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ON THE PERCOLATION OF WATER FROM A CYLINDRICAL RESERVOIR INTO THE SURROUNDING SOIL

1. Introduction. The paper deals with the following physical problem. Suppose we are given a cylindrical reservoir filled with water, surrounded by a dry soil. The bottom of the reservoir touches a horizontal impermeable layer having a plane upper boundary. In such a situation the flow percolates from the reservoir into the surrounding unsaturated region. Obviously, the water-table and the reach of the moisted region are time-varying. If we suppose that the reservoir is continuously refilled with water (so that its height is constant) and that the surrounding region is sufficiently great, the described process may be observed during a large time interval. If we want to describe the time-dependence of this phenomenon, we are led to the non-stationary percolation problem, which is considered in the sequel. The described problem is closely related to a method of cleaning copper ore by plunging it into a large reservoir with water (the so-called flotation method). After the cleaning process the water in the reservoir contains much toxic substances and it is very important to know in what manner the impurities percolate into the surrounding region. The report [2] was the first step in studying this problem and the present paper contains a more detailed study.

Our investigations are based on some physical simplifications. We suppose that the soil is homogeneous and isotropic. Thus the coefficients of hydraulic conductivity K and of permeability of the soil m are constant scalar values. After neglecting the effects of capillarity we can assume that the water-table divides the soil into two regions: a saturated one and a dry one. We use the hydraulic model of filtration (called also the *Dupuit approximation*; see [6] and [7]). In this model the non-stationary percolation is described by Boussinesq's equation

$$\frac{1}{2}\Delta_{x,y}h^2 = \frac{m}{K}h_t. \tag{1}$$

Here the upper boundary of the impermeable layer is taken as the (x, y)-plane and h denotes the height of the saturated region (thus the surface $z = h(x, y, t)$ is the water-table at time). $\Delta_{x,y}$ denotes, as usually, the plane Laplace operator. We assume that the z-axis is at the same time the axis of the reservoir whose radius is equal to one. Accordingly we have the first boundary condition $h|_{r=1} = \text{const}$ or, for simplicity, we assume that

$$h|_{r=1} = 1. \tag{2}$$

We may expect that in the described model the percolation process has a radial symmetry. So we are interested only in the solutions of (1) of the form

$$h = h(r, t) \qquad (r = \sqrt{x^2+y^2}).$$

After introducing polar coordinates in the plane xy we obtain the Boussinesq equation in the form

$$\frac{1}{2}(h^2)_{rr} + \frac{1}{2r}(h^2)_r = h_t \tag{3}$$

(for simplicity, in the sequel we put $m/K = 1$).

To obtain the second boundary condition we suppose that the reach of the saturated region is finite at each moment $t > 0$. This is rather natural from the physical point of view if we know that water percolates with a finite speed and that at the beginning the soil surrounding the reservoir is dry. If we denote by $r_0(t)$ the reach of the infiltrating water at the moment t, the second boundary condition claims

$$h|_{r=r_0(t)} = 0. \tag{4}$$

In our considerations we deal at first with approximate solutions of the problem. We seek namely the function h in a simplified form (5). It will be shown in Section 2 that the problem reduces in this case to the non-linear integral equation (16) (with the parameter A defined by (13)) which can be uniquely solved by the method of successive approximations (see Section 4). The solution $u(\tau)$ describes the free surface of the saturated region after changing the scale on the r-axis by means of substitutions (6) and (10). After solving the integral equation we obtain the approximate solution p_A of our basic problem (see formula (19) and Theorem 1). In Section 5 we give (formula (48)) an estimate of the p.c. error between p_A and the exact solution. This error tends to zero with $r_0 \to \infty$ if the percolation process satisfies conditions (C_1)-(C_3). It is shown that these conditions are satisfied in a simplified model if the function h depends linearly on r. In this model, estimate (48) has a particularly simple form and may be used in numerical calculations.

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2. Reduction of the boundary-value problem to an integral equation. We ask about the solutions of (3) in the domain $1 < r < r_0(t)$, $t > 0$, which satisfy the boundary conditions (2) and (4). As we do not know the free boundary $r_0(t)$, we have here two unknown functions: h and r_0. In the sequel we deal only with solutions of a special form

$$h(r,t) = p(s), \tag{5}$$

where

$$s = \frac{r}{r_0(t)}. \tag{6}$$

We make also one further assumption about the character of our percolation process, namely we suppose that

$$r_0(t)\dot{r}_0(t) = c, \tag{7}$$

where c is a positive constant. Since it is supposed that at the beginning the soil is dry, we have the initial condition $r_0(0) = 1$ and the integration of (7) yields

$$r_0(t) = \sqrt{1+2ct}.$$

Substitution (6) allows us to consider the ordinary differential equation

$$(p')^2 + pp'' + \frac{1}{s}pp' + csp' = 0 \quad \left(b < s < 1,\ b = \frac{1}{r_0(t)}\right) \tag{8}$$

with the boundary conditions

$$p(b) = 1 \quad \text{and} \quad p(1) = 0 \tag{9}$$

as describing the percolation of water. Introducing new variables τ and $q(\tau)$ by means of

$$s = b^\tau \quad \text{and} \quad q(\tau) = p(s) \quad (0 < \tau < 1) \tag{10}$$

we obtain the new boundary value problem

$$(q^2)'' = 2BA^{-\tau}q', \tag{11}$$

$$q(0) = 0, \quad q(1) = 1, \tag{12}$$

where

$$A = r_0^2(t) \tag{13}$$

and

$$B = -c\ln b. \tag{14}$$

Let us now integrate twice the equation (11). If we change once more the unknown function

$$u(\tau) = B^{-1} q(\tau), \tag{15}$$

we obtain, after using the first of the boundary conditions (12), the integral equation

$$u^2(\tau) = 2\int_0^\tau A^{-\sigma}\big(1+(\tau-\sigma)\ln A\big)u(\sigma)\,d\sigma \qquad (0 \leqslant \tau \leqslant 1). \tag{16}$$

To satisfy the second of the boundary conditions (12) we have to assume that

$$u(1) = B^{-1}. \tag{17}$$

Suppose that equation (16) has the solution $u_A(\tau)$ for every $A > 1$ (it follows from the physical meaning of the parameter A that this inequality has to be satisfied). If we put

$$B = [u_A(1)]^{-1}, \quad b = A^{-1/2}, \quad c = \frac{2B}{\ln A} \tag{18}$$

and

$$p_A(s) = Bu_A(\tau)\Big|_{\tau = \frac{-2\ln s}{\ln A}}, \tag{19}$$

then the function p_A is the solution of (8), (9). In this sense the integral equation (16) is equivalent to the boundary-value problem (8), (9) if we fix the value of the parameter A and introduce the constants B, b, and c by means of (18).

3. Properties of the solutions of the integral equation. Our next step is the investigation of equation (16) following the methods used in [3]. We are going to prove its solvability by means of the Banach fixed-point theorem (see [4], p. 323). We suppose from now on that $A > 1$.

LEMMA 1. *If a non-negative function f satisfies the integral equation* (16) *for $\tau \in [0, 1)$, then*

(i) *f is continuous in $[0, 1)$,*

(ii) *there exists* $\lim\limits_{\tau\to 1-} f(\tau) < \infty$,

(iii) *f may be extended to a continuous solution of* (16) *in the interval* $[0, 1]$.

Proof. The kernel of (16) may be estimated from below by A^{-1}. Since the lemma is evidently true for the trivial solution $f = 0$, we may assume that f does not vanish identically. Thus it follows from (16) that f is integrable in each interval $[0, \tau]$ with $\tau < 1$ and, therefore, both f^2 and f are continuous for $0 \leqslant \tau < 1$. Let us introduce now the function

$$\psi(\tau) = \sup_{0\leqslant\sigma\leqslant\tau} f(\sigma).$$

It follows from (16) that $\psi(\tau)$ is finite for $0 \leqslant \tau < 1$ and

$$f^2(\tau) \leqslant 2(1+\ln A)\psi(\tau) \qquad (0 \leqslant \tau < 1). \tag{20}$$

As ψ is a non-decreasing function, it follows from (20) that

$$\psi(\tau) \leqslant 2(1+\ln A).$$

Thus f is bounded in $[0,1)$ and one may take the limit with $\tau \to 1-$ on both sides of (16). This completes the proof.

LEMMA 2. *If f is a continuous solution of* (16) *for* $0 \leqslant \tau \leqslant 1$, *then*
(i) *f is non-negative,*
(ii) *there exists a number $a \in [0,1]$ such that*

$$f(\tau) = 0 \quad \textit{for } 0 \leqslant \tau \leqslant a,$$
$$f(\tau) > 0 \quad \textit{for } a < \tau \leqslant 1.$$

Proof. Let us write

$$E = \{\tau \in [0,1]\colon f(\tau) = 0\}.$$

It is easy to see that E is closed and $0 \in E$. We show that E has the following property:

(D) For each $\tau_1, \tau_2 \in E$ $(\tau_1 < \tau_2 < 1)$, the intersection $E \cap (\tau_1, \tau_2)$ is not empty.

It follows from the integral equation and the continuity of f that f^2 is differentiable in $(0,1)$. It has its minimum at every point of E, therefore

$$\frac{df^2(\tau)}{d\tau} = 0 \qquad (\tau \in E,\ 0 < \tau < 1)$$

or, in another form (after calculating the derivative),

$$\int_0^\tau A^{-\sigma} f(\sigma)\,d\sigma = 0 \qquad (\tau \in E,\ 0 < \tau < 1).$$

This yields

$$\int_{\tau_1}^{\tau_2} A^{-\sigma} f(\sigma)\,d\sigma = 0 \qquad (\tau_1, \tau_2 \in E \setminus \{1\}),$$

therefore, it follows from the continuity of f that it vanishes for some $\tau_3 \in (\tau_1, \tau_2)$. Thus (D) is proved.

Now there are two possibilities: E is the whole interval $[0,1]$ or $E \setminus \{1\} = [0, a]$, where $a < 1$. In the first case, $a = 1$ and f vanishes identically. In the second case we have

$$f^2(1) = 2\int_a^1 A^{-\sigma}\big(1+(1-\sigma)\ln A\big) f(\sigma)\,d\sigma \tag{21}$$

and thus $f(1)$ does not vanish since, in the opposite case, f had to change its sign in $(a, 1)$ which is impossible. So the integral on the right-hand side of (21) is positive and f takes on positive values in $(a, 1]$. If we go back to the calculations of Section 1, it is readily seen what is the physical meaning of vanishing the number a in Lemma 2. If namely u is the solution of (16) such that

$$u(\tau) > 0 \quad \text{for } \tau > 0, \tag{C}$$

then the corresponding solution $p(s)$ of equation (8), given by formula (19), is positive for $s < 1$. In other words, $r_0(t)$ is exactly the reach of percolating water. Therefore, in the sequel we deal only with solutions of (16) satisfying condition (C).

LEMMA 3. *Every continuous solution of* (16) *satisfying* (C) *has the following properties*:

(i) $f \in C^2[0, 1]$,

(ii) $f'(\tau) > 0$ *for* $0 < \tau \leqslant 1$,

(iii) $f'(0) = 1$,

(iv) $f''(\tau) < 0$ *for* $0 \leqslant \tau \leqslant 1$,

(v) *the inequality*

$$\frac{1-A^{-\tau}}{\ln A} \leqslant f(\tau) \leqslant \tau \tag{22}$$

holds for $0 \leqslant \tau \leqslant 1$.

Proof. It follows from the integral equation and from our assumptions that f^2 is continuously differentiable in $(0, 1]$, and so is f. Differentiation of (16) yields, after dividing by f,

$$f'(\tau) = A^{-\tau} + \frac{\ln A}{f(\tau)} \int_0^\tau A^{-\sigma} f(\sigma)\, d\sigma \qquad (0 < \tau \leqslant 1), \tag{23}$$

so (ii) is true. Therefore, f increases in $(0, 1]$ and it follows from (23) that

$$A^{-\tau} \leqslant f'(\tau) \leqslant A^{-\tau} + \ln A \int_0^\tau A^{-\sigma}\, d\sigma \qquad (0 < \tau \leqslant 1).$$

Calculating the integral on the right-hand side we obtain

$$A^{-\tau} \leqslant f'(\tau) \leqslant 1 \qquad (0 < \tau \leqslant 1). \tag{24}$$

So $f'(0)$ exists and (iii) is true. After integrating both sides of (24) we obtain (22). It remains to prove (i) and (iv). Since f is of class C^1, it follows from the integral equation that it is of class $C^2(0, 1]$. Differentiation of (23) gives

$$f''(\tau) = -\ln A \frac{f'(\tau)}{f^2(\tau)} \int_0^\tau A^{-\sigma} f(\sigma)\,d\sigma \qquad (0 < \tau \leqslant 1), \tag{25}$$

therefore (iv) holds for $\tau > 0$. To investigate $f''(0)$ let us take the limit in (25) as $\tau \to 0+$. Applying the de l'Hôspital formula we obtain

$$\lim_{\tau \to 0+} f''(\tau) = \frac{-\ln A}{2} < 0.$$

This completes the proof.

4. Solvability of the integral equation. We introduce the notation

$$(Lf)(\tau) = \int_0^\tau A^{-\sigma}\big(1 + (\tau - \sigma)\ln A\big) f(\sigma)\,d\sigma \quad \text{and} \quad T(f) = \sqrt{2Lf}.$$

For two arbitrary functions f_j ($j = 1, 2$) the inequality $f_1 \leqslant f_2$ means that $f_1(\tau) \leqslant f_2(\tau)$ for $0 \leqslant \tau \leqslant 1$. It is easy to show that

$$Lf_1 \leqslant Lf_2 \quad \text{and} \quad T(f_1) \leqslant T(f_2) \quad \text{if } f_1 \leqslant f_2;$$

the operators L and T are monotone. Let us denote by P the set of all continuous functions satisfying (22). We prove

LEMMA 4. *T maps the set P into itself.*

Proof. Let us put

$$g(\tau) = \frac{1 - A^{-\tau}}{\ln A}, \quad G(\tau) = \tau,$$

and suppose that the inequality $g \leqslant f \leqslant G$ holds. Then

$$T(g) \leqslant T(f) \leqslant T(G) \tag{26}$$

and we have to prove that $g \leqslant T(g)$ and $T(G) \leqslant G$. The elementary calculation shows that

$$T(g)(\tau) = \sqrt{2}\left(\frac{(1 - A^{-\tau})^2}{(2\ln A)^2} + \int_0^\tau \frac{(1 - A^{-\sigma})^2}{2\ln A}\,d\sigma\right)^{1/2}$$

and this yields the left-hand side of (26). Similarly, after calculating the integral, we have

$$T(G)(\tau) = \frac{\sqrt{2}}{\ln A}(A^{-\tau} + \tau\ln A - 1)^{1/2}.$$

Now the right-hand side of (26) is equivalent to

$$\tau^2(\ln A)^2 - 2(A^{-\tau} + \tau\ln A - 1) \geqslant 0 \qquad (0 \leqslant \tau \leqslant 1). \tag{27}$$

Denoting the left-hand side of (27) by $\varphi(\tau)$ we have

$$\varphi(0) = 0 \tag{28}$$

and

$$\varphi'(\tau) = 2(\ln A)\psi(\tau),$$

where

$$\psi(\tau) = \tau\ln A + A^{-\tau} - 1. \tag{29}$$

As $\psi(0) = 0$ and $\psi'(\tau) > 0$ for $\tau > 0$, the function ψ is positive for $\tau \in (0, 1]$, and so is $\varphi'(\tau)$. Therefore, φ is increasing and this fact together with (28) gives (27). The proof is complete.

Let us write now

$$\varrho(f_1, f_2) = \sup_{0 \leqslant \tau \leqslant 1} \frac{\ln A}{\psi(\tau)} |f_1(\tau) - f_2(\tau)|, \tag{30}$$

where $\psi(\tau)$ is given by (29). In a simple way we can prove the following

Lemma 5. *The set P is a complete metric space with the distance ϱ defined by* (30).

We prove now the crucial step in our considerations.

Lemma 6. *The mapping T is a contraction in the space P, namely*

$$\varrho\big(T(f_1), T(f_2)\big) \leqslant \frac{1}{2}\varrho(f_1, f_2) \quad (f_1, f_2 \in P). \tag{31}$$

Proof. We write (31) in the equivalent form

$$|T(f_1) - T(f_2)| \leqslant \frac{\psi}{2\ln A}\varrho(f_1, f_2). \tag{32}$$

It follows from Lemma 4 that

$$T(f_1) + T(f_2) \geqslant \frac{2g}{\ln A} \tag{33}$$

with $g(\tau) = 1 - A^{-\tau}$ $(0 \leqslant \tau \leqslant 1)$. We have also

$$|f_1 - f_2| \leqslant \frac{\psi}{\ln A}\varrho(f_1, f_2)$$

and, therefore, according to the monotonicity of L,

$$|Lf_1 - Lf_2| \leqslant \frac{\varrho(f_1, f_2)}{\ln A} L\psi. \tag{34}$$

Since

$$|T(f_1) - T(f_2)| = \frac{2|Lf_1 - Lf_2|}{T(f_1) + T(f_2)},$$

by (33) and (34) we obtain

(35) $$|T(f_1)-T(f_2)| \leqslant \varrho(f_1, f_2)\frac{L\psi}{g}.$$

To prove (32) it is enough to show that

(36) $$\frac{L\psi}{g} \leqslant \frac{\psi}{2\ln A}.$$

After integrating by parts the integral $L\psi$ we obtain the equivalent form of (36):

(37) $$1-2\tau(\ln A)A^{-\tau}-A^{-2\tau} \geqslant 0 \quad (0 \leqslant \tau \leqslant 1).$$

Denoting by $v(\tau)$ the left-hand side of (37) we have

(38) $$v'(\tau) = 2(\ln A)A^{-\tau}\psi(\tau) \quad (0 \leqslant \tau \leqslant 1).$$

It has been remarked in the proof of Lemma 4 that ψ is a non-negative function in $[0, 1]$. Thus v is non-decreasing in this interval and $v(0) = 0$. Therefore, (37) is true and this completes our proof.

It follows from Lemmas 4-6 that for given $A > 1$ the mapping T has exactly one fixed point in P. This fixed point is the unique solution u_A of (16) satisfying condition (C) and can be computed by the method of successive approximations (see [4], p. 323).

Going back to our basic problem we have now the following

THEOREM 1. *Let $A > 1$ be arbitrarily fixed and let us introduce notation* (18). *Then the function p_A given by* (19) *is the unique solution of* (8), (9), *and it has the following properties*:

(i) $p_A \in C^2[b, 1]$,

(ii) p_A *is positive and decreasing in* $[b, 1)$,

(iii) $p_A(1) = -2/\ln A$.

The theorem follows immediately from the proved properties of the integral equation (16).

5. Final remarks. Since the solution $p_A(s)$ of equation (8) depends on time by means of the variable s and of the parameter A (according to (13)), it is readily seen that the function h_A defined by (5) with p replaced by p_A is not an exact solution of the Boussinesq equation (3). We are going now to estimate the error which arises if we treat the function h_A as describing the water-table in the Boussinesq model.

Let us suppose that the solution h of (3) has (in new variables s and A) the form

$$h(r, t) = P_A(s).$$

Then for the function P_A we obtain the ordinary differential equation

(39) $$R(P_A) = 2A\frac{\partial P_A}{\partial A}\frac{dA}{dt},$$

where R is the differential operator on the left-hand side of (8). Denoting by $F(s; A)$ the right-hand side of (39), after simple calculations we have

$$F(s; A) = 2cs\sqrt{A}\,h_r + 2Ah_t. \tag{40}$$

Let us put

$$z = (P_A^2 - p_A^2)'; \tag{41}$$

then equations (8) and (39) yield the identity

$$z' + \frac{1}{s} z = Q \tag{42}$$

with

$$Q = F + 2cs(p_A' - P_A'). \tag{43}$$

It follows from the physical meaning of the function P_A that it satisfies the same boundary conditions (9) as p_A. Therefore, z is the solution of the first order differential equation (42) with the initial condition $z|_{s=1} = 0$ and it may immediately be written in the form

$$z = -\frac{1}{s}\int_s^1 \sigma Q(\sigma)\,d\sigma. \tag{44}$$

Going back to notation (41) and using the boundary condition at the point $s = b$, after integrating (44) we obtain

$$P_A^2 - p_A^2 = -\int_b^s \frac{1}{\varrho}\int_\varrho^1 \sigma Q(\sigma)\,d\sigma\,d\varrho. \tag{45}$$

We estimate the right-hand side of (45). Let us write

$$a = \sup s|P_A' - p_A'|.$$

Then (40) and (43) after elementary calculations yield

$$\sup|P_A^2 - p_A^2| \leqslant cr_0^2 \sup_{1\leqslant r\leqslant r_0} |h_r| + r_0^3 \sup_{1\leqslant r\leqslant r_0} |h_t| + ca\ln r_0. \tag{46}$$

It follows from Lemma 3 that the solution u_A of the integral equation (16) satisfies the inequality $0 < u'(\tau) \leqslant 1$ and this, according to (18) and (19), gives

$$-c \leqslant sp_A'(s) < 0. \tag{47}$$

We ask now what suppositions concerning the function h have to be made in order that P_A may satisfy an analogue of (47). Obviously, it follows from the definitions of the function P_A and of the variable s that $P_A' = r_0 h_r$ and, therefore, (47) is valid with p_A replaced by P_A if we assume that

$$-c \leqslant rh_r < 0 \quad (1 \leqslant r \leqslant r_0,\ t > 0). \tag{C$_1$}$$

In this case we have $a \leqslant c$ and (46) may be rewritten in the form

$$r_0^{-2}\sup|P_A^2 - p_A^2| \leqslant c\sup|h_r| + r_0\sup|h_t| + c^2 r_0^{-2}\ln r_0. \tag{48}$$

Inequality (48) gives an estimate of the error which arises after replacing the exact solution P_A of Boussinesq's equation by the approximate one p_A. This error may be arbitrarily small for sufficiently large r_0 if we make further assumptions concerning the character of the described percolation process, namely, for $r_0 \to \infty$,

$$\sup_{1\leqslant r\leqslant r_0} |h_r| \to 0 \tag{C$_2$}$$

and

$$r_0 \sup_{1\leqslant r\leqslant r_0} |h_t| \to 0. \tag{C$_3$}$$

In order to conclude our considerations we examine a simplified model of the free surface of the saturated region which satisfies (C_1)-(C_3). We

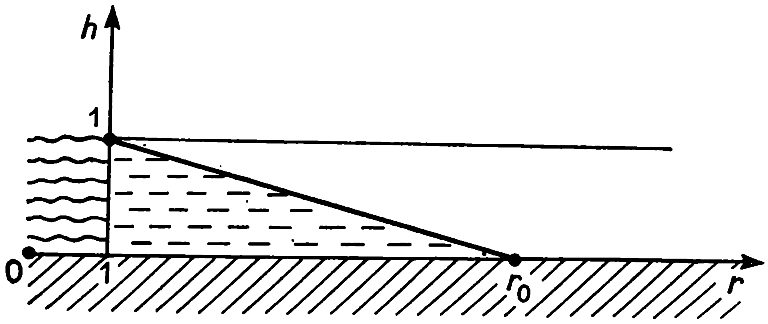

Fig. 1

suppose namely that this surface is linear, thus its vertical section is the straight line through the points (1, 1) and $(r_0, 0)$ in the plane (r, h) (see Fig. 1). In this case we have

$$h(r, t) = \frac{r - r_0}{1 - r_0} \qquad (1 \leqslant r \leqslant r_0),$$

so

$$h_r = \frac{1}{1 - r_0} \quad \text{and} \quad h_t = \frac{r-1}{(1-r_0)^2} r_0.$$

Therefore

$$\sup|h_t| \leqslant \frac{c}{r_0(r_0 - 1)} \quad \text{and} \quad rh_r = \frac{r}{1 - r_0},$$

so conditions (C_1)-(C_3) are satisfied if $c \geqslant 1$. Estimate (48) is then valid in the form

$$r_0^{-2}\sup|P_A^2 - p_A^2| \leqslant 2cr_0^{-1} + c^2 r_0^{-2}\ln r_0$$

and may be used in numerical calculations.

Remark(1). After suitable change of the independent variable and of the unknown function, (16) may be brought to the form of a non-linear convolution equation. Let us put namely

$$x = \tau \ln A \quad \text{and} \quad w(x) = A^{\tau}(\ln A)u(\tau).$$

A simple calculation shows that (16) is equivalent to

$$w^2 = K*w, \tag{49}$$

where

$$K(x) = \begin{cases} 2(1+x)e^{2x} & \text{for } x \geqslant 0, \\ 0 & \text{for } x < 0, \end{cases}$$

and the boundary condition (17) yields

$$w|_{x=\ln A} = \frac{2}{c}.$$

The equation of form (49) has interesting properties which have been studied independently of its physical meaning (see [5]).

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(1) This remark is due to Prof. C. Ryll-Nardzewski.

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O PRZESIĄKANIU WODY ZE ZBIORNIKA WALCOWEGO DO OTACZAJĄCEGO GO GRUNTU

STRESZCZENIE

W pracy badane jest zagadnienie nawilżania ośrodka porowatego przez ciecz zebraną w zbiorniku o kształcie walcowym. Zagadnienie to związane jest z metodą flotacyjną oczyszczania rud miedzi, polegającą na zanurzaniu ich w zbiorniku wypełnionym wodą. Z punktu widzenia ochrony środowiska niezwykle ważna jest znajomość procesu przenikania zanieczyszczeń ze zbiornika do otaczającego go gruntu. Przy założeniu prawa Darcy'ego oraz hipotezy hydraulicznej (tzw. przybliżenie Dupuit) powierzchnia swobodna cieczy nawilżającej opisana jest przez nieliniowe równanie różniczkowe cząstkowe Boussinesq'a. W pracy badane są własności analityczne pewnej klasy przybliżonych rozwiązań problemu, które można otrzymać rozwiązując jednowymiarowe nieliniowe równanie całkowe. Podano również oszacowanie błędu, jaki popełnia się zastępując rozwiązanie dokładne przez rozwiązanie przybliżone.