_1 < \ldots < a_n < 1, \quad 0 < \beta < 1,$$

$$\int_{a_i}^{a_{i+1}} f_i(x_i)\,dx_i = 1, \quad i = 1, \ldots, n, \; a_{n+1} = 1, \; \int_{a_1}^{1} g(y)\,dy+\beta = 1.$$

Let S_r^A be a strategy of player A in Γ_{rn}, $0 \leqslant r \leqslant k$, defined as follows: Player A takes his j-th noisy action at the moment c_{k-r+j}, $j = 1, \ldots, r$, with probability 1 and his i-th silent action at the moment $x_i \in [a_i, a_{i+1})$ according to the density function $f_i(x_i)$, $i = 1, \ldots, n$.

It is easy to see that $S_r^A \in A_r$.

Let $S^A = S_k^A$ by definition.

Let S_r^B be a strategy of player B in Γ_{rn}, $0 \leqslant r \leqslant k$, defined by induction with respect to the number r of noisy actions of player A in the following manner:

1° Player B takes the strategy S_0^B if he chooses randomly a moment y for his action from the interval $[a_1, 1]$ according to the probability measure described by the function $g(y)$ and the constant β so that

$$P\{y \in [a_1, t)\} = \int_{a_1}^{t} g(y)\,dy, \quad t \in [a_1, 1) \text{ and } P\{y = 1\} = \beta.$$

2° Let us assume that the strategy S_r^B for player B for some $r, r \leqslant p < k$, has been defined.

3° Using assumption 2° we define the strategy S_{p+1}^B.

Player B chooses randomly a moment y for his action from the interval $(c_{k-p}, c_{k-p}+\varepsilon_{k-p})$ according to a continuous probability distribution function $H_{p+1}(y)$ and takes his action at y under the condition that the first noisy action of A has not been undertaken yet. In the opposite case, player B gives up his action at y and, afterwards, follows the strategy S_p^B considering the second noisy action of A as the first one. It is clear that

$$S_r^B \in B_r \quad \text{and} \quad S_r^B = [H_r(y), \overline{\eta_r(y)}],$$

where $\eta_r(y) = [y, \{S_{r-w}^B\}]$, $S_{r-w}^B \in B_{r-w}$, $w = 1, \ldots, r$. Let $S^B = S_k^B$ by definition.

Notice that by (1)-(4) we have

$$K[S_r^A;\overline{\eta_r(y)}] = \int_{a_1}^{a_2}\dots\int_{a_n}^{1} K[(\bar{c}_{k,k-r},\bar{x}_n);\overline{\eta_r(y)}]\prod_{i=1}^{n} f_i(x_i)\,dx_i, \tag{9}$$

where $\bar{c}_{k,k-r} = (c_{k-r+1},\dots,c_k)$,

$$K[(\bar{z}_r,\bar{x}_n);S_r^B] = \int_{c_{k-r+1}}^{c_{k-r+1}+\varepsilon_{k-r+1}} K[(\bar{z}_r,\bar{x}_n);(y,\{S_{r-w}^B\})]\,dH_r(y). \tag{10}$$

3. Proof of optimality for S^A and of ε-optimality for S^B. In this section we prove that

$$K[S^A;\eta_k]\geqslant 1-2Q(c_1) \quad \text{for every } \eta_k\in B_k, \tag{11}$$

$$K[F_k;S^B]\leqslant 1-2Q(c_1)+\varepsilon \tag{12}$$

$$\text{for every } F_k\in A_k \text{ and for fixed } \varepsilon>0.$$

For this purpose we use the results obtained by Styszyński in [1] who proved

$$K[\bar{x}_n;S_0^B] = \int_{a_1}^{1} K[\bar{x}_n;y]g(y)\,dy+\beta K[\bar{x}_n;1]\leqslant 1-2Q(a_1) \tag{13}$$

$$\text{for every } \bar{x}_n\in\bar{X}_n,$$

$$K[S_0^A;y] = \int_{a_1}^{a_2}\dots\int_{a_n}^{1} K[\bar{x}_n;y]\prod_{i=1}^{n} f_i(x_i)\,dx_i\geqslant 1-2Q(a_1) \tag{14}$$

$$\text{for every } y\in[0,1].$$

Now we show, using induction with respect to the number r of noisy actions of player A, that inequality (11) is true. In other words, we have to show that the following inequalities hold:

$$K[S_r^A,\eta_r]\geqslant 1-2Q(c_{k-r+1}) \quad \text{for every } \eta_r\in B_r \text{ and } r,\ 0\leqslant r\leqslant k.$$

Proof. 1° By (1), (3), (14), and (6) we have

$$K[S_0^A;\eta_0]\geqslant 1-2Q(a_1).$$

2° Let us assume that

$$K[S_{r-1}^A;\eta_{r-1}]\geqslant 1-2Q(c_{k-r+2})$$

$$\text{for every } \eta_{r-1}\in B_{r-1} \text{ and some } r,\ 1<r\leqslant k.$$

3° We show that

$$K[S_r^A;\eta_r]\geqslant 1-2Q(c_{k-r+1}) \quad \text{for every } \eta_r\in B_r.$$

Let

$$\eta_r = [G_r(y),\overline{\eta_r(y)}],$$

where $\overline{\eta_r(y)} = [y,\{\eta_{r-w}(v)\}]$, $\eta_{r-w}(v)\in B_{r-w}(v)$, $1\leqslant w<r$, $v<y$.

We consider the following cases:

(a) $y < c_{k-r+1}$.

Then from (1) and (2) we obtain

$$K[S_r^A; \overline{\eta_r(y)}] = 1-2Q(y) > 1-2Q(c_{k-r+1})$$

by properties of $Q(t)$.

(b) $y = c_{k-r+1}$.

Using relations (9) and (2) we have

$$K[S_r^A; \overline{\eta_r(y)}] = 1-2Q(c_{k-r+1})+P(c_{k-r+1})Q(c_{k-r+1}) > 1-2Q(c_{k-r+1}).$$

(c) $c_{k-r+1} < y$.

Using the inductive hypothesis and (9), (2), and (7), we see that

$$\begin{aligned} K[S_r^A; \overline{\eta_r(y)}] &= P(c_{k-r+1}) + [1-P(c_{k-r+1})]K[S_{r-1}^A; \eta_{r-1}(c_{k-r+1})] \\ &\geqslant P(c_{k-r+1}) + [1-P(c_{k-r+1})][1-2Q(c_{k-r+2})] = 1-2Q(c_{k-r+2}). \end{aligned}$$

Hence we obtain

$$K[S_r^A; \overline{\eta_r(y)}] \geqslant 1-2Q(c_{k-r+1}).$$

In such a way by (1) we get

$$K[S_r^A; \eta_r] = \int_y K[S_r^A; \overline{\eta_r(y)}]dG_r(y) \geqslant 1-2Q(c_{k-r+1}).$$

This completes the proof of relation (11).

Now we show that inequality (12) is valid. For this purpose we prove that the inequality

$$K[(\bar{z}_{k-l}, \bar{x}_n); S_{k-l}^B] \leqslant 1-2Q(c_{l+1}) + \frac{\varepsilon}{2^l} \tag{15}$$

holds for every $(\bar{z}_{k-l}, \bar{x}_n) \in \overline{Z_{k-l}X_n}$ and for every l, $0 \leqslant l \leqslant k$. This will be shown by induction with respect to the number l.

Proof. 1° Let $l = k$. Then, by (13),

$$K[(\bar{z}_{k-k}, \bar{x}_n); S_0^B] \leqslant 1-2Q(a_1) < 1-2Q(c_{k+1}) + \frac{\varepsilon}{2^k},$$

where $(\bar{z}_{k-k}, \bar{x}_n) = \bar{x}_n$.

2° Let us assume that

$$K[(\bar{z}_{k-r}, \bar{x}_n); S_{k-r}^B] \leqslant 1-2Q(c_{r+1}) + \frac{\varepsilon}{2^r} \tag{16}$$

for some r, $0 < r \leqslant k$ and for every $(\bar{z}_{k-r}, \bar{x}_n) \in \overline{Z_{k-r}X_n}$.

To state that (15) holds for $l = r-1$ we consider the following three cases: $z_1 < c_r$, $c_r \leqslant z_1 \leqslant c_r + \varepsilon_r$, and $c_r + \varepsilon_r < z_1$.

(a) $z_1 < c_r$.

Then, by (10), (2), (16), and (7) we have

$$K[(\bar{z}_{k-r+1}, \bar{x}_n); S^B_{k-r+1}] = P(z_1) + [1-P(z_1)]K[(\bar{z}_{k-r+1,1}, \bar{x}_n); S^B_{k-r}]$$
$$\leqslant P(z_1) + [1-P(z_1)]\left[1-2Q(c_{r+1}) + \frac{\varepsilon}{2^r}\right]$$
$$= 1-2Q(c_{r+1})[1-P(z_1)] + [1-P(z_1)]\frac{\varepsilon}{2^r}$$
$$< 1-2Q(c_{r+1})[1-P(c_r)] + \frac{\varepsilon}{2^{r-1}} = 1-2Q(c_r) + \frac{\varepsilon}{2^{r-1}}.$$

(b) $c_r \leqslant z_1 \leqslant c_r + \varepsilon_r$.

At first we show that

$$L = P(z_1) + [1-P(z_1)]K[(\bar{z}_{k-r}, \bar{x}_n); S^B_{k-r}] \leqslant 1-2Q(c_r) + \frac{\varepsilon}{2^{r-1}}.$$

By the inductive hypothesis and by (8) and (7) we have

$$L \leqslant P(z_1) + [1-P(z_1)]\left[1-2Q(c_{r+1}) + \frac{\varepsilon}{2^r}\right] < 1-2Q(c_{r+1})[1-P(z_r)] + \frac{\varepsilon}{2^r}$$
$$< 1-2Q(c_{r+1})[1-P(c_r+\varepsilon_r)] + \frac{\varepsilon}{2^r}$$
$$\leqslant 1-2Q(c_{r+1})\left[1-P(c_r) - \frac{\varepsilon}{2^{r+1}}\right] + \frac{\varepsilon}{2^r}$$
$$< 1-2Q(c_{r+1})[1-P(c_r)] + \frac{\varepsilon}{2^r} + \frac{\varepsilon}{2^r} = 1-2Q(c_r) + \frac{\varepsilon}{2^{r-1}}.$$

Using (10) and (2) we obtain

$$K[(\bar{z}_{k-r+1}, \bar{x}_n); S^B_{k-r+1}] = \int_{c_r}^{c_r+\varepsilon_r} K[(\bar{z}_{k-r+1}, \bar{x}_n); (y, S^B_{k-r+1})]\,dH_{k-r+1}(y)$$
$$= \int_{c_r}^{z_1} [1-2Q(y)]\,dH_{k-r+1}(y) +$$
$$+ \int_{z_1}^{c_r+\varepsilon_r} \{P(z_1) + [1-P(z_1)]K[(\bar{z}_{k-r+1}, \bar{x}_n);\ S^B_{k-r}]\}\,dH_{k-r+1}(y)$$
$$\leqslant \int_{c_r}^{z_1} \left[1-2Q(c_r) + \frac{\varepsilon}{2^{r-1}}\right] dH_{k-r+1}(y) +$$
$$+ \int_{z_1}^{c_r+\varepsilon_r} \left[1-2Q(c_r) + \frac{\varepsilon}{2^{r-1}}\right] dH_{k-r+1}(y)$$
$$= -12Q(c_r) + \frac{\varepsilon}{2^{r-1}}.$$

(c) $c_r + \varepsilon_r < z_1$.

We have

$$K[(\bar{z}_{k-r+1}, \bar{x}_n);\ S^B_{k-r+1}] = \int_{c_r}^{c_r+\varepsilon_r} K[(\bar{z}_{k-r+1}, \bar{x}_n); Q(y)]\,dH_{k-r+1}(y)$$

$$= \int_{c_r}^{c_r+\varepsilon_r} [1-2Q(y)]\,dH_{k-r+1}(y)$$

$$< \int_{c_r}^{c_r+\varepsilon_r} [1-2Q(c_r)]\,dH_{k-r+1}(y)$$

$$< 1-2Q(c_r)+\frac{\varepsilon}{2^{r-1}}.$$

This completes the proof of relation (15) and, thereby, of inequality (12).

It follows from (11) and (12) that the game has the value

$$v = 1-2Q(c_1),$$

the strategy S^A of player A is optimal and the strategy S^B of player B is ε-optimal.

Reference

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GRA CZASOWA TYPU k GŁOŚNYCH I n CICHYCH AKCJI PRZECIW JEDNEJ AKCJI GŁOŚNEJ

STRESZCZENIE

Rozpatrzono grę czasową, w której gracz A ma k akcji głośnych i n akcji cichych, a gracz B ma jedną akcję głośną ($k > 0$, $n > 1$). Gracz A podejmuje swoje akcje w kolejności: głośne, ciche.

Funkcje sukcesu $P(t)$ i $Q(t)$ odpowiednio dla graczy A i B oznaczają prawdopodobieństwo odniesienia sukcesu, gdy akcja podejmowana jest przez gracza w chwili t, $t \in [0, 1]$.

Wypłatę dla gracza A określa funkcja

$$K[(\bar{z}_k, \bar{x}_n); y] = \Pr\{A \text{ sam odniesie sukces}\} - \Pr\{B \text{ sam odniesie sukces}\},$$

jeśli gracz A podejmuje akcje w chwilach określonych przez wektor $(\bar{z}_k, \bar{x}_n)$, gracz B zaś w chwili y.

Zadaniem gracza A jest maksymizowanie średniej wypłaty, a zadaniem B — minimizowanie jej.

W pracy skonstruowano strategie mieszane dla obu graczy i udowodniono ich optymalność.