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ALGORITHM 26

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LIMIT DISTRIBUTION OF THE NUMBER OF ITEMS IN THE QUEUEING SYSTEM $E_2/E_2/n$

1. Procedure declaration. The procedure *limdist* calculates the limit probabilities of the number of items in a queueing system in which interarrival times and service times are independent random variables with the Erlangian distribution of order 2.

Data:

n — number of service channels,

mi — parameter such that *2/mi* is the expected value of the service time,

lambda — parameter such that *2/lambda* is the expected value of the interarrival time.

Results:

P[0:n] — array of the limit probabilities (*P[i]* is the probability of *i* items being in the system, $i = 0, 1, \ldots, n$),

E — expected value of the number of items being in the system.

Other parameters:

sleGJ — global procedure with heading: **procedure** *sleGJ* (*n*, *x*, *a*, *singsyst*); **value** *n*; **integer** *n*; **array** *x*, *a*; **label** *singsyst*;

The procedure *sleGJ* (see [2]) solves the system of *n* linear equations

$$\sum_{l=1}^{n} a_{kl}x_l = a_{k,n+1} \quad (k = 1, 2, \ldots, n), \tag{1}$$

where *n* is the number of equations and unknowns, *x[1:n]* is the array of the solutions $x_1, x_2, \ldots, x_n$ (the array *x* must be of size at least *[1:(n+2)↑2/4]*), *a[1: n+1]* is an array in which the *k*-th execution ($k = 1, 2, \ldots, n$) of the instructions of the procedure *oneequation* (see below) places the coefficients $a_{k1}, a_{k2}, \ldots, a_{k,n+1}$ of the *k*-th equation, *singsyst* is the label of the instruction (outside of the procedure *sleGJ*) to which the program jumps if the matrix of the system is singular, and *oneequation* is the procedure declared in the body of *limdist*.

```
procedure limdist(n,mi,lambda,P,E);
 value n,mi,lambda;
 integer n;
 real mi,lambda,E;
 array P;
 begin
  integer N,i,j,k,m;
  real e,p;
  N:=(n+1)×(n+2);
  begin
   array a[1:N+1],X[1:(N+2)×(N+2)÷4];
   procedure ortim(l,q);
    value l;
    integer l;
    array q;
    comment procedure ortim places in the array q[0:l]
        the elements of row i×(i1)+m of the transposed
        transition intensity matrix Q(n)(see [1])-;
    begin
     j:=i×(i-1)+m;
     for k:=1 step 1 until l do
      q[k]:=.0;
     if i≥2∧m≠1∧m≠i+1
      then q[j-1]:=((if m≤i then 0 else i)+i-m+1)×mi;
     q[j]:=(if i≤n∨m≤i then -lambda else .0)-(i-1)×mi;
     if m≠i
      then q[j-(if m<i then i-1 else i)]:=lambda;
     if i≤n
      then q[j+i+i+(if m≤i then 1 else 2)]:=(if m≤i then m
                                          else m-i)×mi
```

```
 end ortim;
procedure oneequation;
 comment the j-th execution of the procedure oneequation
     places in the array a[1:N+1] the coefficients
     of equation j of the system according to (3) in
     [1] for j≠6 and the coefficients of the sixth
     equation of the system according to (5) in [1];
 begin
  if i×(i-1)+m≠6
   then
   begin
    ortim(N,a);
    a[N+1]:=.0
   end i×(i-1)+m≠6
   else
   for k:=1 step 1 until N+1 do
    a[k]:=1.0;
  if m<i+i
   then m:=m+1
   else
   begin
    i:=i+1;
    m:=1
   end m≥i+i
 end oneequation;
comment insert here the body of procedure sleGJ which
     solves a system of linear equations using
     procedure oneequation;
i:=m:=1;
sleGJ(N,X,a,singsyst);
```

```
e:=0;
j:=1;
for i:=0 step 1 until n do
 begin
  k:=j+i+i+1;
  p:=0;
  for m:=j step 1 until k do
   p:=p+X[m];
  P[i]:=p;
  e:=e+i×p;
  j:=k+1
 end i;
E:=e
end a,X
end limdist
```

2. Method used and certification. The used method is described in paper [1].

The procedure *limdist* has been verified on several examples. The results obtained by the Odra computer (for $n = 1, 2, \ldots, 6$) have been presented in Table 1 of [1].

3. Correction added in proof. The comment in the middle of page 548 should read as follows:

comment procedure *ortim* places in the array $q[0:1]$ the elements of row $i \times (i-1)+m$ of the transposed transition intensity matrix $Q(n)$ (see [1]);

References

[1] Maria Jankiewicz, *Explicit formulas for the transition intensities in the queueing system* $E_2/E_2/n$, Zastosow. Matem. 13 (1972) p. 187-198.

[2] S. Paszkowski, *Rozwiązywanie układu równań liniowych, Procedura sleGJ* in *Biblioteka programów maszyny cyfrowej Odra 1204, Odra-Algol, Procedury 1204-VIII-10*, Wrocław 1970.

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ALGORYTM 26

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GRANICZNY ROZKŁAD LICZBY JEDNOSTEK W SYSTEMIE MASOWEJ OBSŁUGI $E_2/E_2/n$

STRESZCZENIE

Procedura *limdist* oblicza graniczne prawdopodobieństwa liczby jednostek w systemie obsługi masowej, w którym odstępy między kolejnymi zgłoszeniami jednostek oraz czasy obsługi są niezależnymi zmiennymi losowymi o rozkładzie Erlanga rzędu 2.

Dane:

n — liczba niezależnych linii obsługi,

mi — parametr, dla którego *2/mi* jest wartością oczekiwaną czasów obsługi,

lambda — parametr, dla którego *2/lambda* jest wartością oczekiwaną odstępów między kolejnymi zgłoszeniami jednostek.

Wyniki:

P[*0*:*n*] — tablica granicznych prawdopodobieństw stanów systemu (*P*[*i*] jest prawdopodobieństwem tego, że w systemie znajduje się *i* jednostek, $i = 0, 1, \ldots, n$).

E — oczekiwana liczba jednostek w systemie.

Metoda użyta w procedurze *limdist* została opisana w [1]. Do rozwiązania układu równań liniowych wykorzystano procedurę *sleGJ* (patrz [2]), co wymaga umieszczenia w programie procedury *oneequation*. Procedurę *limdist* sprawdzono na maszynie Odra 1204; wyniki obliczeń znajdują się w tablicy 1 w [1].