ZASTOSOWANIA MATEMATYKI
APPLICATIONES MATHEMATICAE
XV, 1 (1976)

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ALGORITHM 42

ENCLOSURE OF A POINT TO THE MINIMUM SPANNING TREE

1. Procedure declaration. Given the minimum spanning tree $F(Z)$ over the set Z of k points $P_1, P_2, \ldots, P_k$ and given k distances of the point P_{k+1} from the points of the set Z, procedure *Minden* calculates the minimum spanning tree for the set of $k+1$ points.

Data:

k — number of points in $F(Z)$;
$L[1:k-1]$ — integer array of links in $F(Z)$;
$c[1:k-1]$ — real array of the distances occurring in $F(Z)$;
$d[1:k]$ — real array of the distances of point P_{k+1} from the points of Z, i.e. $d[i] = d(P_{k+1}, P_i)$.

Results:

$L[1:k]$ — integer array of links in the newly formed minimum spanning tree;
$c[1:k]$ — real array of the distances occurring in the newly formed minimum spanning tree.

2. Method used. Let be given the minimum spanning tree $F(Z)$ over the set $Z = \{P_1, P_2, \ldots, P_k\}$, i.e. let be given the distances $c_1, c_2, \ldots, c_{k-1}$ on the links $P_1P_{L_1}, P_2P_{L_2}, \ldots, P_{k-1}P_{L_{k-1}}$, and also the distances $d_1, d_2, \ldots, d_k$ of the point P_{k+1} from the points of the set Z, i.e. the distance of the point P_{k+1} from the point $P_i \in Z$ equals d_i. Procedure *Minden* calculates the minimum spanning tree of the set $Z \cup \{P_{k+1}\}$ using the following theorem (see [1], p. 17):

For the link P_iP_j connecting the points P_i and P_j of the set X to belong to the minimum spanning tree $F(X)$ it is necessary and sufficient that there exist sets U and V with the following properties: $X = U \cup V$, $P_i \in U$, $P_j \in V$ and $d(P_i, P_j) = d(U, V)$.

Connecting the point P_i with the point P_{L_i} (for $i = 1, 2, \ldots, k$), one obtains the link of the minimum spanning tree $F(Z \cup \{P_{k+1}\})$ having the distance d_i.

```
procedure Minden(k,d,L,c);
 value k;
 integer k;
 integer array L;
 array c,d;
 begin
  integer g,i,i1,j,k1,nr,p,rg,rj,tj;
  real ck,dj,R;
  integer array r,t[1:k];
  p:=k+1;
  R:=ck:=c[k]:=d[k];
  nr:=k;
  k1:=k-1;
  for j:=1 step 1 until k1 do
   begin
    r[j]:=j;
    dj:=d[j];
    if dj<R
     then
     begin
      R:=dj;
      nr:=j
     end dj<R;
    if L[j]=k
     then
     begin
      if c[j]<ck
       then
       begin
        ck:=c[j];
```

```
      p:=j
     end c[j]<ck
    end L[j]=k;
   t[j]:=L[j];
   d[j]:=c[j];
   c[j]:=dj
  end j;
 t[k]:=p;
 d[k]:=ck;
 L[nr]:=k+1;
 c[nr]:=R;
 r[k]:=nr;
 r[nr]:=k;
 for i:=k1 step -1 until 1 do
  begin
   R:=2×c[r[1]];
   i1:=i+1;
   for j:=1 step 1 until i do
    begin
     rj:=r[j];
     tj:=t[rj];
     dj:=d[rj];
     for g:=i1 step 1 until k do
      begin
       rg:=r[g];
       if tj=rg
        then
        begin
         if dj<R
          then
```

```
             begin
              R:=dj;
              p:=j;
              nr:=rg
             end dj<R
           end tj=rg
           else
            if t[rg]=rj
             then
             begin
              if d[rg]<R
               then
               begin
                R:=d[rg];
                p:=j;
                nr:=rg
               end d[rg]<R
             end t[rg]=rj
         end g;
        if c[rj]<R
         then
         begin
          R:=c[rj];
          p:=j;
          nr:=k+1
         end c[rj]<R
       end j;
      j:=r[p];
      r[p]:=r[i];
      r[i]:=j;
      L[j]:=nr;
      c[j]:=R
    end i
  end Minden
```

3. Certification. For the data

i	1	2	3	4	5	6	7	8	9
$L[i]$	8	1	4	6	6	2	2	9	–
$c[i]$	3.6	1.4	21.2	4.1	1.4	3.0	4.1	5.0	–
$d[i]$	6.4	6.4	28.3	9.0	10.2	8.9	4.0	8.2	4.1

the following results were obtained:

i	1	2	3	4	5	6	7	8	9
$L[i]$	2	7	4	6	6	2	10	1	10
$c[i]$	1.4	4.1	21.2	4.1	1.4	3.0	4.0	3.6	4.1

Reference

[1] G. Trybuś, *Numeryczne aspekty wyznaczania dystansów*, WSE Wrocław, Doctor Dissertation.

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Received on 20. 11. 1973

ALGORYTM 42

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DOŁĄCZENIE PUNKTU DO NAJKRÓTSZEGO DENDRYTU

STRESZCZENIE

Procedura *Minden* służy do obliczania najkrótszego dendrytu dla zbioru $(k+1)$-punktowego, gdy dany jest najkrótszy dendryt $F(Z)$ zbioru k-punktowego Z oraz k odległości dowolnego punktu P_{k+1} od punktów zbioru Z.

Dane:

k – liczba punktów w dendrycie $F(Z)$;
$L[1:k-1]$ – tablica połączeń w dendrycie $F(Z)$;
$c[1:k-1]$ – tablica długości krawędzi w dendrycie $F(Z)$;
$d[1:k]$ – tablica odległości punktu P_{k+1} od punktów $P_1, P_2, \ldots, P_k$ zbioru Z, gdzie $d[i] = d(P_{k+1}, P_i)$.

Wyniki:

$L[1:k]$ – tablica połączeń w najkrótszym dendrycie $(k+1)$-punktowym;
$c[1:k]$ – tablica długości krawędzi w najkrótszym dendrycie $(k+1)$-punktowym.

Obliczenia, wykonane na maszynie cyfrowej Odra 1204, wykazały poprawność procedury.