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ON THE EXISTENCE OF THE WEAK SOLUTION OF A BOUNDARY VALUE PROBLEM ARISING IN THE THEORY OF WATER PERCOLATION

1. Introduction. Statement of the problem. Consider a process of percolation of water from a reservoir into a porous medium. We suppose that: the water reservoir and the porous medium are bounded from below by a horizontal impermeable layer whose upper boundary is the (x, y)-plane, the reservoir is of the form of a cylinder with radius equal to a, and it is continuously refilled by water. Therefore, we can assume that the level of the reservoir is constant and equal to b. We assume also that the porous material occupies all of the infinite layer with height greater than or equal to b outside of the reservoir. Our investigations are based on some physical simplifications. We assume that the porous medium is homogeneous and isotropic, which implies that the hydraulic conductivity coefficient K and the permeability coefficient m of the medium are constant scalar values. Suppose that at the time $t = 0$ water begins to flow through walls of the reservoir into the unsaturated porous medium. We look for the surface which is a boundary of the saturated region at the time $t > 0$. In the hydraulic model of filtration, called also the *Dupuit approximation* (see [1], [8] and [9]), this surface is described by the Boussinesq equation

$$\frac{1}{2}\Delta u^2 = \frac{m}{K} u_t, \tag{1}$$

where $u = u(x, y, t)$ denotes the height of the column of water at the point (x, y) at the time t, and Δ is the Laplace operator in the space variables (x, y). Note that knowledge of the form of the surface $z = u(x, y, t)$ for $t > 0$ is very important in prediction of the spread of impurities which may be dissolved in the water. Such a situation arises by cleaning the ores of copper by means of the so-called flotation method.

In this paper we assume that the described process of percolation has radial symmetry, so we look for the solution of (1) in the form $u = u(r, t)$, where $r = \sqrt{x^2+y^2}$. We examine this process in the time interval $[0, T]$

with $T > 0$, which leads to the problem described by the equations

$$\frac{1}{2}(u^2)_{rr}+\frac{1}{2r}(u^2)_r = u_t \quad \text{in } D=(a,\infty)\times(0,T], \tag{2}$$

$$u(a,t) = b \quad \text{in } [0,T],$$

$$u(r,0) = \begin{cases} b & \text{for } r=a, \\ 0 & \text{for } r>a, \end{cases} \tag{3}$$

where a, b and T are some fixed positive constants. Here equation (2) is obtained from (1) by introducing the polar coordinates in the (x, y)-plane and putting, for simplicity, $m/K = 1$.

Formulas (1) and (2) are degenerate non-linear parabolic equations: they are parabolic for $u > 0$, but for $u = 0$ they are not. In general, boundary value problems for equations of this kind need not have classical solutions (see [4]) and, therefore, we interpret the solutions of (2)-(3) in a suitable generalized sense.

A large class of degenerate parabolic equations, including equation (2), was studied in [2]. In that paper the existence theorem was proved for a certain class of weak solutions of initial boundary value problems, but under assumptions about the initial boundary data stronger yet than ours. The data (3) do not satisfy the compatibility condition at $r = a$, which is prescribed in [2].

The existence and regularity properties of a certain class of approximative solutions of problem (2)-(3) were examined in [3].

Definition. By a *weak solution* of problem (2)-(3) in D we mean the function $u = u(r, t)$ which has the following properties:

(i) u is non-negative in $\bar{D} = [a, \infty)\times[0, T]$ and continuous in $(a, \infty)\times[0, T]$,

(ii) u satisfies conditions (3),

(iii) u satisfies the integral identity

$$\iint_D \left\{f_t u+\left[f_{rr}-\left(\frac{1}{r}f\right)_r\right]\frac{u^2}{2}\right\}dr\,dt+\int_0^T f_r(a,t)\frac{b^2}{2}\,dt = 0 \tag{4}$$

for all functions $f \in C_0^2(\bar{D})$ which vanish for $r = a$ and $t = T$.

We prove the following result:

THEOREM 1. *There exists a weak solution u of problem* (2)-(3) *in D, bounded in $\bar{D}$. Moreover, $u^2/2$ has a weak* (*distributional*) *derivative with respect to r in $(a, \infty)\times(0, T)$ which is essentially bounded on every compact subset of $(a, \infty)\times[0, T]$.*

We shall obtain the weak solution u described in Theorem 1 as the limit as $n \to \infty$ of classical solutions u_n of the sequence of first boundary value problems, i.e.,

$$\frac{1}{2}(u^2)_{rr} + \frac{1}{2r}(u^2)_r = u_t \quad \text{in } (a, a+n)\times(0, T], \tag{5}$$

$$\begin{aligned} u(a, t) &= u(a+n, t) = b && \text{in } [0, T], \\ u(r, t) &= \sqrt{2\tilde{v}_n(r)} && \text{in } [a, a+n], \end{aligned} \tag{6}$$

where $\tilde{v}_n = \tilde{v}_n(r)$ are sufficiently changed, smooth, strictly positive functions (see Section 2 of this paper).

Our method of the proof is similar to that given in [6], where the degenerate parabolic equation $u_t = \varphi(x, u)_{xx}$ was studied. In [6] this method is applied to the problems whose initial boundary value data satisfy the compatibility conditions; however, we are able to apply it here because of the particular kind of non-compatibility of conditions (3).

2. Auxiliary lemmas. In this section we consider the quasilinear equation

$$v_{rr} = \frac{1}{\sqrt{2v}} v_t - \frac{1}{r} v_r$$

obtained from (2) by substituting $v = u^2/2$ under the additional assumption that $u > 0$.

Let $\{\tilde{v}_n\}$ be a sequence of strictly positive $C^\infty([a, \infty))$-functions such that, for $n \geqslant 3$,

$$\begin{aligned} &\tilde{v}_n(r) = \begin{cases} b^2/2n & \text{in } [a+1/n, a+n-2], \\ b^2/2 & \text{in } [a, a+1/2n] \text{ and } [a+n-1, \infty), \end{cases} \\ &\tilde{v}_n(r) \geqslant \tilde{v}_{n+1}(r) \quad \text{in } [a, \infty), \end{aligned} \tag{7}$$

and, for every $\delta > 0$ and for some $K_\delta > 0$,

$$\left|\frac{d\tilde{v}_n}{dr}\right| \leqslant K_\delta \quad \text{in } [a+\delta, \infty) \tag{7'}$$

uniformly with respect to n. Note that $\{\tilde{v}_n\}$ converges to the function equal to zero in (a, ∞) and $b^2/2$ for $r = a$.

Consider the sequence of first boundary value problems

$$v_{rr} = \frac{1}{\sqrt{2v}} v_t - \frac{1}{r} v_r \quad \text{in } (a, a+n)\times(0, T], \tag{8}$$

$$\begin{aligned} v(a, t) &= v(a+n, t) = \frac{b^2}{2} && \text{in } [0, T], \\ v(r, 0) &= \tilde{v}_n(r) && \text{in } [a, a+n] \end{aligned} \tag{9}$$

for $n \geqslant 3$, where $\tilde{v}_n$ are defined above. Note that if v_n is a solution of (8)-(9), then $u_n = \sqrt{2v_n}$ is the solution of problem (5)-(6).

For $r_1, r_2 \in R$ we put

$$D_{r_1,r_2} = (r_1, r_2) \times (0, T]$$

and

$$\Gamma_{r_1,r_2} = \{r_1\} \times [0, T] \cup (r_1, r_2) \times \{0\} \cup \{r_2\} \times [0, T].$$

LEMMA 1. *For each $n \geqslant 3$ there exists a unique solution $v_n = v_n(r, t)$, $v_n \in C^{2,1}(\bar{D}_{a,a+n})$, of problem* (8)-(9) *in $D_{a,a+n}$. Moreover,*

$$\frac{b^2}{2n} \leqslant v_n \leqslant \frac{b^2}{2} \ \text{in } \bar{D}_{a,a+n} \quad \text{and} \quad (v_n)_r \in C^{2,1}(D_{a,a+n}).$$

Proof. Let $A_n = A_n(s)$ denote a $C^\infty(R)$-function such that

$$A_n(s) = \frac{1}{\sqrt{2s}} \ \text{in} \left[\frac{b^2}{2n}, \frac{b^2}{2}\right] \quad \text{and} \quad \frac{1}{2b} \leqslant A_n \leqslant \frac{n}{b} \ \text{for } s \in R.$$

Consider the equation

$$v_{rr} = A_n(v) v_t - \frac{1}{r} v_r \tag{8'}$$

in $D_{a,a+n}$ with conditions (9) on $\Gamma_{a,a+n}$. According to Theorem 1 of [7], for each $n \geqslant 3$ there exists a unique solution $v_n = v_n(r, t)$ of problem (8')-(9) such that $v_n \in C^{2,1}(\bar{D}_{a,a+n})$ and $|v_n| \leqslant b^2/2$ in $\bar{D}_{a,a+n}$. Moreover, by Lemma 2 of [7], $(v_n)_r \in C^{2,1}(D_{a,a+n})$. We assert that $v_n \geqslant b^2/2n$ in $\bar{D}_{a,a+n}$. Set

$$z_n = \left(v_n - \frac{b^2}{2n}\right) \exp[-t].$$

Then $z_n \geqslant 0$ on $\Gamma_{a,a+n}$. If $z_n < 0$ at any point of $D_{a,a+n}$, then it must have a negative minimum at some point (r_0, t_0) of that set. At (r_0, t_0) we have

$$A_n\left(z_n \exp[t_0] + \frac{b^2}{2n}\right)(z_n)_t - (z_n)_{rr} \leqslant 0$$

which contradicts

$$A_n\left(z_n \exp[t] + \frac{b^2}{2n}\right)(z_n)_t - (z_n)_{rr} = -A_n\left(z_n \exp[t] + \frac{b^2}{2n}\right) z_n,$$

since

$$A_n\left(z_n \exp[t] + \frac{b^2}{2n}\right) z_n < 0 \quad \text{at } (r_0, t_0).$$

Thus $z_n \geqslant 0$ in $\bar{D}_{a,a+n}$ and the assertion is proved. Since

$$\frac{b^2}{2n} \leqslant v_n \leqslant \frac{b^2}{2} \quad \text{in } \bar{D}_{a,a+n},$$

the solution of problem (8′)-(9) is also the unique classical solution of the original problem (8)-(9).

LEMMA 2. *Let v_n be the solution of problem* (8)-(9) *and let a and β be real numbers such that $a < a < \beta$. Then there exists a constant $C_{a,\beta} > 0$, independent of n, such that $|(v_n)_r| \leqslant C_{a,\beta}$ in $\bar{D}_{a,\beta}$.*

Proof. It suffices to consider only the case $n > \beta$. Let $\tilde{a}$, $\tilde{\beta}$ and γ be real numbers such that $a < \tilde{a} < \tilde{\beta} < \beta$ and

$$\gamma \geqslant \sup_{r \in [\tilde{a},\tilde{\beta}]} \left[-(\tilde{a}+\tilde{\beta}-2r) \pm \frac{(r-\tilde{a})(\tilde{\beta}-r)}{r} \right].$$

Consider a function

$$w_n(r,t) = (r-\tilde{a})(\tilde{\beta}-r)|(v_n)_r| + \exp[\gamma v_n] \tag{10}$$

which is positive and continuous on $\bar{D}_{\tilde{a},\tilde{\beta}}$. Assume that w_n attains its maximum value at the point $(r_0, t_0) \in \bar{D}_{\tilde{a},\tilde{\beta}}$. If $(r_0, t_0) \in \Gamma_{\tilde{a},\tilde{\beta}}$ or $(v_n)_r(r_0, t_0) = 0$, then

$$\max_{\bar{D}_{\tilde{a}\tilde{\beta}}} w_n \leqslant \left(\frac{\tilde{\beta}-\tilde{a}}{2}\right)^2 K_{\tilde{a}} + \exp\left[\gamma \frac{b^2}{2}\right],$$

where $K_{\tilde{a}}$ is defined by (7′). Hence $|(v_n)_r| \leqslant C_1$ in the original rectangle $\bar{D}_{a,\beta}$.

Let $(r_0, t_0) \in D_{\tilde{a},\tilde{\beta}}$ and $(v_n)_r(r_0, t_0) \neq 0$. At (r_0, t_0) we have

$$\sqrt{2v_n}(w_n)_{rr} + \frac{\sqrt{2v_n}}{r}(w_n)_r - (w_n)_t \leqslant 0 \tag{11}$$

and

$$(w_n)_r = 0. \tag{12}$$

Differentiating (10) with respect to r and t, we obtain

$$(w_n)_r = \pm(r-\tilde{a})(\tilde{\beta}-r)(v_n)_{rr} + \{\gamma \exp[\gamma v_n] \pm (\tilde{a}+\tilde{\beta}-2r)\}(v_n)_r, \tag{13}$$

$$(w_n)_{rr} = \pm(r-\tilde{a})(\tilde{\beta}-r)(v_n)_{rrr} + \{\gamma \exp[\gamma v_n] \pm 2(\tilde{a}+\tilde{\beta}-2r)\}(v_n)_{rr} + \gamma^2 \exp[\gamma v_n](v_n)_r^2 \mp 2(v_n)_r, \tag{14}$$

$$(w_n)_t = \pm(r-\tilde{a})(\tilde{\beta}-r)(v_n)_{rt} + \gamma \exp[\gamma v_n](v_n)_t \tag{15}$$

in $D_{\tilde{a},\tilde{\beta}}$, where the lower sign is set if $(v_n)_r(r_0, t_0) < 0$, and the upper one if $(v_n)_r(r_0, t_0) > 0$. If we write (8) in the form

$$(v_n)_t = \sqrt{2v_n}(v_n)_{rr} + \frac{\sqrt{2v_n}}{r}(v_n)_r \tag{16}$$

and differentiate this equation with respect to r, we obtain

$$(17)\qquad (v_n)_{tr} = \sqrt{2v_n}(v_n)_{rrr} + \left[(\sqrt{2v_n})_r + \frac{\sqrt{2v_n}}{r}\right](v_n)_{rr} + \frac{\sqrt{2v_n}}{r}(v_n)_r$$

in $D_{\tilde{a},\tilde{\beta}}$. By (11), (12) and (13)-(17), we get the inequality

$$\left[(\tilde{a}+\tilde{\beta}-2r)\mp\frac{(r-\tilde{a})(\tilde{\beta}-r)}{r}+\gamma\exp[\gamma v_n]\right](v_n)_r^2 \leqslant \frac{2v_n}{(r-\tilde{a})(\tilde{\beta}-r)}F_n(r,t)(v_n)_r$$

which holds at (r_0, t_0). Here

$$F_n(r,t) = \Big\{2(\tilde{a}+\tilde{\beta}-2r)[\gamma\exp[\gamma v_n]\pm(\tilde{a}+\tilde{\beta}-2r)] + \left[\pm 2-\frac{\tilde{a}+\tilde{\beta}-2r}{r}\pm\frac{(r-\tilde{a})(\tilde{\beta}-r)}{r^a}\right](r-\tilde{a})(\tilde{\beta}-r)\Big\}.$$

Since there exist positive constants C_2 and C_3 dependent on a and β such that

$$C_2 \leqslant \tilde{a}+\tilde{\beta}-2r\pm\frac{(r-\tilde{a})(\tilde{\beta}-r)}{r}+\gamma\exp[\gamma v_n]$$

and

$$|F_n(r,t)| \leqslant C_3,$$

we have

$$|(v_n)_r| \leqslant \frac{C_4}{(r-\tilde{a})(\tilde{\beta}-r)} \quad \text{at } (r_0, t_0),$$

where $C_4 = C_3 b^2/C_2$. Hence

$$\max_{\bar{D}_{\tilde{a},\tilde{\beta}}} w_n \leqslant C_4 + \exp\left[\gamma\frac{b^2}{2}\right] = C_5$$

and $|(v_n)_r| \leqslant C_6$ in the original rectangle $\bar{D}_{a,\beta}$. Thus $C_{a,\beta} = \max(C_1, C_6)$.

Lemma 3. *If v_n and v_{n+1} are solutions of* (8)-(9), *then*

$$v_n(r,t) \geqslant v_{n+1}(r,t) \quad in\ \bar{D}_{a,a+n}.$$

Proof. Set $w_n = v_n - v_{n+1}$. We have $w_n \geqslant 0$ on $\Gamma_{a,a+n}$ and

$$\sqrt{2v_n}(w_n)_{rr} = (w_n)_t - \frac{1}{\sqrt{2v_{n+1}}}\frac{1}{\sqrt{2\theta_n}}(v_{n+1})_t w_n - \frac{\sqrt{2v_n}}{r}(w_n)_r$$

in $D_{a,a+n}$, where $\theta_n = \theta_n(r, t)$ is a suitable intermediate value between $v_n(r, t)$ and $v_{n+1}(r, t)$. Moreover,

$$\left| \frac{1}{\sqrt{2v_{n+1}}} \frac{1}{\sqrt{2\theta_n}} (v_{n+1})_t \right| \leqslant L_n$$

in $\bar{D}_{a,a+n}$ for some constant $L_n > 0$. Set $p_n = w_n \exp[-2L_n t]$. By the argument used in Lemma 1 we prove that $p_n \geqslant 0$ in $\bar{D}_{a,a+n}$ (we omit further details). Hence $w_n \geqslant 0$ in $\bar{D}_{a,a+n}$.

We need also the following well-known lemma:

LEMMA 4. *Let $A = (r_1, r_2) \times (t_1, t_2)$ and let f be an integrable function in A. If for $n \geqslant 1$ there exist $C^1(\bar{A})$-functions f_n such that*

$$\lim_n \iint_A f_n \varphi \, dr\, dt = \iint_A f \varphi \, dr\, dt \quad \text{for all } \varphi \in C_0^\infty(A)$$

and $|(f_n)_r| \leqslant C$ in $\bar{A}$ for some positive constant C, uniformly with respect to n, then there exists a weak derivative $f_r \in L^\infty(\bar{A})$.

For the proof of this lemma note that

$$F(\varphi) = \iint_A f \varphi_r \, dr\, dt$$

is a continuous linear functional on the subspace $C_0^\infty(A)$ of $L^1(A)$. Therefore, there exists a function $g \in L^\infty(A)$ such that

$$F(\varphi) = \iint_A g \varphi \, dr\, dt \quad \text{for all } \varphi \in C_0^\infty(A)$$

and the assertion is valid.

3. Proof of Theorem 1. Let v_n for $n \geqslant 3$ be solutions of problem (8)-(9) given in Lemma 1. Set

$$u_n = \sqrt{2v_n}. \tag{18}$$

We prove that the sequence $\{u_n\}$ converges in $\bar{D}$ as $n \to \infty$ and the limit function $u = \lim u_n$ is the weak solution of problem (2)-(3) described in Theorem 1. It follows from Lemmas 1 and 3 that the sequence $\{v_n\}$ is non-increasing and bounded from below by a function equal to zero. Thus $\{v_n\}$ converges everywhere in $\bar{D}$, and hence the same is valid for the sequence $\{u_n\}$. If we put $v = \lim v_n$, then we have $u = \sqrt{2v}$. It is easy to verify that u is non-negative and bounded in $\bar{D}$, and u satisfies conditions (3). Fix any rectangle $D_{\alpha,\beta}$, where $a < \alpha < \beta$. By Lemma 2, the functions v_n are Lipschitz continuous with respect to r in $\bar{D}_{\alpha,\beta}$, uniformly with respect to n and t. In view of Theorem 1 in [5], in every subrectangle $\bar{D}_{\alpha',\beta'} \subset D_{\alpha,\beta}$ the functions v_n are Hölder continuous with respect to t, uniformly with respect to n and r. Therefore, the limit function v is continuous in $(a, \infty)\times[0, T]$ and the same is valid for u. Let u_n be defined by (18). It follows from Lemma 1 that if $n \geqslant 3$, then u_n is the classical solution of the following problem:

$$\frac{1}{2}(u^2)_{rr}+\frac{1}{2r}(u^2)_r = u_t \quad \text{in } (a, a+n)\times(0, T],$$
$$u(a, t) = u(a+n, t) = b \quad \text{in } [0, T],$$
$$u(r, 0) = \sqrt{2\tilde{v}_n(r)} \quad \text{in } [a, a+n].$$

Therefore, if $f \in C_0^2(\bar{D})$ and f vanishes for $r = a$ and $t = T$, then we have

$$\iint_D \left\{f_t u_n + \left[f_{rr} - \left(\frac{1}{r}f\right)_r\right]\frac{u_n^2}{2}\right\}dr\,dt + \int_0^T f_r(a, t)\frac{b^2}{2}dt + \int_a^\infty f(r, 0)\sqrt{2\tilde{v}_n(r)}\,dr = 0$$

for sufficiently large n. Applying the Lebesgue dominated convergence theorem, we infer that the function u satisfies the integral identity (4).

Finally, by Lemma 4, for any rectangle $D_{\alpha,\beta}$, where $a < \alpha < \beta$, there exists a weak derivative $(u^2/2)_r \in L^\infty(D_{\alpha,\beta})$, which yields the existence of a weak derivative $(u^2/2)_r$ on the whole half-stripe $(a, \infty)\times(0, T)$, bounded on every compact subset of $(a, \infty)\times[0, T]$.

4. Final remarks. In previous sections we have constructed the weak solution u of problem (2)-(3) as a limit of the non-increasing sequence of functions $u_n = \sqrt{2v_n}$ (see (18)), where v_n are solutions of certain quasilinear equation. Note that, by Dini's theorem, $u_n \to u$ uniformly in every compact subset of $(a, \infty)\times[0, T]$. It is possible to replace functions v_n in (18) by solutions of certain linear parabolic equations. This fact can be useful for an approximative solution of the considered problem.

Fix any $n \geqslant 3$. Let v_n be the solution of problem (8)-(9) in $D_{a,a+n} = (a, a+n)\times(0, T]$. Define the following sequence of functions: $v_{n,0} = \tilde{v}_n$, where $\tilde{v}_n$ is defined by (7), (7′), and $v_{n,m}$ is a solution of the problem

$$v_{rr} = \frac{1}{\sqrt{2v_{n,m-1}}}v_t - \frac{1}{r}v_r \quad \text{in } D_{a,a+n}, \tag{19}$$

$$v(a, t) = v(a+n, t) = \frac{b^2}{2} \quad \text{in } [0, T],$$
$$v(r, 0) = \tilde{v}_n(r) \quad \text{in } [a, a+n]. \tag{20}$$

We have the following

THEOREM 2. *For each* $m \geqslant 1$ *there exists a unique classical solution* $v_{n,m}$ *of problem* (19)-(20). *The sequence* $\{v_{n,m}\}_{m=1}^{\infty}$ *converges uniformly in* $\bar{D}_{a,a+n}$ *and*

$$\lim_{m\to\infty} v_{n,m} = v_n.$$

For the proof of this theorem see [7].

Set $u_{n,m} = \sqrt{2v_{n,m}}$. We can choose the subsequence $\{u_{n,m_n}\}$ such that

$$\lim_{n\to\infty} u_{n,m_n} = u.$$

The functions u_{n,m_n} tend to u uniformly on every compact subset of $(a, \infty)\times[0, T]$.

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O ISTNIENIU SŁABEGO ROZWIĄZANIA ZAGADNIENIA BRZEGOWEGO POJAWIAJĄCEGO SIĘ W TEORII PRZESĄCZANIA WODY

STRESZCZENIE

W pracy badane jest zagadnienie nawilżania ośrodka porowatego przez ciecz wypełniającą zbiornik o kształcie walca. Zagadnienie to prowadzi do pewnego problemu brzegowego, z danymi brzegowymi nie spełniającymi warunku zgodności, dla nieliniowego równania różniczkowego cząstkowego Boussinesq'a. W pracy udowodniono istnienie słabego rozwiązania tego problemu i otrzymano pewne własności typu regularności tego rozwiązania.