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## GROUND-WAVE PERTURBATION OVER A TRANSITION ZONE BETWEEN TWO DIFFERENT SECTIONS

### LIST OF PRINCIPAL SYMBOLS

- $E, H$  = electric and magnetic field strength, respectively  
 $H_0^{(2)}, H_1^{(2)}$  = Hankel function of the second kind and of the  
zero and first order, respectively  
 $d$  = width of the transition zone  
 $f$  = frequency  
 $p$  = electric moment of the dipole  
 $w$  = attenuation function  
 $x$  = distance along the path (see Fig. 2)  
 $Z$  = field impedance  
 $Z_0$  = intrinsic impedance of free space =  $(\mu_0/\epsilon_0)^{1/2}$   
 $Z_s$  = surface impedance  
 $\omega$  = angular frequency  
 $\mu_0, \epsilon_0$  = absolute permeability and permittivity of free  
space  
 $\epsilon, \epsilon_r$  = absolute and relative permittivity of the soil  
 $\sigma$  = conductivity of the soil  
 $\epsilon'$  = complex absolute permittivity of the soil  
=  $\epsilon'_r \epsilon_0 = \epsilon - j\sigma/\omega$   
 $\epsilon'_r$  = complex relative permittivity of the soil =  $\epsilon_r -$   
 $-j\sigma/(\omega\epsilon_0)$   
 $\lambda_0$  = free-space wavelength  
 $\gamma_0$  = free-space propagation coefficient =  $\omega(\mu_0\epsilon_0)^{1/2}$   
=  $2\pi/\lambda_0$   
 $\zeta = \gamma_0 x = 2\pi x/\lambda_0$   
 $\delta = \gamma_0 d = 2\pi d/\lambda_0$

Subscript  $t$  at a vector denotes its tangential component.  
Subscripts  $a$  and  $b$  denote the zone with transmitter ( $a$ ),  
and the zone past the boundary ( $b$ ), respectively.

The time factor  $\exp(j\omega t)$  is suppressed.

Rationalized M.K.S. units are used throughout.

**1. Introduction.** Because of the irregularity of the earth surface and the electrical inhomogeneity of the ground and the atmosphere, the problem of radio wave propagation over the real earth represents one of

the most complex issues of mathematical physics. In a somewhat arbitrary manner the problem is generally subdivided into a number of simpler specialized cases which are solved by application of elaborate mathematical methods under approximations which often are stated not sufficiently clear or even are not mentioned at all. The impact of advanced mathematics then makes one believe oneself to be confronted by rigorous solutions. On hand of an example of ground-wave propagation across a boundary between two different sections of the earth surface we shall discuss in the present paper some of these approximations.

The adopted method of approach allows a clear and visual presentation of the extent of the field perturbation appearing across and in the vicinity of the transition zone.

The paper may also be of some interest from a purely formal point of view, as it puts forward a novel method with a direct physical meaning which may be of a potential use for a class of diffraction problems on bodies of finite conductivity.

The preliminary numerical results of the research were submitted to C.C.I.R. and U.R.S.I. [5], [6]. For a number of reasons the research was brought to completion only recently. The presented theory and the preliminary numerical results are due to one of the authors (Z.G.); the final computational program was the responsibility of the other of the authors (L.S.).

**2. The character of the electrical inhomogeneity of the earth.** The only parts of the earth surface which from the point of view of radio wave propagation may be regarded as truly homogeneous are seas. The influence of sea on radio wave propagation may be thus described in a unique manner by means of a single complex parameter. Mathematically this may be expressed by ascribing to the sea surface a unique value of the surface impedance or by introducing other equivalent boundary conditions. A detailed discussion of this problem may be found for instance in a monograph by Godziński [4].

In case of land the inhomogeneity of soil is of most casual and complex character. For instance when performing measurements of electrical conductivity of soil one is often surprised to find values differing by one order of magnitude at places only a few meters apart. Also in the vertical direction the distribution of  $\sigma$  and  $\epsilon_r$  is usually of a complex character. Moreover, only very seldom is this distribution of the form of a parallel stratification. In general the strata — if present — are oblique, there are lens-shaped inclusions etc. As discussed in [4], the influence of the soil on radio wave propagation depends in such cases on the direction of propagation. As a consequence, it is in principle not possible to characterize the earth by a unique value of surface impedance. In the case of small-

scale earth inhomogeneities at large distances from the transmitter or receiver the differences of impedance are statistically averaged and as a consequence some "effective" values of conductivity and permittivity or surface impedance can be introduced; some aspects of this problem have been studied by Feinberg ([1], p. 121-136, and [2], p. 381-404). If the inhomogeneous area lies near to the receiver the process of averaging does not come into action. As a consequence it is in principle impossible in such a case to characterize the corresponding area of the earth surface by a unique value of the surface impedance.

The most pronounced difference of electrical parameters will be found at the sea coast. At the same time this is the case of greatest practical importance. Sharp contrast between sea and land comes into appearance only in cases of a steep coast, cliff etc.; thus the inhomogeneity of electrical parameters is in such cases accompanied by a drastic surface irregularity.

In the case of a "flat" coast where the surface irregularity is rather small the variation of electrical parameters extends over some distance. From an electrical point of view such a coast is a structure similar to that shown in Fig. 1. Radio wave propagation past such a coast represents

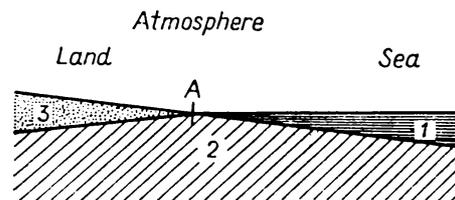


Fig. 1. Cross-section through a coast-line  
1 - sea water, 2 - wet soil, 3 - dry soil

a complex diffraction problem by a three-wedge system: sea water (1), wet soil (2), dry soil (3). Because of the large attenuation inside the sea water and the soil, the waves reflected from the boundaries 1/2 and 2/3 are so weak at large distances from the edge line *A*, that they can be completely disregarded; as a consequence the corresponding sections exhibit there properties characteristic of a homogeneous medium. At large distances from the coast line the sections of the radio wave path may thus be characterized by corresponding unique surface impedances.

The variation of impedance across the transition zone is in the present case gradual, with some doubts as to the situation in the close vicinity of the edge line *A*. In the theoretical case the edges of all three wedges are "sharp" as the model assumes a discontinuous change of electrical parameters across the whole surface of the boundaries 1/2 and 2/3 up to the edge line *A*. In such a case the field most probably would show a singularity at the edge line *A*. In practical cases, however, due to a gradually changing content of moisture, constant movement of waves, surf etc.

the electrical parameters of the media change continuously in the vicinity to the line  $A$ , i.e. the boundaries 1/2 and 2/3 are blurred there. This makes one believe that most probably no field singularity is to be expected at the edge line  $A$  and thus the variation of effective electrical parameters is, in practical cases, gradual across the whole transition zone.

When considering the variation of effective electrical parameters across the transition zone, we must take into account that due to a complex configuration of media these parameters will in principle show dependence on the direction of wave propagation. For instance, when using the impedance concept this will not be that of a unique *surface* impedance but the impedances will depend more or less on the direction of wave propagation [4].

From a practical point of view we must also take into account that even the "flat" sea-coast from Fig. 1 in fact represents a definite irregularity of the earth's surface. When comparing flat coast theory with results of practical experiments this must be kept in mind as a considerable part of the field disturbance at a "flat" coast is certainly caused by the diffraction effects at the rising ground ([1], p. 178-186, [2], p. 368-381, [13]).

In the existing theoretical investigations the above doubts and complications in general are disregarded without any comment as to the ensuing errors.

**3. Mathematical formulation of the problem.** Because present research is concerned with the field in the immediate vicinity of the boundary only, it is admissible to use the flat earth approximation and to assume a homogeneous atmosphere. Accordingly the  $xy$ -plane will be taken as the plane of the earth's surface with the  $z$ -axis directed upwards towards the atmosphere, characterized by a constant scalar permittivity.

It will be assumed that the earth surface consists of three zones of different electrical properties. The zone  $a$  extends to the left of the line  $x = 0$  (Fig. 2); the earth is there homogeneous with constant surface impedance  $Z_s = Z_a$ . To the right of the line  $x = d$  there extends the zone  $b$  with constant surface impedance  $Z_s = Z_b$ . In the transition zone  $t$ , of width  $d$ , the effective electrical parameters of the soil change gradually from values characteristic of the zone  $a$  to that characteristic of the zone  $b$ .

The transmitting aerial (aerial 1) is a short vertical dipole of electric moment  $p$  situated just over the earth surface at the point  $A_1$  at a large distance  $r_0$  from the boundary. The electromagnetic field produced by the aerial 1 will be denoted by  $E_1, H_1$ .

We shall consider radio wave propagation across the boundary at normal incidence; the direction of propagation will thus be assumed parallel to the  $x$ -axis. In view of the large distance from aerial 1 to the boundary, the wavefronts over the zones  $b$  and  $t$  in the neighbourhood

of the  $x$ -axis are parallel to the  $y$ -axis. As a consequence the magnetic field  $\mathbf{H}_1$  just over the corresponding parts of the earth surface is also parallel to the  $y$ -axis.

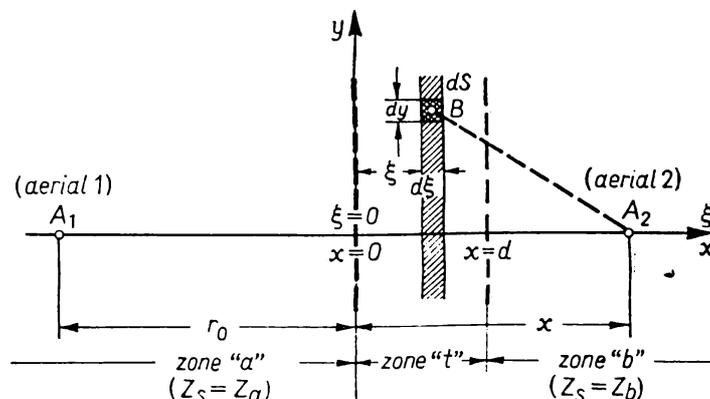


Fig. 2. Radio wave path

To make possible a close control of all approximations the analysis must be founded on a rigorous basis. Out of a number of equivalent methods of approach we shall choose in the present paper the vector integral equation (6) from the paper by Godziński [4]. To this purpose a second, auxiliary and fictitious problem will be introduced: the same atmosphere as before, a plane homogeneous earth of surface impedance  $Z_a$  over all three zones  $a$ ,  $b$  and  $t$ , and a transmitting aerial (aerial 2) in the form of a short vertical dipole of electric moment  $p$  situated just over the earth's surface at the point  $A_2$  at a distance  $x$  from the beginning of the transition zone. This auxiliary fictitious field will be denoted by  $\mathbf{E}_2, \mathbf{H}_2$ . We then have

$$(1) \quad E_{1z} = E_{2z} - \frac{j}{\omega p} \int_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{S}$$

where  $E_{1z}$  is the vertical component of the electric field  $\mathbf{E}_1$  at the point  $A_2$ ,  $E_{2z}$  is the vertical component of the electric field  $\mathbf{E}_2$  at the point  $A_1$ , and  $d\mathbf{S}$  is a vectorial element of the earth surface, directed towards the atmosphere. The integral in (1) extends in principle over the entire earth surface. However, due to the properties of Fresnel zones only parts of the earth surface near the  $x$ -axis are of practical importance.

Due to the assumption that the permittivity of the atmosphere is a scalar quantity the ionosphere is excluded from consideration; as a consequence the fields satisfy the reciprocity theorem. Consequently  $E_{2z}$  may be considered as the vertical component of the field which would be produced at the point  $A_2$  by the dipole 1 over a fictitious homogeneous earth of surface impedance  $Z_a$ . The surface integral in (1) thus represents

the change of field caused by the presence of zones  $t$  and  $b$  with electrical properties different from those of the zone  $a$ .

In the triple products  $(\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S}$  only tangential components of field vectors are of importance; introducing a unit vertical vector  $\mathbf{n}$  directed towards the atmosphere we obtain

$$(\mathbf{E}_1 \times \mathbf{H}_2) \cdot d\mathbf{S} = -(\mathbf{E}_{1t} \times \mathbf{n}) \cdot \mathbf{H}_{2t} dS$$

and

$$(\mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{S} = -(\mathbf{E}_{2t} \times \mathbf{n}) \cdot \mathbf{H}_{1t} dS.$$

In the case of a homogeneous earth of surface impedance  $Z_s$  the tangential components of electromagnetic field satisfy a relation

$$\mathbf{E}_t \times \mathbf{n} = Z_s \mathbf{H}_t$$

which is valid everywhere except in the immediate vicinity of the points with field singularities. For the field  $\mathbf{E}_2$  we thus have

$$(2) \quad (\mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{S} = -Z_a \mathbf{H}_{2t} \cdot \mathbf{H}_{1t} dS.$$

This formula is valid over the whole surface  $S$  except at very small distances from the point  $A_2$  where due to field singularity produced by the aerial 2 the field impedance is no longer constant and equal to surface impedance  $Z_a$ .

In the case of the field  $\mathbf{E}_1$  we may write similarly

$$(3) \quad (\mathbf{E}_1 \times \mathbf{H}_2) \cdot d\mathbf{S} = -Z_1 \mathbf{H}_{1t} \cdot \mathbf{H}_{2t} dS.$$

Over the zone  $a$  and  $b$  the field impedance  $Z_1$  is equal to surface impedance  $Z_a$  and  $Z_b$ , respectively. Over the transition zone  $t$  the impedance  $Z_1$  changes in general gradually from value  $Z_a$  to the value  $Z_b$  as has been discussed in Section 2.

Whereas the impedances  $Z_1 = Z_a$  and  $Z_1 = Z_b$  over the zones  $a$  and  $b$  are the corresponding unique surface impedances which do not depend on the direction of wave propagation, the impedances  $Z_1$  over the elements of the transition zone depend in general — as we have mentioned in Section 2 — on the direction of wave propagation. As a consequence, in cases of oblique propagation across the transition zone  $Z_1$  may depend on the angle of propagation. For a very gradual transition this effect most probably may be neglected; for a rapid transition the magnitude of this effect is, however, by no means obvious.

Substituting from (2) and (3) into (1) we obtain

$$(4) \quad E_{1z} = E_{2z} - \frac{j}{\omega p} \int_S (Z_a - Z_1) \mathbf{H}_{1t} \cdot \mathbf{H}_{2t} dS.$$

Over the zone  $a$  we have  $Z_1 = Z_a$ ; the surface integral from (4) extends thus over the zones  $t$  and  $b$  only.

The classical method of solution of integral equation (4) or of other equivalent equations is based on a number of approximations:

(a) vectors  $\mathbf{H}_{1t}$  and  $\mathbf{H}_{2t}$  are approximately assumed parallel;  
 (b) fields are expressed by means of conventional attenuation functions which in principle describe the variation of the Hertzian vector and not that of the electromagnetic field;

(c) due to the conventional approximations of the stationary phase method the shape of Fresnel zones is approximated by ellipses for which transmitter and receiver are at two ends of the ellipse long axis instead at its focii.

Whereas all these approximations may be accepted at somewhat larger distances from the boundary they certainly are inadmissible in its immediate vicinity ([3], p. 449). Thus the classical mixed-path theories ([2], [12]) give results valid in principle at somewhat larger distances from the boundary only; the accuracy of these methods at small distances may be estimated from the discussion in Section 9.

**4. Method of solution of the basic integral equation.** The integral equation (4) is the basic equation of present research. It is rigorous except one questionable approximation consisting in application for the field  $E_2$  of a constant surface impedance  $Z_a$  for the whole surface  $S$  including the points in the immediate vicinity to  $A_2$ . Inspection of classical theories for homogeneous earth seems to confirm the applicability of this approximation; consequently it is generally adopted.

In order to perform integration in (4) we shall divide the earth's surface into narrow stripes parallel to the  $y$ -axis and of width  $d\xi$  which we subdivide into surface elements  $dS = d\xi dy$  (Fig. 2). According to the reciprocity theorem the field  $\mathbf{H}_{2t}$  which is generated at the point  $B$  by a dipole of electric moment  $p$  situated at the point  $A_2$  has the same magnitude as the field  $\mathbf{H}'_{2t}$  which the same dipole situated at  $B$  would produce at  $A_2$ . When displacing the dipole from  $A_2$  to  $B$  we must change the sign of the scalar product in (4) because this change means a reversal of the direction of magnetic field. Thus, considering that the field  $\mathbf{H}_{1t}$  has only the  $y$ -component, we obtain  $\mathbf{H}_{1t} \cdot \mathbf{H}_{2t} = -H_{1y} H'_{2y}$ . The contribution from the surface element  $dS$  to the vertical component of the resulting electric field is therefore

$$(5) \quad dE_z = -Z_0 \left[ \left( \frac{j}{\omega p} \frac{Z_1 - Z_a}{Z_0} H_{1y} d\xi dy \right) H'_{2y} \right].$$

The field  $dE_z$  from (5) represents elementary field which is to be added to  $E_{2z}$  when computing  $E_{1z}$ . The factor inside the round brackets

is dimensionless. As a consequence the expression in the square brackets may be considered as the  $y$ -component of the magnetic field produced at the point  $A_2$  by a vertical dipole of moment

$$(6) \quad dp = \frac{j}{\omega} \frac{Z_1 - Z_a}{Z_0} H_{1y} d\xi dy$$

situated at the point  $B$  over a homogeneous plane earth of surface impedance  $Z_a$ .

The physical sense of this result is very simple. The influence of the earth surface on radio wave propagation is due to electric currents and charges induced in the earth surface by the primary wave. These currents and charges constitute secondary sources of radiation. The final field is a superposition of the primary field, the secondary field, the tertiary field caused by the secondary radiation, etc. In case of a homogeneous earth these secondary sources etc. are of definite nature and distribution. Any surface element of a different surface impedance represents thus some additional (difference) source as compared with the previous homogeneous case. This is just the situation expressed by (4)-(6): the difference between the field  $E_{1z}$  for the inhomogeneous earth and the field  $E_{2z}$  for the homogeneous earth is due to the presence of some additional sources which when placed over the homogeneous earth are equivalent to the influence of a change of electrical properties of the ground.

The situation expressed by (5) and (6) occurs for all surface elements  $dS$  from the shaded strip in Fig. 2. Thus the contribution to the field from the whole strip ( $\Delta E_z$ ) amounts to the product of  $(-Z_0)$  and the  $y$ -component of a magnetic field generated by a continuous array of vertical dipoles situated along the strip. The distribution of these dipoles is homogeneous with a constant density  $g$  of electric moments per unit length

$$(7) \quad g = \frac{dp}{dy} = \frac{j}{\omega} \frac{Z_1 - Z_a}{Z_0} H_{1y} d\xi.$$

If we remember that a dipole is an arrangement of two equal and opposite charges, then it follows that the described continuous distribution of dipoles represents in fact a double line source of constant density  $g$ . This line source is parallel to the  $y$ -axis and of a vertical polarization.

Because of symmetry properties of the described line source the magnetic field generated by it is parallel to the  $y$ -axis. We may thus restate our previous formulation so that  $\Delta E_z$  may be considered as the product of  $(-Z_0)$  and the magnetic field ( $\Delta H'_y$ ) generated by our double line source over a homogeneous earth of surface impedance  $Z_a$ . Such product, however, is just the vertical component of the electric field produced by the line

source

$$(8) \quad E_z = -Z_0 H'_y;$$

this follows for instance at once from (42) in [4]. Again, this result is valid in principle all over  $S$  except in the immediate vicinity to the line source.

We thus see that the physical content of (4) is quite simple: the field perturbation generated by a strip of a surface impedance different than that for the remaining part of the path is the same as the field produced by a suitable vertically polarized double line source. Knowing the primary field it is thus possible to calculate in a more or less direct manner the field perturbation caused by a change of electrical properties of a conducting body.

The proposed method of approach is especially straightforward in cases when the change of electrical properties is comparatively small. Such a situation may be found for instance if we consider the important problem of diffraction by a body of large but finite conductivity. The field existing in the case of perfect conductor could then be used as the reference solution; at the surface of the diffracting body is then  $Z_s = 0$ . Transition to the body of the same shape but of finite conductivity means replacing  $Z_s = 0$  by a suitable non-zero value. This change of boundary conditions is especially small for metals; in such a case the final solution could be probably obtained by calculating the first order perturbation only. Should this method prove convenient from the computational point of view it could perhaps help to solve a number of important problems of diffraction by real conductors.

**5. Computation of the resultant field of line sources.** In the present investigation we are interested in the field structure at the immediate neighbourhood of the transition zone only. Consequently, it is sufficient to derive expressions for the field of double line sources at very small numerical distances. At such distances the attenuation of waves comes not yet in appearance and the field is practically the same as over a perfectly conducting plane. The field is thus twice the field the considered double line source would produce in a homogeneous unbounded atmosphere, or — which for all practical applications is the same — in free space.

In order to solve the problem we may conveniently introduce a vertical Hertzian vector  $\mathbf{H}$ . The Green function for the considered free-space problem (Fig. 3) is equal to (see [8])

$$G(\varrho/\varrho_0) = -j\pi H_0^{(2)}(\gamma_0 R),$$

where  $H_0^{(2)}$  is the Hankel function of the second kind of order zero.

The only component of the Hertzian vector ( $\Pi_z$ ) and the magnetic field  $\mathbf{H}'$  of the double line source are given by formulae

$$\Pi_z = [g/(4\pi\epsilon_0)]G(\rho/\rho_0) \quad \text{and} \quad \mathbf{H}' = j\omega\epsilon_0 \text{curl} \Pi.$$

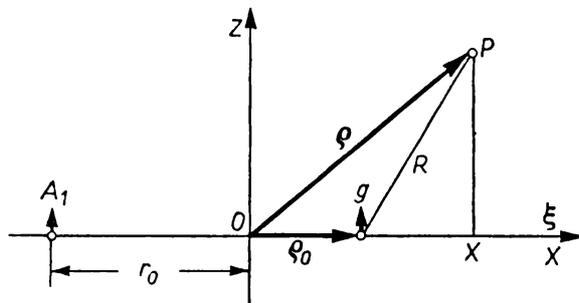


Fig. 3. Geometry of the double line source field

$A_1$  - primary source,  $g$  - double line source,  $P$  - point of observation

As expected, this gives  $H'_x = H'_z = 0$ . For the  $y$ -component of  $\mathbf{H}'$  we obtain

$$(9) \quad H'_y = \frac{g\omega\gamma_0}{4} \frac{x-\xi}{R} H_1^{(2)}(\gamma_0 R),$$

where  $H_1^{(2)}$  is the Hankel function of the second kind of first order. As stated previously this field is to be multiplied by a factor of 2 and calculated for the points  $P$  lying just over the plane  $xy$ . Taking into account (7) and (9) we get for the contribution from the elementary strip

$$(10) \quad \Delta E_z = (-Z_0) \left[ \frac{j\gamma_0}{2} \frac{Z_1 - Z_a}{Z_0} H_{1y} \frac{x-\xi}{|x-\xi|} H_1^{(2)}(\gamma_0 |x-\xi|) d\xi \right].$$

Integrating (10) over the zones  $t$  and  $b$  and adding the resulting value to  $E_{2z}$  we obtain the final formula for  $E_{1z}$  as

$$(11) \quad E_{1z} = E_{2z} - j \frac{\gamma_0 Z_0}{2} \int_0^\infty \frac{Z_1 - Z_a}{Z_0} H_{1y} \frac{x-\xi}{|x-\xi|} H_1^{(2)}(\gamma_0 |x-\xi|) d\xi.$$

This equation may be transformed by introduction of the conventional attenuation function  $w_1(r)$

$$(12) \quad E_{1z} = \frac{\omega^2 \mu_0}{2\pi} \frac{p}{r} w_1(r) \exp(-j\gamma_0 r),$$

where  $r = r_0 + x$  is the distance from the transmitter to the point of observation (point  $A_2$ ). Next it is necessary to calculate  $H_{1y}$ . In the case of a homogeneous earth equation (8) makes it possible to obtain

$H_y$  at once from  $E_z$ . Applying formula (8) also in the present case of an inhomogeneous earth we have from (12)

$$(13) \quad H_{1y} = -\frac{1}{Z_0} \frac{\omega^2 \mu_0}{2\pi} \frac{p}{r} w(r) \exp(-j\gamma_0 r).$$

It must be stressed, however, that in the case of an inhomogeneous earth with rapid changes of electrical properties application of (8) may be connected with considerable errors (see for instance Section 7 in [4]). Substituting from (13) into (11) and denoting by  $w_a$  the attenuation function for the homogeneous earth of surface impedance  $Z_a$  we obtain

$$(14) \quad \frac{w_1(r_0+x)}{r_0+x} = \frac{w_a(r_0+x)}{r_0+x} + \frac{j\gamma_0}{2} \int_0^\infty \frac{Z_1-Z_a}{Z_0} \frac{w_1(r_0+\xi)}{r_0+\xi} \exp[j\gamma_0(x-\xi)] \frac{x-\xi}{|x-\xi|} H_1^{(2)}(\gamma_0|x-\xi|) d\xi.$$

In the present investigation we are interested in the field in the immediate vicinity to the boundary only. Consequently  $(r_0+x)/(r_0+\xi) \simeq 1$ ; the distance factors  $(r_0+x)$  and  $(r_0+\xi)$  in (13) may be thus approximately omitted.

In the first-order approximation the attenuation function  $w_1(r_0+\xi)$  under the integral sign may be assumed as constant and equal to the value it would show over a homogeneous earth of surface impedance  $Z_a$  at a distance  $r_0$  (see Fig 2):

$$w_1(r_0+\xi) \simeq w_a(r_0).$$

The final results will show when this first-order approximation is admissible.

Taking into account that  $x \ll r_0$  we may also put

$$w_a(r_0+x) \simeq w_a(r_0);$$

this approximation is not objectionable. Under these approximations (14) takes on the final form

$$(15) \quad \frac{w_1(r_0+x)}{w_a(r_0)} = 1 + \frac{j\gamma_0}{2} \int_0^\infty \frac{Z_1-Z_a}{Z_0} \exp[j\gamma_0(x-\xi)] \frac{x-\xi}{|x-\xi|} H_1^{(2)}(\gamma_0|x-\xi|) d\xi.$$

The integral equation (14) has thus been transformed into an integral formula which may be evaluated by directed quadrature.

**6. Variation of impedance across the transition zone.** As discussed in Section 2 variation of impedance is in the case of a flat coast most probably gradual over the whole transition zone. Introducing dimensionless contrast parameters  $\mathfrak{z}$  and  $\mathfrak{z}_1$

$$(16) \quad \begin{aligned} \mathfrak{z} &= \exp(-j\pi/4)(Z_b - Z_a)/Z_0, \\ \mathfrak{z}_1 &= \exp(-j\pi/4)(Z_1 - Z_a)/Z_0 \end{aligned}$$

we thus have

$$(17) \quad (Z_1 - Z_a)/Z_0 = \mathfrak{z}_1 \exp(+j\pi/4).$$

In particular, over the zone  $a$  where  $Z_1 = Z_a$

$$\mathfrak{z}_{1a} = 0,$$

and over the zone  $b$  where  $Z_1 = Z_b$

$$(18) \quad \mathfrak{z}_{1b} = \mathfrak{z}.$$

Over the transition zone  $t$  we shall assume for simplicity a linear variation of impedance

$$\mathfrak{z}_{1t} = \mathfrak{z}(\xi/d).$$

For homogeneous ground of given electrical parameters the surface impedance  $Z_s$  may be calculated [4] as a good approximation from formula

$$(19) \quad Z_s = Z_0(\varepsilon_r')^{-1/2} = Z_0[\varepsilon_r - j\sigma/(\omega\varepsilon_0)]^{-1/2}.$$

In case of a well conducting ground and a not too high frequency  $\varepsilon_r \ll \sigma/(\omega\varepsilon_0)$ . We may thus omit  $\varepsilon_r$  in (19) which gives for the contrast parameter approximate formula

$$\begin{aligned} \mathfrak{z} &= (\omega\varepsilon_0)^{1/2} [(1/\sqrt{\sigma_b}) - (1/\sqrt{\sigma_a})] \\ &= 7.45 \times 10^{-6} \sqrt{f} [(1/\sqrt{\sigma_b}) - (1/\sqrt{\sigma_a})]. \end{aligned}$$

For well conducting path sections the contrast parameter is thus real. Values of  $|\mathfrak{z}|$  for number of cases are shown in Table 1.

The assumption of a linear variation of impedance across the transition zone which has been adopted in the present research is for purely mathematical reasons in principle inadmissible. This is caused by the basic demand characteristic of all theories of the problem that all field vectors be continuous together with their first and second derivatives; the discontinuities if present must be excluded by suitably deforming the surface of integration. The demand of field continuity means in our case that impedance  $Z_1$  together with its first and second derivatives be continuous

TABLE 1. The magnitude of the contrast parameter  $\beta$

Type of boundary	Frequency $f$		
	100 kHz	500 kHz	2 MHz
dry ground/sea	0.073	0.164	0.328
wet ground/sea	0.022	0.05	0.10
dry ground/wet ground	0.051	0.114	0.228

Conductivities: dry ground  $\sigma = 10^{-3} \Omega^{-1}m^{-1}$ , wet ground  $\sigma = 10^{-2} \Omega^{-1}m^{-1}$ , sea  $\sigma = 4 \Omega^{-1}m^{-1}$

across the transition zone. Instead of linear variation with constant slope (Fig. 4, full line) a variation with a changing slope (Fig. 4, dashed line) should thus in principle be assumed. However, inspection of (15) and

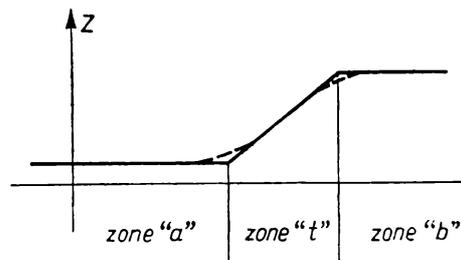


Fig. 4. Variation of impedance  $Z$  across the transition zone  $t$   
 ————— linear variation  
 - - - - - variation consistent with premises of theory

consideration of final numerical results show that for the range of parameters used in the present research the errors connected with the assumption of a linear impedance variation are completely negligible.

**7. Transformation of final formula to dimensionless form.** Substituting from (17) and (18) into (15) and introducing dimensionless distances

$$(20) \quad \zeta = \gamma_0 x = 2\pi x / \lambda_0, \quad \delta = \gamma_0 d = 2\pi d / \lambda_0, \quad u = \gamma_0 (\xi - x)$$

we obtain

$$(21) \quad \frac{w_1(r_0 + x)}{w_a(r_0)} = 1 + \beta \left\{ \frac{\exp(-j\pi/4)}{2\delta} \left[ \int_{-\zeta}^{-\zeta+\delta} |u| \exp(-ju) H_1^{(2)}(|u|) du + \right. \right. \\ \left. \left. + \zeta \int_{-\zeta}^{-\zeta+\delta} \frac{u}{|u|} \exp(-ju) H_1^{(2)}(|u|) du + \delta \int_{-\zeta+\delta}^{\infty} \frac{u}{|u|} \exp(-ju) H_1^{(2)}(|u|) du \right] \right\}.$$

The expression in braces is a function of parameters  $\zeta$  and  $\delta$  only; we shall denote it by  $W(\zeta, \delta)$ . Equation (21) then takes the final form

$$(22) \quad \frac{w(r_0 + x)}{w_a(r_0)} = 1 + \mathfrak{z}W(\zeta, \delta),$$

where the subscript 1 in  $w_1(r_0 + x)$  has been suppressed as now superfluous.

Equation (22) shows that field variations in the immediate vicinity to the boundary depend on the product of two factors: the contrast parameter  $\mathfrak{z}$  and the function  $W$  which in turn depends on the width of the transition zone (parameter  $\delta$ ) and location of the point of observation (parameter  $\zeta$ ).

The computation of the function  $W(\zeta, \delta)$  may be facilitated by introducing four auxiliary functions  $A_r(u)$ ,  $B_r(u)$ ,  $A_t(u)$  and  $B_t(u)$  defined as follows:

$$\begin{aligned} A_r(u) &= \frac{1}{2}e^{-j\pi/4} \int e^{-ju} H_1^{(2)}(u) du \\ &= -\frac{1}{2}e^{-j\pi/4} e^{-ju} [(1 + ju) H_0^{(2)}(u) - u H_1^{(2)}(u)]; \\ B_r(u) &= \frac{1}{2}e^{-j\pi/4} \int_0^u u e^{-ju} H_1^{(2)}(u) du; \\ A_t(u) &= \frac{1}{2}e^{-j\pi/4} \int e^{+ju} H_1^{(2)}(u) du \\ &= -\frac{1}{2}e^{-j\pi/4} e^{+ju} [(1 - ju) H_0^{(2)}(u) - u H_1^{(2)}(u)]; \\ B_t(u) &= \frac{1}{2}e^{-j\pi/4} \int_0^u u e^{+ju} H_1^{(2)}(u) du. \end{aligned}$$

We then have:

over the zone  $a$ , i.e. in front of the boundary

$$(23) \quad \begin{aligned} W(\zeta, \delta) &= \frac{1}{\delta} [B_r(-\zeta + \delta) - B_r(-\zeta) - (-\zeta + \delta)A_r(-\zeta + \delta) - \zeta A_r(-\zeta)]; \end{aligned}$$

over the transition zone  $t$

$$(24) \quad W(\zeta, \delta) = \frac{1}{\delta} [B_r(\delta - \zeta) + B_t(\zeta) - (\delta - \zeta)A_r(\delta - \zeta) - \zeta A_t(\zeta)];$$

over the zone  $b$ , i.e. past the boundary

$$(25) \quad W(\zeta, \delta) = \frac{1}{\delta} [B_t(\zeta) - B_t(\zeta - \delta) + (\zeta - \delta)A_t(\zeta - \delta) - \zeta A_t(\zeta)].$$

At the boundaries of the transition zone the auxiliary functions exhibit singularities. However, when multiplied by the corresponding factors from equations (23)-(25) they give a continuous and free from singularities variation of  $W(\zeta, \delta)$  across the boundaries.

**8. Numerical results.** The numerical computations were made difficult by the fact that  $W(\zeta, \delta)$  is obtained as a small difference of comparatively large functions. In order to achieve a reasonable precision of final results it was thus necessary to perform calculations of the auxiliary functions  $A_r, B_r, A_t$  and  $B_t$  with great accuracy.

All calculations were carried out on a digital computer (Elliott 803) in two modes of programming. The first one was based on Autocode Mark 3 because of its simplicity in programming. Since any arithmetic operation in Autocode is carried out with accuracy not exceeding  $2^{-29}$  ( $\simeq 10^{-8.7}$ ), the final results of the computations of the function  $W(\zeta, \delta)$  were off the desired degree of accuracy. In order to exploit the total possibility of the computer the second mode of programming was based on Basic 803 Instruction Code which assures operational accuracy of the range  $2^{-39}$ . This mode, however, involves certain difficulties since each number  $x$  participating in any arithmetic operation must be in the range  $-1 \leq x < 1$ . Most steps in the present problem required calculations to be done with numbers exceeding unity in magnitude. It was therefore generally necessary to scale all numbers by a suitable factor, to examine all results of addition and division, and to re-scale all results of multiplications.

The functions  $H_k^{(2)}(u)$  were expressed as

$$H_k^{(2)}(u) = J_k(u) - jY_k(u),$$

where  $J_k(u)$  and  $Y_k(u)$  denote Bessel and Neumann functions, respectively. Hence the real and imaginary parts of  $A_r(u)$ ,  $A_t(u)$  functions and also the integrands  $b_r(u)$ ,  $b_t(u)$  in the formulas

$$B_r(u) = \int_0^u b_r(u) du, \quad B_t(u) = \int_0^u b_t(u) du$$

may be expressed in a general shorthand form

$$F = F[u, \sin u, \cos u, J_0(u), Y_0(u), J_1(u), Y_1(u)].$$

The values of the  $J_k(u)$  and  $Y_k(u)$  functions were determined from the known series expansions in which all the terms of the range less than  $10^{-9}$  were neglected. As the function  $Y_k(u)$  has a logarithmic singularity near  $u = 0$  a function  $V_k(u) = uY_k(u)$ , properly adapted, was evaluated instead of the first one.

The values of the  $A_r(u)$ ,  $A_t(u)$ ,  $B_r(u)$ ,  $B_t(u)$  were computed in the range 0.1 (0.1) 16.0. In order to perform numerical integrations the  $b_r(u)$ ,  $b_t(u)$  functions – and therefore the  $J_k(u)$ ,  $uY_k(u)$  and associated trigonometric functions – were computed in the range 0.0 (0.02) 16.0.

In any interval  $(a, a + 0.1)$  the integrands  $b_r(u)$ ,  $b_t(u)$  were replaced by an interpolating polynomial of order five. The integrations were performed by the use of the Newton-Cotes formula which may be written in the following form:

$$\int_a^{a+0.1} f(x) dx = \frac{0.1}{288} \{19[f(a) + f(a + 0.1)] + 75[f(a + 0.02) + f(a + 0.08)] + 50[f(a + 0.04) + f(a + 0.06)]\}.$$

The values of  $W(\zeta, \delta)$  were calculated for the following values of  $\delta$ : 0.3, 0.6, 1.0, 1.5, 2.0, 3.1, 5.0, 6.3, 8.0, 9.5, and 11.0. Besides,  $W(\zeta, \delta)$  was computed for additional twelve values of parameters  $\zeta, \delta$ ; this was needed when drawing the plot from Fig. 7b.

In the process of computation almost the total store of the computer was used (4096 locations). 3604 locations of the store were occupied by the values of  $b_r(u)$ ,  $b_t(u)$  functions, and the rest by the program and sub-routines. The program in blocks is shown in Fig. 5.

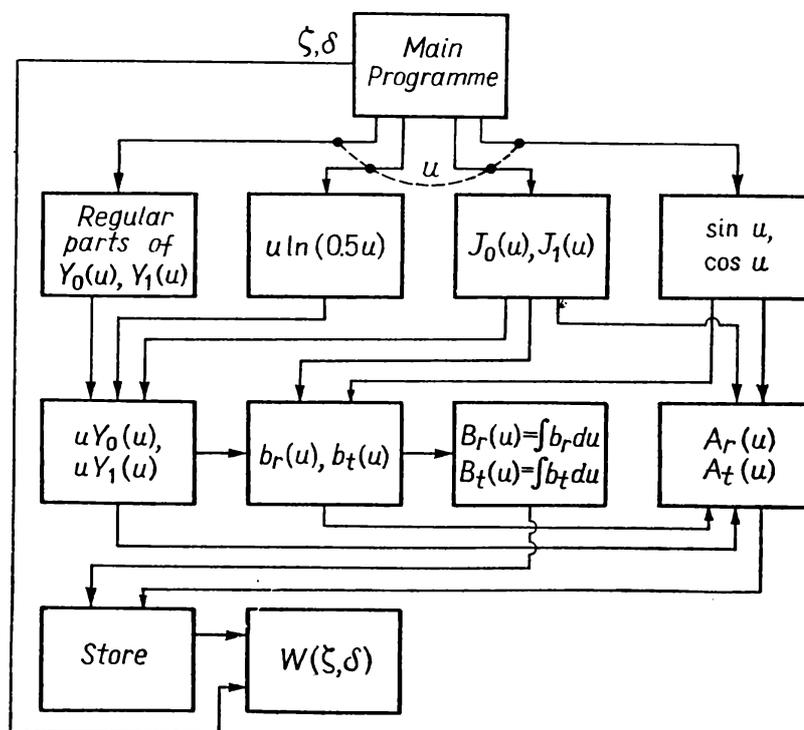


Fig. 5. Block diagram of computation programme

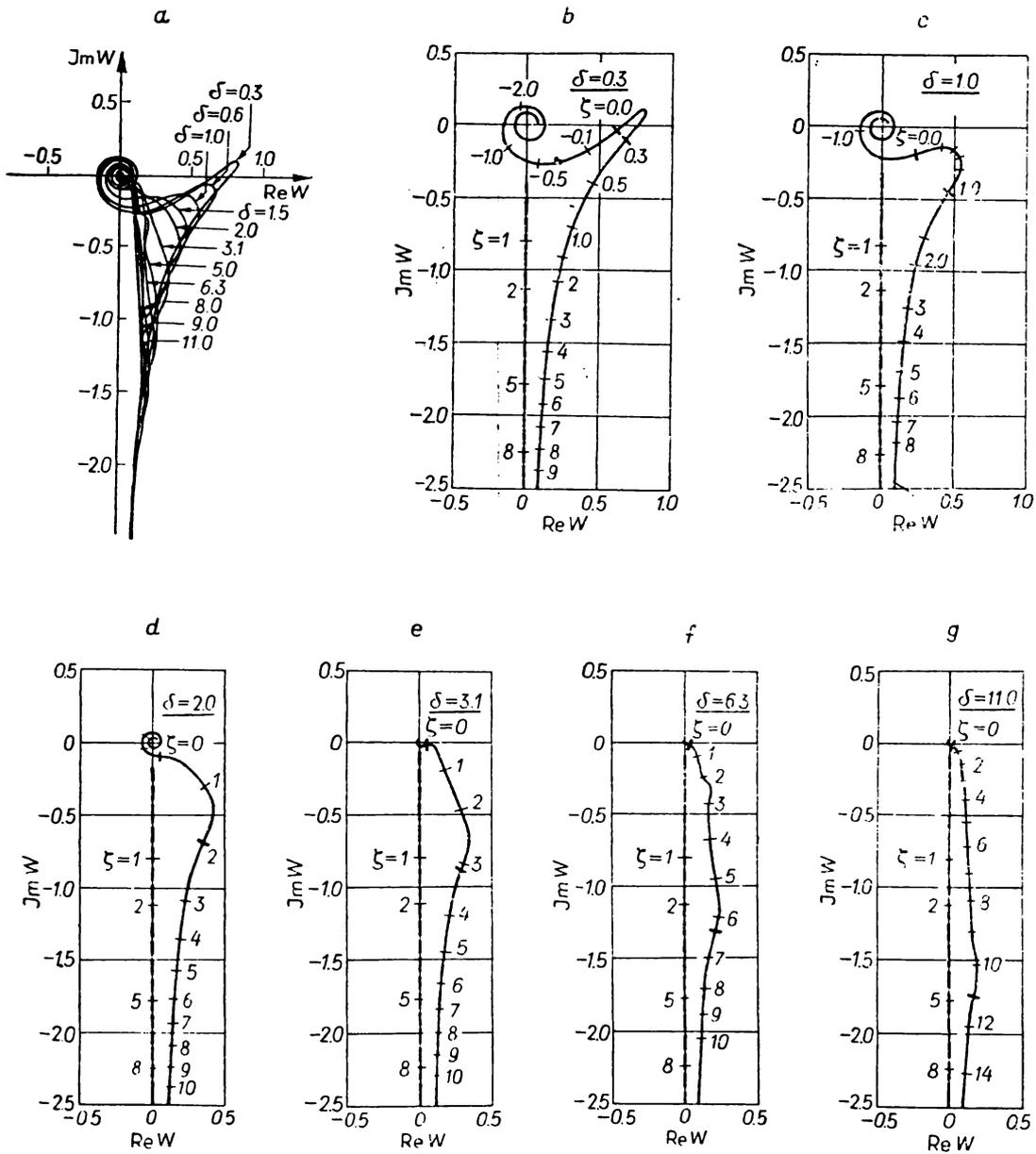


Fig. 6. Plots of  $W(\zeta, \delta) = f(\zeta)$  for  $\delta =$  parameter

— plots according to (22)  
 - - - - - variation according to classical theories in compliance with (27)  
 The ends of the transition zone are indicated by heavy marks

The results of computations in form of polar diagrams are summarized in Fig. 6a and shown in more detail for six values of  $\delta$  in Figs. 6b-g.

The behaviour of  $W(\zeta, \delta)$  for large negative  $\zeta$  (i.e. at comparatively large distances in front of the boundary) is shown in detail for one value of  $\delta$  only (Fig. 7a). For other values of  $\delta$  the character of  $W(\zeta, \delta)$  for negative  $\zeta$  is similar (see Fig. 6) but magnitude of the effect is different; this may be inferred from Fig. 7b.

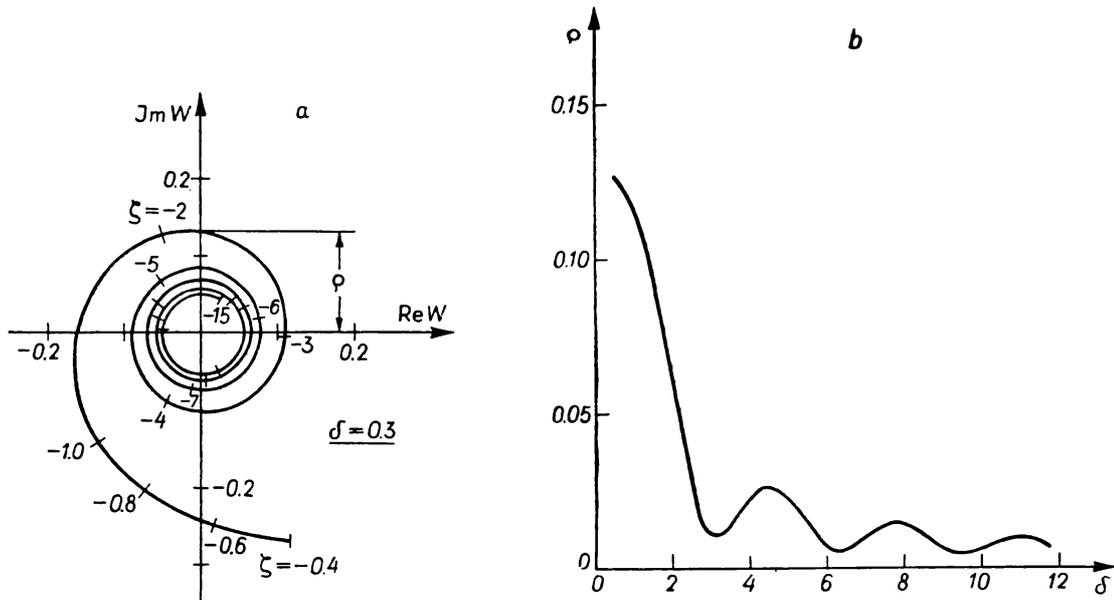


Fig. 7. Behaviour of  $W(\zeta, \delta)$  for large negative values of  $\zeta$

(a)  $W(\zeta, \delta) = f(\zeta)$  for  $\delta = 0.3$

(b)  $\rho = f(\delta)$  (parameter  $\rho$  is explained in Fig. 7a)

An example of field variations is shown in Fig. 8. For the purpose of illustration a dry land-sea path of a pronounced contrast was assumed. The electrical parameters of the ground are  $\sigma = 10^{-3} \Omega^{-1} \text{m}^{-1}$  and  $\epsilon_r = 4$  and that of the sea  $\sigma = 4 \Omega^{-1} \text{m}^{-1}$  and  $\epsilon_r = 80$ . Frequency is 1 MHz, the width parameter  $\delta = 1$ ; according to (20) the width of the transition zone is thus  $d = \delta \lambda_0 / (2\pi) \simeq 48$  m. The contrast parameter  $\mathfrak{z}$  calculated according to (16) and (19) is  $\mathfrak{z} = 0.229^{173^\circ 38'}$ .

The variations of  $w(r_0 + x)/w_a(r_0)$  as calculated from (22) are shown in Fig. 8a in form of a polar diagram; Fig. 8a shows thus at the same time changes of magnitude and phase of the field. In Fig. 8b we have a plot of  $|w(r_0 + x)/w_a(r_0)|$  as a function of  $\zeta$ ; this diagram gives a conventional picture of changes of the field with distance.

### 9. General character of field variations across the transition zone.

As follows from Fig. 8 and the plots of  $W(\zeta, \delta)$  from Fig. 6 the field in front of the boundary has an oscillatory component. This behaviour is of an obvious physical nature and is connected with the fact that over the zone  $a$  the secondary waves propagate in the backward direction. As a consequence the primary and the secondary waves show rapidly changing phase differences. When interfering these two waves produce a final field changing from point to point in a manner characteristic of a standing wave. In accord with this explanation the periodicity of the fluctuations should approximately be equal to  $\lambda_0/2$ ; expressed in  $\zeta$ -units the period should be equal to about  $\pi$ . This is in agreement with the

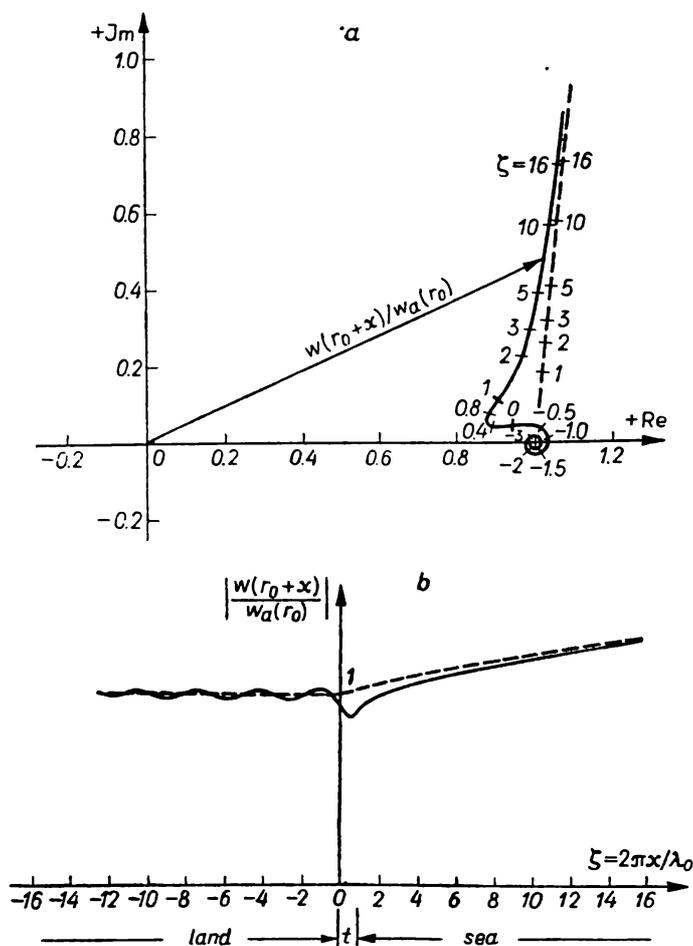


Fig. 8. Field variation across land-sea boundary (transmitter on land section;  $\delta = 1$ ;  $f = 1$  MHz)

t — transition zone. The ends of the transition zone are indicated by heavy marks  
 ————— plot according to (22)  
 - - - - - variation according to classical theories in compliance with (27)

results of numerical calculations (see for instance Fig. 7a) and a formula which may be derived for  $W(\zeta, \delta)$  for large negative values of  $\zeta$ .

With increasing distance from the boundary the plots of  $W(\zeta, \delta)$  are for negative  $\zeta$  of a shape of a tightening spiral. This means that fluctuations diminish when moving away from the boundary. It may be shown that at large distances from the boundary the amplitude of fluctuations decreases as  $(-\zeta)^{-1/2}$ , i.e. is inversely proportional to the square root of distance. This is, after all, obvious from purely physical reasons: the elementary secondary waves have the nature of cylindrical waves and must decrease with distance so as to carry a constant power across cross-sections which increase in proportion to distance. As a consequence the electromagnetic field of these waves must be inversely proportional to the square root of distance.

For a given contrast between the two sections of a path the largest intensity of the return wave and consequently the largest oscillations of field strength occur for a narrow transition zone; for wide transition zones they are much smaller (see Fig. 7b). For certain widths of the transition zone the return wave is very weak. As follows from Fig. 7b the corresponding minima occur for  $\delta = 2\pi d/\lambda_0 = n\pi$  ( $n = 1, 2, \dots$ ). We thus see that in the cases of transition zones of the widths  $d = n\lambda_0/2$  there occurs approximate "matching" of the zone  $b$  with the zone  $a$ . The field in front of the boundary shows then practically no intensity fluctuations. This fact may perhaps serve as an explanation of discrepancies in the observations of field behaviour in the vicinity to a boundary where some authors reported presence of field fluctuations whereas others did not notice this phenomenon.

The magnitude of field fluctuations — especially for wide transition zones — is not large. As follows from Figs. 8 and 7b and Table 1 the amplitude of field oscillations does not exceed at the most a few per cent and that of the phase about one or two degrees; the fluctuations of such magnitude may appear in the neighbourhood of a sea coast. In the cases of overland propagation the field perturbation is generally much smaller, in proportion to the actual value of the contrast parameter  $\beta$ . From a practical point of view the fluctuations are then in many cases completely negligible, in particular for somewhat wider transition zones.

The field variations over the transition zone are in general gradual except at small values of  $\delta$  when there comes into appearance an increasingly strong local field perturbation over the transition zone (Fig. 6a).

When  $\delta \rightarrow 0$  this perturbation develops into field singularity ([9], [10] and [2], p. 374-375). The case represents then formally a discontinuous change of surface impedance. However, this formal limiting process  $\delta \rightarrow 0$  is not equivalent to the consideration of an abrupt physical boundary. As discussed in Section 2 in the case of an abrupt real boundary there exist fundamental doubts as to the physical nature of the considered singularity. In general, singularities in current and charge distribution are there to be expected; besides, the impedance would certainly not be constant up to the very boundary. Moreover, an abrupt coast represents in practice always a definite surface irregularity which makes the problem the more complex. There also comes into action an additional factor of purely formal character: taking the limit  $\delta \rightarrow 0$  we must allow for the fact that the field over the transition zone is increasingly different from the primary field which would there exist in case of a homogeneous path. Consequently, it is not admissible to use a first order perturbation approach which was employed in the present research like in other similar investigations. Summing up the above remarks we are led to the conclusion

that as yet the problem of an abrupt physical boundary must be considered as an open question.

Past the boundary, i.e. over the zone  $b$ , the primary and the secondary waves propagate in the same direction; their phase relations are thus approximately constant. As a consequence, the resultant field changes monotonically with distance and shows no fluctuations. For not too wide transition zones at distances  $x \gg \lambda_0/(2\pi)$  an asymptotic expansion of  $W(\zeta, \delta)$  yields

$$(26) \quad w(r_0 + x)/w_a(r_0) = 1 - j(2/\pi)^{1/2} 3\zeta^{1/2}.$$

Absence of the parameter  $\delta$  in (26) may be easily explained by the comparatively small contribution from the transition zone as compared with much larger contributions from the parts of the zone  $b$  up to the point of observation.

Introducing a numerical distance  $\varrho = sx$ , where

$$s = -j \frac{\gamma_0}{2(\varepsilon_r' + 1)} = -j \frac{\gamma_0}{2} \left( \frac{Z_s}{Z_0} \right)^2,$$

we obtain from (16) and (26)

$$(27) \quad \frac{w(r_0 + x)}{w_a(r_0)} = 1 - j \frac{2}{\sqrt{\pi}} (\sqrt{s_b} - \sqrt{s_a}) \sqrt{x}.$$

This is, however, a formula of classical mixed-path theories for points at small numerical distances past the boundary (see for instance equations (56) and (34) in [3]). At larger distances past the boundary the present theory goes thus asymptotically over into classical theory. This settles the question raised in Section 3 as to the validity of classical theories of mixed-path propagation and at the same time furnishes a verification of the present theory.

The asymptotic behaviour of the attenuation function according to (27) has been plotted in Figs. 6 and 8 by means of a broken line. It may be seen from these figures that beginning at about a wavelength from the middle of the transition zone the classical theories show practically negligible errors. If in the considered practical applications the small field perturbations existing in the neighbourhood of the boundary are of no importance then for not too narrow transition zones the classical mixed-path theories prove to be sufficiently accurate up to the boundary and across it.

**10. Conclusions.** As the discussion has shown, in the cases of comparatively broad transition zones the results of the present research and of other similar investigations ([2], p. 368-381, [7], [9]-[11], [13], [14]) are valid with considerable accuracy. The field perturbations are then rather small. Much larger effects are to be expected in the neighbourhood of an abrupt coast. This much more interesting and important problem, however, presents serious theoretical difficulties and doubts. It awaits still a reliable solution and the existing theoretical results must so far be considered with considerable caution.

#### References

- [1] Е. Л. Фейнберг, *Распространение радиоволн вдоль реальной поверхности*, in *Исследования по распространению радиоволн*, Сборник 2, АН СССР, Москва 1948, p. 97-215.
- [2] — *Распространение радиоволн вдоль земной поверхности*, АН СССР, Москва 1961.
- [3] Z. Godziński, *The use of equivalent secondary sources in the theory of ground-wave propagation over an inhomogeneous earth*, Proc. IEE, Part C, 105 (1958), p. 448-464.
- [4] — *The surface impedance concept and the structure of radio waves over real earth*, Proc. IEE, Part C, 108 (1961), p. 362-373.
- [5] — *Amplitude and phase changes of ground-wave at the boundary between two different sections of the surface of the earth*, C.C.I.R. Study Groups, Geneva 1962, Document V/52.
- [6] — *Electromagnetic field at the transition zone between two different sections of earth surface*, U.R.S.I. Symposium on Electromagnetic Theory and Antennas, Copenhagen 1962, p. 67-72.
- [7] Ю. К. Калинин, *Возмущение поля плоской радиоволны неоднородностями земной поверхности*, Радиотехника и Электроника 3 (1958), p. 557-561.
- [8] P. M. Morse, H. Feshbach, *Methods of theoretical physics*, McGraw-Hill, New York 1953, p. 811.
- [9] J. R. Wait, *Amplitude and phase of the low-frequency ground wave near a coastline*, J. Res. NBS 58 (1957), p. 237-242.
- [10] — *Oblique propagation of groundwaves across a coastline*, Part I, J. Res. NBS 67 D (1963), p. 617-624.
- [11] — *Oblique propagation of groundwaves across a coastline*, Part III, J. Res. NBS 68 D (1964), p. 291-296.
- [12] — *Electromagnetic surface waves*, in *Recent advances in radio research*, ed. by J. A. Saxton, Academic Press, New York 1965.
- [13] J. R. Wait, C. M. Jackson, *Oblique propagation of groundwaves across a coastline*, Part II, J. Res. NBS 67 D (1963), p. 625-630.
- [14] J. R. Wait, K. P. Spies, *Propagation of radio waves past a coastline with a gradual change of surface impedance*, IEEE Trans. AP-12 (1964), p. 570-575.

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**Z. GODZIŃSKI i L. STASIEŃSKI (Wrocław)****ROZCHODZENIE SIĘ FALI PRZYZIEMNEJ NAD STREFĄ PRZEJŚCIOWĄ  
MIĘDZY DWOMA RÓŻNYMI ODCINKAMI TRASY****STRESZCZENIE**

W pracy przeprowadzono dyskusję zagadnienia zaburzenia pola fali przyziemnej w okolicy strefy przejściowej pomiędzy dwu różnymi odcinkami trasy. Zastosowana metoda analizy daje jasny obraz sytuacji fizycznej oraz wielkości efektu. Pozwoliła ona równocześnie na przeprowadzenie krytycznej dyskusji konwencjonalnych przybliżeń. Użyta metoda może również mieć pewne znaczenie przy dyskusji zjawisk dyfrakcji fal elektromagnetycznych na ciałach o skończonej przewodności. Pokazano, że w przypadku stopniowego przejścia między dwu odcinkami trasy przed granicą dwu różnych stref występują niewielkie fluktuacje pola o charakterze fali stojącej. Fluktuacje te nie przekraczają kilku procent w wielkości i jednego do dwu stopni w fazie. Największe fluktuacje występują w przypadku wąskich stref przejściowych. Dla określonych szerokości strefy przejściowej fluktuacje wykazują głębokie minima, to jest występuje dopasowanie obu odcinków trasy. Po przekroczeniu strefy przejściowej pole zmienia się monotonicznie i na odległościach większych niż około długość fali jest praktycznie takie samo jak przewidują to konwencjonalne teorie tras mieszanych. Przeprowadzono krytyczną dyskusję ważnego zagadnienia bardzo wąskiej strefy przejściowej. Stwierdzono, że zagadnienie to stanowi ciągle jeszcze otwarty problem naukowy, który do obecnej chwili nie znalazł zadowalającego rozwiązania.

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