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TABLES OF THE FUNCTIONS OF THE PARABOLIC CYLINDER FOR NEGATIVE INTEGER PARAMETERS

The function of the parabolic cylinder $D_{-n}(z)$, where the parameter $-n$ is a negative integer, is needed for the calculations of the exact distributions of Shor's u_n statistic [4], as we have shown in the paper [3]. Tables of the functions $D_n(z)$ (cf. [2], p. 101-109) are, as far as we know, given for non-negative integers n only. That is why we decided to present additional tables for negative values of the parameter n. The values of the function $D_{-n}(z)$ are calculated from two basic formulas, for $n = 1$ and $n = 2$,

$$D_{-1}(z) = \exp\left(-\frac{1}{4}z^2\right)\sqrt{\frac{\pi}{2}}\left[1-\Phi\left(\frac{z}{\sqrt{2}}\right)\right],$$

(1)

$$D_{-2}(z) = \exp\left(-\frac{1}{4}z^2\right)\sqrt{\frac{\pi}{2}}\left\{z\left[1-\Phi\left(\frac{z}{\sqrt{2}}\right)\right]-\sqrt{\frac{2}{\pi}}\exp\left(-\frac{1}{2}z^2\right)\right\},$$

where Φ is the Laplace function, and a recurrence formula is derived for higher numbers $n = 3, 4, \ldots, 20$.

We have

$$H_{\nu+1}(z)-2zH_\nu(z)+2\nu H_{\nu-1}(z) = 0, \qquad \operatorname{Re}\nu < 0, \tag{2}$$

where H_ν is the Hermite function of the first kind. For any arbitrary p the function of the parabolic cylinder can be expressed by H_ν as follows:

$$D_p(z) = 2^{-p/2}e^{-z^2/4}H_p\left(\frac{z}{\sqrt{2}}\right). \tag{3}$$

Thus, from (2) and (3) we have the recurrence formula

$$D_{\nu-1}(z) = D_\nu(z)\frac{z}{\nu}-\frac{1}{\nu}D_{\nu+1}(z),$$

and, finally,

$$D_{-n}(z) = \left(-D_{-n+1}(z)z+D_{-n+2}(z)\right)/(n-1), \qquad n = 3, 4, \ldots \tag{4}$$

The function $D_p(z)$ can be expressed by the Whittaker function (cf. [5], p. 122-133)

$$D_p(z) = 2^{1/4+p/2} z^{-1/2} W_{1/4+p/2,-1/4}(z^2),$$

as well as by the Kummer function. i.e., the confluent hypergeometric function (cf. [1], p. 536-540 and p. 552-554)

$$D_p(z) = 2^{p/2}\exp\left(-\frac{1}{4}z^2\right)\left[\frac{\sqrt{\pi}}{\Gamma\left(\frac{(1-p}{2}\right)}\,{}_1F_1\left(-\frac{p}{2};\frac{1}{2};\frac{1}{2}z^2\right) - \right.$$

$$\left. - \frac{\sqrt{2\pi}\,z}{\Gamma\left(-\frac{1}{2}p\right)}\,{}_1F_1\left(\frac{1-p}{2};\frac{3}{2};\frac{1}{2}z^2\right)\right].$$

The values of the function $D_{-n}(x)$ for $x = 0.00(0.05)3.00$ given in the table have been calculated on the computer Odra 1204 at the

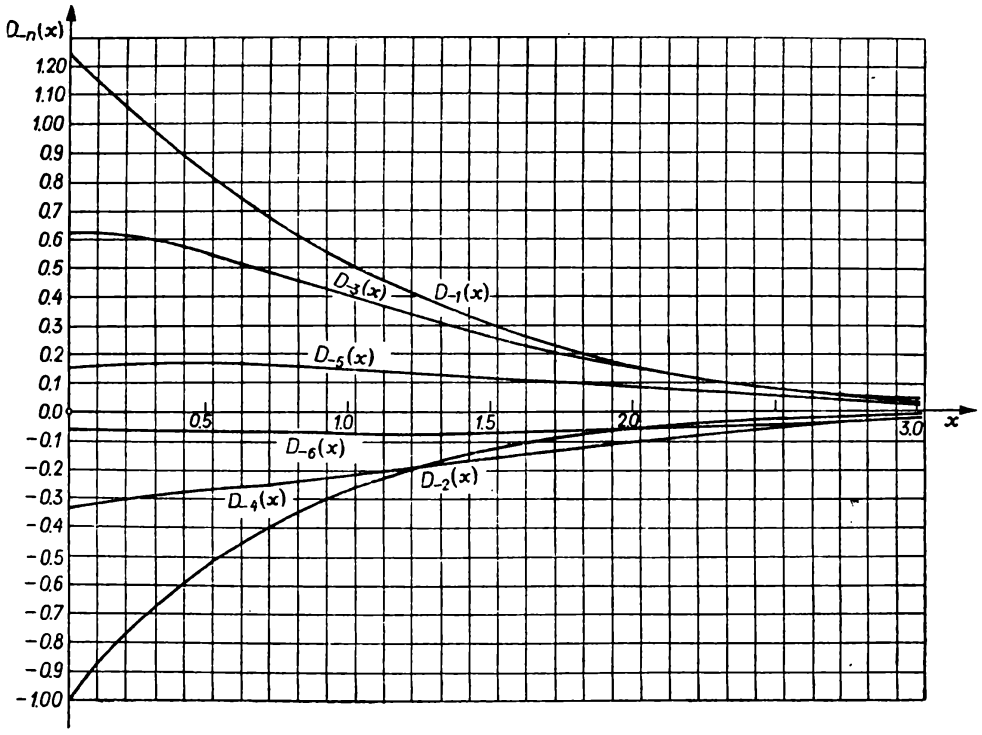

Fig. 1

Cracow University Computing Center. The curves in Fig. 1 give an idea about the range of the functions $D_{-n}(x)$ for $n = 1, 2, 3, 4, 5, 6$.

The binary floating-point arithmetics of the computer does not guarantee quite exact results of the calculations. We have $x = m \cdot 2^c$, where m is the mantissa, $m \in [-1, -\frac{1}{2}] \cup [+\frac{1}{2}, +1)$, and c is the integer exponent of the number x.

An error of calculations arises when the mantissa is rounded off to a value from the defined interval, so that it takes N significant bits of the computer word. For the Odra 1204 is $N = 37$, because 38 bits of the computer word are provided for the mantissa, the first bit being the sign bit.

In [6] round-off errors for the four elementary operations are estimated and the round-off errors of any functions can be derived from those. The symbol fl will be used to denote that there is a floating-point operation under consideration. Thus

$$\mathrm{fl}(x_1 \begin{matrix}+\\ -\\ \times\\ :\end{matrix} x_2) \equiv (x_1 \begin{matrix}+\\ -\\ \times\\ :\end{matrix} x_2)(1+\varepsilon),$$

where ε is the relative error of the result, $|\varepsilon| \leqslant 2^{-37}$. Hence, a floating-point operation gives results which are different $1+\varepsilon$ times from the exact ones.

Let us analyse the error of the recurrence formulas (1), (2) and (4) for $n = 3, 4, \ldots, 20$. We suppose that the calculations of the values D_{-1} and D_{-2} are exact to the extent of a representation error, i.e., the modulus of the relative error for each of the values is equal to 2^{-37} of less

$$D^*_{-1} = D_{-1}(1+\varepsilon), \qquad D^*_{-2} = D_{-2}(1+\varepsilon),$$

where D_{-n} is the exact value, and D^*_{-n} is the calculated value. In addition, let us suppose that ε is always the same, since we take under consideration its upper bound only. We have

$$D^*_{-n} = \{[(-D^*_{-n+1}\cdot z)(1+\varepsilon) + D^*_{-n+2}](1+\varepsilon)/(n-1)\}(1+\varepsilon), \qquad |\varepsilon| \leqslant 2^{-37},$$

and

$$|D^*_{-n} - D_{-n}| \leqslant |(D_{-n+1}\cdot z - D_{-n+2} - D^*_{-n+1}\cdot z - 3D^*_{-n+1}\cdot\varepsilon\cdot z -$$
$$- 3D^*_{-n+1}\cdot\varepsilon^2\cdot z - D^*_{-n+1}\cdot\varepsilon^3\cdot z + D_{-n+2} + 2D_{-n+2}\cdot\varepsilon + D_{-n+2}\cdot\varepsilon^2)/(n-1)|.$$

We neglect the components which have powers ε^2 and ε^3, because they are too small and will not influence the results. Hence,

$$|D^*_{-n} - D_{-n}| \leqslant \frac{z}{n-1}|D_{-n+1} - D^*_{-n+1}| + \frac{1}{n-1}|D_{-n+2} - D^*_{-n+2}| +$$
$$+ \frac{3|\varepsilon|}{n-1}|D^*_{-n+1}|\cdot|z| + \frac{2|\varepsilon|}{n-1}|D^*_{-n+2}|.$$

If we take $|z| \leqslant 3$ and $M = \max|D_{-n}|$, $n = 1, 2, \ldots, 20$, we obtain

$$|D^*_{-n} - D_{-n}| \leqslant \frac{3}{n-1}|D_{-n+1} - D^*_{-n+1}| + \frac{1}{n-1}|D_{-n+2} - D^*_{-n+2}| + \frac{11}{n-1}|\varepsilon|M.$$

Since $|D_{-1}-D^*_{-1}| \leqslant |\varepsilon|M$ and $|D_{-2}-D^*_{-2}| \leqslant |\varepsilon|M$, there is

$$|D_{-3}-D^*_{-3}| \leqslant \frac{3|\varepsilon|M}{2} + \frac{|\varepsilon|M}{2} + \frac{11}{2}|\varepsilon|M < 8|\varepsilon|M.$$

If $n > 3$, it may be observed that

$$|D_{-n}-D^*_{-n}| \leqslant |D_{-n+1}-D^*_{-n+1}| + |D_{-n+2}-D^*_{-n+2}| + 10|\varepsilon|M,$$

therefore,

$$|D_{-4}-D^*_{-4}| \leqslant 8|\varepsilon|M + |\varepsilon|M + 10|\varepsilon|M < 20|\varepsilon|M,$$

$$|D_{-5}-D^*_{-5}| < 20|\varepsilon|M + 8|\varepsilon|M + 10|\varepsilon|M < 40|\varepsilon|M,$$

$$|D_{-6}-D^*_{-6}| < 40|\varepsilon|M + 20|\varepsilon|M + 10|\varepsilon|M < 80|\varepsilon|M$$

and

$$|D_{-n}-D^*_{-n}| < (2^{n-4}|\varepsilon|M \cdot 2^{n-5}|\varepsilon|M + |\varepsilon|M)10 < 2^{n-3}\cdot 10|\varepsilon|M \quad \text{for } n \geqslant 5.$$

Finally, we obtain the absolute error estimate of the result as follows:

$$|D_{-20}-D^*_{-20}| < 2^{17}\cdot 10\cdot 2^{-37}M = 2^{-20}\cdot 10M.$$

We wish to express our thanks to Mr. B. Owczarek and Mrs. J. Sulińska from the Cracow University Computing Center for their help in the computations and in the error analysis.

Notice for the tables. Numbers $(-k)$ in the brackets denote that the table value is to be multiplied by the factor 10^{-k}.

During proof-reading the following errors in the tables have been found:

D_{-12} (1.30) should equal $-4.140258_{10}-4$,

D_{-12} (1.55) should equal $-5.035676_{10}-4$.

x	$D_{-1}(x)$	$D_{-2}(x)$	$D_{-3}(x)$	$D_{-4}(x)$	$D_{-5}(x)$
0.00	1.253314	-1.000000	6.266571/-1/	-3.333333/-1/	1.566643/-1/
0.05	1.204087	-9.391709/-1/	6.255227/-1/	-3.234823/-1/	1.604242/-1/
0.10	1.156368	-8.818664/-1/	6.222771/-1/	-3.146980/-1/	1.634367/-1/
0.15	1.110101	-8.278757/-1/	6.171411/-1/	-3.068156/-1/	1.657909/-1/
0.20	1.065238	-7.770022/-1/	6.103193/-1/	-2.996887/-1/	1.675643/-1/
0.25	1.021735	-7.290626/-1/	6.020006/-1/	-2.931876/-1/	1.688244/-1/
0.30	9.795489/-1/	-6.838866/-1/	5.923574/-1/	-2.871979/-1/	1.696292/-1/
0.35	9.386399/-1/	-6.413152/-1/	5.815501/-1/	-2.816193/-1/	1.700292/-1/
0.40	8.989785/-1/	-6.011981/-1/	5.697288/-1/	-2.763632/-1/	1.700685/-1/
0.45	8.605307/-1/	-5.633963/-1/	5.570295/-1/	-2.713532/-1/	1.697846/-1/
0.50	8.232668/-1/	-5.277796/-1/	5.435783/-1/	-2.665229/-1/	1.692099/-1/
0.55	7.871639/-1/	-4.942237/-1/	5.294935/-1/	-2.618150/-1/	1.683729/-1/
0.60	7.521905/-1/	-4.626169/-1/	5.148803/-1/	-2.571817/-1/	1.672973/-1/
0.65	7.183279/-1/	-4.328489/-1/	4.998398/-1/	-2.525816/-1/	1.660045/-1/
0.70	6.855542/-1/	-4.048180/-1/	4.844634/-1/	-2.479808/-1/	1.645125/-1/
0.75	6.538442/-1/	-3.784319/-1/	4.688341/-1/	-2.433525/-1/	1.628371/-1/
0.80	6.231833/-1/	-3.535972/-1/	4.530305/-1/	-2.386739/-1/	1.609924/-1/
0.85	5.935504/-1/	-3.302305/-1/	4.371232/-1/	-2.339284/-1/	1.589906/-1/
0.90	5.649292/-1/	-3.082502/-1/	4.211772/-1/	-2.291032/-1/	1.568425/-1/
0.95	5.372989/-1/	-2.875833/-1/	4.052515/-1/	-2.241908/-1/	1.545582/-1/
1.00	5.106430/-1/	-2.681578/-1/	3.894004/-1/	-2.191861/-1/	1.521466/-1/
1.05	4.849455/-1/	-2.499048/-1/	3.736728/-1/	-2.140870/-1/	1.496160/-1/
1.10	4.601878/-1/	-2.327620/-1/	3.581130/-1/	-2.088954/-1/	1.469745/-1/
1.15	4.363537/-1/	-2.166678/-1/	3.427608/-1/	-2.036142/-1/	1.442293/-1/
1.20	4.134263/-1/	-2.015648/-1/	3.276520/-1/	-1.982491/-1/	1.413877/-1/
1.25	3.913864/-1/	-1.874008/-1/	3.128187/-1/	-1.928081/-1/	1.384572/-1/
1.30	3.702156/-1/	-1.741260/-1/	2.982897/-1/	-1.873008/-1/	1.354452/-1/
1.35	3.499032/-1/	-1.616822/-1/	2.840871/-1/	-1.817333/-1/	1.323568/-1/
1.40	3.304262/-1/	-1.500298/-1/	2.702339/-1/	-1.761191/-1/	1.292002/-1/
1.45	3.117631/-1/	-1.391293/-1/	2.567503/-1/	-1.704724/-1/	1.259838/-1/
1.50	2.939020/-1/	-1.289298/-1/	2.436484/-1/	-1.648008/-1/	1.227124/-1/

x	$D_{-1}(x)$	$D_{-2}(x)$	$D_{-3}(x)$	$D_{-4}(x)$	$D_{-5}(x)$
1.55	2.768234/-1/	-1.193924/-1/	2.309408/-1/	-1.591169/-1/	1.193930/-1/
1.60	2.605020/-1/	-1.104892/-1/	2.186424/-1/	-1.534390/-1/	1.160362/-1/
1.65	2.449295/-1/	-1.021668/-1/	2.067523/-1/	-1.477694/-1/	1.126430/-1/
1.70	2.300760/-1/	-9.440764/-2/	1.952845/-1/	-1.421304/-1/	1.092266/-1/
1.75	2.159219/-1/	-8.717979/-2/	1.842433/-1/	-1.365352/-1/	1.057950/-1/
1.80	2.024537/-1/	-8.044142/-2/	1.736241/-1/	-1.309883/-1/	1.023508/-1/
1.85	1.896526/-1/	-7.416024/-2/	1.634245/-1/	-1.254985/-1/	9.889919/-2/
1.90	1.774924/-1/	-6.831892/-2/	1.536492/-1/	-1.200841/-1/	9.545225/-2/
1.95	1.659501/-1/	-6.289671/-2/	1.442993/-1/	-1.147601/-1/	9.202040/-2/
2.00	1.550122/-1/	-5.785507/-2/	1.353612/-1/	-1.095258/-1/	8.860319/-2/
2.05	1.446554/-1/	-5.317543/-2/	1.268325/-1/	-1.043941/-1/	8.521009/-2/
2.10	1.348585/-1/	-4.883709/-2/	1.187082/-1/	-9.937477/-2/	8.184880/-2/
2.15	1.256096/-1/	-4.479965/-2/	1.109645/-1/	-9.445779/-2/	7.851219/-2/
2.20	1.168678/-1/	-4.108816/-2/	1.036309/-1/	-8.969202/-2/	7.523833/-2/
2.25	1.086319/-1/	-3.764125/-2/	9.666234/-2/	-8.504383/-2/	7.200274/-2/
2.30	1.008791/-1/	-3.444636/-2/	9.005286/-2/	-8.052265/-2/	6.881374/-2/
2.35	9.358680/-2/	-3.149239/-2/	8.379695/-2/	-7.613841/-2/	6.568055/-2/
2.40	8.673251/-2/	-2.876973/-2/	7.788993/-2/	-7.190185/-2/	6.261360/-2/
2.45	8.029412/-2/	-2.627014/-2/	7.232799/-2/	-6.782457/-2/	5.962455/-2/
2.50	7.426201/-2/	-2.395637/-2/	6.707646/-2/	-6.388251/-2/	5.669568/-2/
2.55	6.860508/-2/	-2.184569/-2/	6.215579/-2/	-6.011432/-2/	5.386185/-2/
2.60	6.331793/-2/	-1.989290/-2/	5.751973/-2/	-5.648140/-2/	5.109284/-2/
2.65	5.838672/-2/	-1.807440/-2/	5.314194/-2/	-5.296684/-2/	4.837602/-2/
2.70	5.377068/-2/	-1.644037/-2/	4.907984/-2/	-4.965197/-2/	4.578504/-2/
2.75	4.947596/-2/	-1.491853/-2/	4.525096/-2/	-4.645289/-2/	4.324910/-2/
2.80	4.546718/-2/	-1.355031/-2/	4.170402/-2/	-4.344052/-2/	4.083437/-2/
2.85	4.174739/-2/	-1.227340/-2/	3.836328/-2/	-4.053625/-2/	3.847290/-2/
2.90	3.829180/-2/	-1.110446/-2/	3.524736/-2/	-3.777394/-2/	3.619795/-2/
2.95	3.508131/-2/	-1.004731/-2/	3.236044/-2/	-3.517020/-2/	3.402813/-2/
3.00	3.210601/-2/	-9.081200/-3/	2.967480/-2/	-3.270187/-2/	3.194510/-2/

0.00	-6.666667/-2/	2.611071/-2/	-9.523810/-3/	3.263839/-3/	-1.058201/-3/
0.05	-6.630071/-2/	2.728987/-2/	-9.666457/-3/	3.471650/-3/	-1.093338/-3/
0.10	-6.620834/-2/	2.834293/-2/	-9.863233/-3/	3.666156/-3/	-1.136650/-3/
0.15	-6.633685/-2/	2.929023/-2/	-1.010434/-2/	3.850735/-3/	-1.186883/-3/
0.20	-6.664031/-2/	3.014872/-2/	-1.038144/-2/	4.028126/-3/	-1.243007/-3/
0.25	-6.707873/-2/	3.093234/-2/	-1.068740/-2/	4.200524/-3/	-1.304170/-3/
0.30	-6.761734/-2/	3.165240/-2/	-1.101615/-2/	4.369656/-3/	-1.369672/-3/
0.35	-6.822590/-2/	3.231805/-2/	-1.136246/-2/	4.536863/-3/	-1.438929/-3/
0.40	-6.887812/-2/	3.293663/-2/	-1.172182/-2/	4.703170/-3/	-1.511455/-3/
0.45	-6.955125/-2/	3.351378/-2/	-1.209035/-2/	4.869305/-3/	-1.586837/-3/
0.50	-7.022558/-2/	3.405379/-2/	-1.246464/-2/	5.035764/-3/	-1.664725/-3/
0.55	-7.088403/-2/	3.455986/-2/	-1.284171/-2/	5.202850/-3/	-1.744808/-3/
0.60	-7.151202/-2/	3.503409/-2/	-1.321892/-2/	5.370681/-3/	-1.826815/-3/
0.65	-7.209690/-2/	3.547791/-2/	-1.359393/-2/	5.539246/-3/	-1.910494/-3/
0.70	-7.262790/-2/	3.589200/-2/	-1.396462/-2/	5.708404/-3/	-1.995611/-3/
0.75	-7.309606/-2/	3.627653/-2/	-1.432907/-2/	5.877916/-3/	-2.081945/-3/
0.80	-7.349355/-2/	3.663121/-2/	-1.468550/-2/	6.047451/-3/	-2.169274/-3/
0.85	-7.381408/-2/	3.695542/-2/	-1.503231/-2/	6.216611/-3/	-2.257381/-3/
0.90	-7.405230/-2/	3.724827/-2/	-1.536796/-2/	6.384929/-3/	-2.346044/-3/
0.95	-7.420421/-2/	3.750870/-2/	-1.569107/-2/	6.551902/-3/	-2.435042/-3/
1.00	-7.426654/-2/	3.773553/-2/	-1.600029/-2/	6.716978/-3/	-2.524141/-3/
1.05	-7.423678/-2/	3.792744/-2/	-1.629437/-2/	6.879566/-3/	-2.613102/-3/
1.10	-7.411346/-2/	3.808321/-2/	-1.657214/-2/	7.039071/-3/	-2.701680/-3/
1.15	-7.389559/-2/	3.820154/-2/	-1.683248/-2/	7.194861/-3/	-2.789619/-3/
1.20	-7.358287/-2/	3.828119/-2/	-1.707433/-2/	7.346299/-3/	-2.876654/-3/
1.25	-7.317592/-2/	3.832118/-2/	-1.729677/-2/	7.492768/-3/	-2.962526/-3/
1.30	-7.267592/-2/	3.832065/-2/	-1.749897/-2/	7.633663/-3/	-3.046970/-3/
1.35	-7.208298/-2/	3.827813/-2/	-1.767978/-2/	7.768229/-3/	-3.129654/-3/
1.40	-7.139986/-2/	3.819333/-2/	-1.783865/-2/	7.895929/-3/	-3.210327/-3/
1.45	-7.062979/-2/	3.806617/-2/	-1.797511/-2/	8.016259/-3/	-3.288742/-3/
1.50	-6.977387/-2/	3.789553/-2/	-1.808817/-2/	8.128472/-3/	-3.364542/-3/

x	$D_{-6}(x)$	$D_{-7}(x)$	$D_{-8}(x)$	$D_{-9}(x)$	$D_{-10}(x)$
1.55	-6.883522/-2/	3.768127/-2/	-1.817731/-2/	8.232013/-3/	-3.437437/-3/
1.60	-6.781939/-2/	3.742454/-2/	-1.824266/-2/	8.326600/-3/	-3.507247/-3/
1.65	-6.672605/-2/	3.712349/-2/	-1.828283/-2/	8.411270/-3/	-3.573492/-3/
1.70	-6.556312/-2/	3.678064/-2/	-1.829860/-2/	8.486033/-3/	-3.636095/-3/
1.75	-6.433527/-2/	3.639695/-2/	-1.828999/-2/	8.550554/-3/	-3.694829/-3/
1.80	-6.304393/-2/	3.597164/-2/	-1.825613/-2/	8.604083/-3/	-3.749275/-3/
1.85	-6.169241/-2/	3.550502/-2/	-1.819667/-2/	8.646108/-3/	-3.799108/-3/
1.90	-6.028868/-2/	3.500012/-2/	-1.811270/-2/	8.676782/-3/	-3.844288/-3/
1.95	-5.883998/-2/	3.445973/-2/	-1.800521/-2/	8.696236/-3/	-3.884763/-3/
2.00	-5.734644/-2/	3.388268/-2/	-1.787311/-2/	8.703613/-3/	-3.920038/-3/
2.05	-5.581495/-2/	3.327179/-2/	-1.771744/-2/	8.699069/-3/	-3.950059/-3/
2.10	-5.425145/-2/	3.262947/-2/	-1.753905/-2/	8.682685/-3/	-3.974743/-3/
2.15	-5.265180/-2/	3.195226/-2/	-1.733559/-2/	8.652973/-3/	-3.993276/-3/
2.20	-5.104327/-2/	3.125559/-2/	-1.711508/-2/	8.613595/-3/	-4.007221/-3/
2.25	-4.941000/-2/	3.052921/-2/	-1.687153/-2/	8.561269/-3/	-4.014932/-3/
2.30	-4.775885/-2/	2.977652/-2/	-1.660641/-2/	8.496406/-3/	-4.016460/-3/
2.35	-4.609754/-2/	2.900163/-2/	-1.632162/-2/	8.419681/-3/	-4.011986/-3/
2.40	-4.443490/-2/	2.820956/-2/	-1.601969/-2/	8.332102/-3/	-4.001860/-3/
2.45	-4.278094/-2/	2.740631/-2/	-1.570377/-2/	8.235069/-3/	-3.986632/-3/
2.50	-4.112434/-2/	2.658442/-2/	-1.536934/-2/	8.125973/-3/	-3.964919/-3/
2.55	-3.949240/-2/	2.576124/-2/	-1.502622/-2/	8.009764/-3/	-3.939013/-3/
2.60	-3.786456/-2/	2.492345/-2/	-1.466650/-2/	7.882045/-3/	-3.906647/-3/
2.65	-3.623266/-2/	2.406543/-2/	-1.428658/-2/	7.740607/-3/	-3.866576/-3/
2.70	-3.465432/-2/	2.322528/-2/	-1.390894/-2/	7.597428/-3/	-3.824666/-3/
2.75	-3.307758/-2/	2.236874/-2/	-1.351309/-2/	7.441217/-3/	-3.775160/-3/
2.80	-3.155535/-2/	2.153156/-2/	-1.312053/-2/	7.283631/-3/	-3.723855/-3/
2.85	-3.003680/-2/	2.067963/-2/	-1.271054/-2/	7.113082/-3/	-3.664758/-3/
2.90	-2.854960/-2/	1.983196/-2/	-1.229461/-2/	6.935792/-3/	-3.600934/-3/
2.95	-2.711064/-2/	1.900075/-2/	-1.188041/-2/	6.755994/-3/	-3.534510/-3/
3.00	-2.570744/-2/	1.817790/-2/	-1.146302/-2/	6.570871/-3/	-3.463959/-3/

x	$D_{-11}(x)$	$D_{-12}(x)$	$D_{-13}(x)$	$D_{-14}(x)$	$D_{-15}(x)$
0.00	3.263839/-4/	-9.620010/-5/	2.719866/-5/	-7.400007/-6/	1.942761/-6/
0.05	3.526316/-4/	-1.009972/-4/	2.980679/-5/	-7.883658/-6/	2.157212/-6/
0.10	3.779821/-4/	-1.067680/-4/	3.238824/-5/	-8.462064/-6/	2.373889/-6/
0.15	4.028768/-4/	-1.133923/-4/	3.499047/-5/	-9.126218/-6/	2.597100/-6/
0.20	4.276727/-4/	-1.207765/-4/	3.765233/-5/	-9.869766/-6/	2.830449/-6/
0.25	4.526567/-4/	-1.288486/-4/	4.040573/-5/	-1.068846/-5/	3.076989/-6/
0.30	4.780557/-4/	-1.375535/-4/	4.327682/-5/	-1.157974/-5/	3.339338/-6/
0.35	5.040488/-4/	-1.468496/-4/	4.628719/-5/	-1.254232/-5/	3.619786/-6/
0.40	5.307752/-4/	-1.567059/-4/	4.945480/-5/	-1.357599/-5/	3.920371/-6/
0.45	5.583382/-4/	-1.670991/-4/	5.279439/-5/	-1.468127/-5/	4.242926/-6/
0.50	5.868126/-4/	-1.780119/-4/	5.631821/-5/	-1.585931/-5/	4.589133/-6/
0.55	6.162494/-4/	-1.894314/-4/	6.003639/-5/	-1.711165/-5/	4.960557/-6/
0.60	6.466770/-4/	-2.013474/-4/	6.395711/-5/	-1.844013/-5/	5.358656/-6/
0.65	6.781067/-4/	-2.137512/-4/	6.808708/-5/	-1.984675/-5/	5.784819/-6/
0.70	7.105332/-4/	-2.266349/-4/	7.243147/-5/	-2.133361/-5/	6.240357/-6/
0.75	7.439574/-4/	-2.399907/-4/	7.699420/-5/	-2.290280/-5/	6.726521/-6/
0.80	7.782870/-4/	-2.538094/-4/	8.177788/-5/	-2.455628/-5/	7.244493/-6/
0.85	8.135385/-4/	-2.680808/-4/	8.678393/-5/	-2.629594/-5/	7.795391/-6/
0.90	8.496369/-4/	-2.827925/-4/	9.201251/-5/	-2.812337/-5/	8.380253/-6/
0.95	8.865191/-4/	-2.979304/-4/	9.746275/-5/	-3.004000/-5/	9.000054/-6/
1.00	9.241119/-4/	-3.134776/-4/	1.031325/-4/	-3.204693/-5/	9.655670/-6/
1.05	9.623323/-4/	-3.294137/-4/	1.090181/-4/	-3.414482/-5/	1.034787/-5/
1.10	1.001092/-3/	-3.457165/-4/	1.151150/-4/	-3.633408/-5/	1.107732/-5/
1.15	1.040292/-3/	-3.623596/-4/	1.214171/-4/	-3.861456/-5/	1.184456/-5/
1.20	1.079828/-3/	-3.793135/-4/	1.279170/-4/	-4.098569/-5/	1.264999/-5/
1.25	1.119593/-3/	-3.965470/-4/	1.346064/-4/	-4.344653/-5/	1.349389/-5/
1.30	1.159472/-3/	-4.140298/-4/	1.414755/-4/	-4.599569/-5/	1.437642/-5/
1.35	1.199326/-3/	-4.317041/-4/	1.485106/-4/	-4.863026/-5/	1.529724/-5/
1.40	1.239039/-3/	-4.495438/-4/	1.557000/-4/	-5.134798/-5/	1.625623/-5/
1.45	1.278494/-3/	-4.675053/-4/	1.630313/-4/	-5.414621/-5/	1.725310/-5/
1.50	1.317528/-3/	-4.855304/-4/	1.704853/-4/	-5.701988/-5/	1.828680/-5/

x	$D_{-11}(x)$	$D_{-12}(x)$	$D_{-13}(x)$	$D_{-14}(x)$	$D_{-15}(x)$
1.55	1.356004/-3/	-5.03576/-4/	1.780445/-4/	-5.996435/-5/	1.935637/-5/
1.60	1.393819/-3/	-5.215780/-4/	1.856954/-4/	-6.297620/-5/	2.046123/-5/
1.65	1.430753/-3/	-5.394758/-4/	1.934074/-4/	-6.604600/-5/	2.159880/-5/
1.70	1.466740/-3/	-5.572321/-4/	2.011695/-4/	-6.917079/-5/	2.276856/-5/
1.75	1.501650/-3/	-5.747925/-4/	2.089614/-4/	-7.234423/-5/	2.396885/-5/
1.80	1.535278/-3/	-5.920704/-4/	2.167504/-4/	-7.555547/-5/	2.519645/-5/
1.85	1.567446/-3/	-6.089893/-4/	2.245063/-4/	-7.879431/-5/	2.644827/-5/
1.90	1.598093/-3/	-6.255149/-4/	2.322143/-4/	-8.205554/-5/	2.772284/-5/
1.95	1.627152/-3/	-6.416100/-4/	2.398577/-4/	-8.533326/-5/	2.901839/-5/
2.00	1.654369/-3/	-6.571614/-4/	2.473910/-4/	-8.861102/-5/	3.032950/-5/
2.05	1.679669/-3/	-6.721255/-4/	2.547939/-4/	-9.188100/-5/	3.165357/-5/
2.10	1.702965/-3/	-6.864517/-4/	2.620428/-4/	-9.513396/-5/	3.298743/-5/
2.15	1.723852/-3/	-6.999598/-4/	2.690638/-4/	-9.834207/-5/	3.432137/-5/
2.20	1.742948/-3/	-7.128824/-4/	2.759408/-4/	-1.015348/-4/	3.566552/-5/
2.25	1.759487/-3/	-7.248888/-4/	2.825405/-4/	-1.046619/-4/	3.700213/-5/
2.30	1.773426/-3/	-7.359400/-4/	2.888407/-4/	-1.077134/-4/	3.832725/-5/
2.35	1.784785/-3/	-7.460210/-4/	2.948278/-4/	-1.106820/-4/	3.963790/-5/
2.40	1.793656/-3/	-7.551486/-4/	3.005011/-4/	-1.135655/-4/	4.093273/-5/
2.45	1.800232/-3/	-7.633818/-4/	3.058764/-4/	-1.163676/-4/	4.221265/-5/
2.50	1.803827/-3/	-7.704079/-4/	3.108206/-4/	-1.190353/-4/	4.345778/-5/
2.55	1.805425/-3/	-7.766224/-4/	3.154843/-4/	-1.216236/-4/	4.468747/-5/
2.60	1.803933/-3/	-7.815338/-4/	3.196600/-4/	-1.240500/-4/	4.587071/-5/
2.65	1.798703/-3/	-7.848309/-4/	3.232088/-4/	-1.262565/-4/	4.698489/-5/
2.70	1.792403/-3/	-7.876503/-4/	3.265882/-4/	-1.284183/-4/	4.809412/-5/
2.75	1.782291/-3/	-7.887690/-4/	3.292838/-4/	-1.303307/-4/	4.912095/-5/
2.80	1.771043/-3/	-7.893432/-4/	3.317670/-4/	-1.321762/-4/	5.013288/-5/
2.85	1.755764/-3/	-7.880624/-4/	3.334785/-4/	-1.337289/-4/	5.104328/-5/
2.90	1.737850/-3/	-7.855182/-4/	3.346544/-4/	-1.350782/-4/	5.188436/-5/
2.95	1.718280/-3/	-7.821305/-4/	3.354637/-4/	-1.362884/-4/	5.267960/-5/
3.00	1.696275/-3/	-7.775258/-4/	3.357377/-4/	-1.372876/-4/	5.340003/-5/

0.00	−4.933338/−7/	1.214226/−7/	−2.901964/−8/	6.745699/−9/	−1.527349/−9/
0.05	−5.327679/−7/	1.364907/−7/	−3.174073/−8/	7.670984/−9/	−1.690752/−9/
0.10	−5.799635/−7/	1.519928/−7/	−3.500958/−8/	8.638545/−9/	−1.888075/−9/
0.15	−6.343855/−7/	1.682661/−7/	−3.880150/−8/	9.671463/−9/	−2.115838/−9/
0.20	−6.957237/−7/	1.855996/−7/	−4.310845/−8/	1.079007/−8/	−2.382445/−9/
0.25	−7.638474/−7/	2.042469/−7/	−4.793583/−8/	1.201283/−8/	−2.681002/−9/
0.30	−8.387692/−7/	2.244356/−7/	−5.329999/−8/	1.335698/−8/	−3.016162/−9/
0.35	−9.206163/−7/	2.463751/−7/	−5.922633/−8/	1.483913/−8/	−3.390528/−9/
0.40	−1.009609/−6/	2.702634/−7/	−6.574790/−8/	1.647570/−8/	−3.807272/−9/
0.45	−1.106039/−6/	2.962902/−7/	−7.290411/−8/	1.828317/−8/	−4.270081/−9/
0.50	−1.210258/−6/	3.246414/−7/	−8.073994/−8/	2.027841/−8/	−4.783113/−9/
0.55	−1.322664/−6/	3.555014/−7/	−8.930526/−8/	2.247885/−8/	−5.350980/−9/
0.60	−1.443688/−6/	3.890543/−7/	−9.865415/−8/	2.490260/−8/	−5.978722/−9/
0.65	−1.573792/−6/	4.254865/−7/	−1.088446/−7/	2.756864/−8/	−6.671802/−9/
0.70	−1.713457/−6/	4.649861/−7/	−1.199381/−7/	3.049682/−8/	−7.436098/−9/
0.75	−1.863179/−6/	5.077441/−7/	−1.319993/−7/	3.370798/−8/	−8.277907/−9/
0.80	−2.023459/−6/	5.539538/−7/	−1.450954/−7/	3.722389/−8/	−9.203921/−9/
0.85	−2.194801/−6/	6.038108/−7/	−1.592965/−7/	4.106738/−8/	−1.022125/−8/
0.90	−2.377706/−6/	6.575118/−7/	−1.746745/−7/	4.526216/−8/	−1.133739/−8/
0.95	−2.572670/−6/	7.152557/−7/	−1.913037/−7/	4.983301/−8/	−1.256027/−8/
1.00	−2.780173/−6/	7.772402/−7/	−2.092596/−7/	5.480554/−8/	−1.389817/−8/
1.05	−3.000672/−6/	8.436607/−7/	−2.286186/−7/	6.020612/−8/	−1.535974/−8/
1.10	−3.234609/−6/	9.147119/−7/	−2.494583/−7/	6.606200/−8/	−1.695403/−8/
1.15	−3.482387/−6/	9.905818/−7/	−2.718563/−7/	7.240092/−8/	−1.869038/−8/
1.20	−3.744378/−6/	1.071453/−6/	−2.958895/−7/	7.925112/−8/	−2.057847/−8/
1.25	−4.020927/−6/	1.157503/−6/	−3.216356/−7/	8.664154/−8/	−2.262829/−8/
1.30	−4.312336/−6/	1.248904/−6/	−3.491712/−7/	9.460145/−8/	−2.485016/−8/
1.35	−4.618769/−6/	1.345786/−6/	−3.785636/−7/	1.031582/−7/	−2.725406/−8/
1.40	−4.940447/−6/	1.448303/−6/	−4.098865/−7/	1.123414/−7/	−2.985076/−8/
1.45	−5.277547/−6/	1.556596/−6/	−4.432124/−7/	1.221808/−7/	−3.265129/−8/
1.50	−5.630005/−6/	1.670738/−6/	−4.785948/−7/	1.327017/−7/	−3.566565/−8/

x	$D_{-16}(x)$	$D_{-17}(x)$	$D_{-18}(x)$	$D_{-19}(x)$	$D_{-20}(x)$
1.55	-5.997781/-6/	1.790808/-6/	-5.160903/-7/	1.439305/-7/	-3.890434/-8/
1.60	-6.380945/-6/	1.916922/-6/	-5.557659/-7/	1.558971/-7/	-4.237901/-8/
1.65	-6.778935/-6/	2.049003/-6/	-5.976347/-7/	1.686167/-7/	-4.609748/-8/
1.70	-7.191823/-6/	2.187166/-6/	-6.417650/-7/	1.821204/-7/	-5.007209/-8/
1.75	-7.619314/-6/	2.331415/-6/	-6.881936/-7/	1.964308/-7/	-5.431302/-8/
1.80	-8.060605/-6/	2.481596/-6/	-7.369104/-7/	2.115575/-7/	-5.882705/-8/
1.85	-8.514908/-6/	2.637553/-6/	-7.879048/-7/	2.275098/-7/	-6.362095/-8/
1.90	-8.981929/-6/	2.799282/-6/	-8.412097/-7/	2.443100/-7/	-6.870520/-8/
1.95	-9.461276/-6/	2.966743/-6/	-8.968485/-7/	2.619776/-7/	-7.408973/-8/
2.00	-9.951335/-6/	3.139511/-6/	-9.547268/-7/	2.804980/-7/	-7.977489/-8/
2.05	-1.045139/-5/	3.317432/-6/	-1.014831/-6/	2.998797/-7/	-8.576759/-8/
2.10	-1.096050/-5/	3.500281/-6/	-1.077123/-6/	3.201244/-7/	-9.207287/-8/
2.15	-1.147553/-5/	3.687111/-6/	-1.141343/-6/	3.411665/-7/	-9.867634/-8/
2.20	-1.199993/-5/	3.879085/-6/	-1.207877/-6/	3.631342/-7/	-1.056196/-7/
2.25	-1.252778/-5/	4.074352/-6/	-1.276181/-6/	3.858755/-7/	-1.128632/-7/
2.30	-1.305773/-5/	4.272502/-6/	-1.346146/-6/	4.093688/-7/	-1.204050/-7/
2.35	-1.358874/-5/	4.473215/-6/	-1.417694/-6/	4.335997/-7/	-1.282449/-7/
2.40	-1.412027/-5/	4.676336/-6/	-1.490793/-6/	4.585688/-7/	-1.363873/-7/
2.45	-1.465257/-5/	4.881966/-6/	-1.565494/-6/	4.843014/-7/	-1.448438/-7/
2.50	-1.517865/-5/	5.087776/-6/	-1.641064/-6/	5.105798/-7/	-1.535534/-7/
2.55	-1.570511/-5/	5.295970/-6/	-1.718226/-6/	5.376358/-7/	-1.625893/-7/
2.60	-1.622092/-5/	5.502820/-6/	-1.795780/-6/	5.651026/-7/	-1.718445/-7/
2.65	-1.671776/-5/	5.705435/-6/	-1.872774/-6/	5.926826/-7/	-1.812307/-7/
2.70	-1.721816/-5/	5.911448/-6/	-1.951710/-6/	6.211703/-7/	-1.909932/-7/
2.75	-1.769422/-5/	6.111253/-6/	-2.029422/-6/	6.495646/-7/	-2.008276/-7/
2.80	-1.816988/-5/	6.313035/-6/	-2.108611/-6/	6.787303/-7/	-2.110029/-7/
2.85	-1.861349/-5/	6.505732/-6/	-2.185578/-6/	7.074794/-7/	-2.211523/-7/
2.90	-1.903619/-5/	6.693081/-6/	-2.261537/-6/	7.361965/-7/	-2.313951/-7/
2.95	-1.944621/-5/	6.877870/-6/	-2.337408/-6/	7.651790/-7/	-2.418256/-7/
3.00	-1.983251/-5/	7.056098/-6/	-2.411812/-6/	7.939742/-7/	-2.523018/-7/

References

[1] J. Antoniewicz, *Tablice funkcji dla inżynierów*, PWN, Warszawa 1969.
[2] Jahnke-Emde-Lösch, *Tafeln höherer Funktionen*, B. G. Teubner, Stuttgart 1966.
[3] J. Murzewski and A. Sowa, *Określenie nominalnych wartości wytrzymałości materiałów budowlanych*, Normalizacja 3 (1972).
[4] Я. В. Шор, *Статистические методы анализа и контроля качества и надежности*, Советское Радио, Москва 1962.
[5] E. T. Whittaker and G. N. Watson, *A course of modern analysis*, Polish ed., Part 2, PWN, Warszawa 1968.
[6] J. H. Wilkinson, *Rounding errors in algebraic processes*, Polish ed., PWN, Warszawa 1967.

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TABLICE FUNKCJI WALCA PARABOLICZNEGO DLA UJEMNYCH PARAMETRÓW CAŁKOWITYCH

STRESZCZENIE

W pracy podano tablice funkcji walca parabolicznego $D_n(x)$ dla wskaźnika n ujemnego całkowitego. Przy opracowaniu tablic korzystano z wzorów (1), (2) i (4). Pracę uzupełniono analizą błędu obliczeń na m. c. Odra 1204.
