ZASTOSOWANIA MATEMATYKI
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ALGORITHM 27

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STATISTICAL PREDICTION BY THE METHOD OF HARMONICAL WEIGHTS

1. Procedure declaration. Procedure *Prognoza* calculates for the given values $x_1, x_2, \ldots, x_n$ of a time series the predicted values $x^*_{n+1}, x^*_{n+2}, \ldots, x^*_{n+p}$ $(0 < p < n)$ and their confidence intervals by the method of harmonical weights presented in [1].

Data:

n — number of terms in the time series,
k — number of points in a segment $(1 < k \leqslant n)$,
p — prediction horizon $(0 < p < n)$,
alfa — confidence level of the confidence intervals,
$x[1:n]$ — array of values of the time series.

Results:

w — mean increment of the trend,
s — standard deviation of the trend increments,
$r[0:p]$ — radii of the confidence intervals for $y[0:p]$,
$y[0:p]$ — array holding the predicted values in $y[1:p]$ and the from the trend calculated value of the last term of the time series in $y[0]$. The array y must be declared as $y[0:n]$ on entry.

2. Method used. Given p, the prediction horizon, the predicted values x^*_{n+j} and the confidence intervals $(x^*_{n+j} - r_j, x^*_{n+j} + r_j)$, associated with them, on the confidence level a, are calculated on the basis of the time series $x_1, x_2, \ldots, x_n$ after the method given in [1]. The quantities $y_j = x^*_{n+j}$ and r_j $(j = 1, 2, \ldots, p)$ are calculated in the following way:

For the sets $X_i = \{x_i, x_{i+1}, \ldots, x_{i+k-1}\}$, where k is a fixed natural number satisfying $1 < k \leqslant n$ and $i = 1, 2, \ldots, n-k+1$, the linear trends

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procedure Prognoza(n,k,p,alfa,x,w,s,r,y);
 value n,k;
 integer n,k,p;
 real alfa,w,s;
 array x,r,y;
 begin
  integer h,i,j,k1,n1;
  real a,b,c,R,xr;
  n1:=n-k+1;
  k1:=k-1;
  w:=s:=.0;
  for j:=1 step 1 until n do
   y[j]:=.0;
  R:=.5×k1;
  c:=k×k1×(k+k1)/6.0;
  for j:=1 step 1 until k1 do
   begin
    s:=s+x[j];
    w:=w+j×x[j]
   end j;
  for j:=1 step 1 until n1 do
   begin
    i:=j+k-1;
    xr:=x[i];
    s:=s+xr;
    w:=w+i×xr;
    c:=c+i×i;
    b:=j+R;
    a:=(w-s×b)/(c-k×b×b);
    b:=s/k-a×b;
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  xr:=x[j];
  s:=s-xr;
  w:=w-j×xr;
  c:=c-j×j;
  for h:=j step 1 until i do
   y[h]:=y[h]+a×h+b
 end j;
if k≤n1
 then
 begin
  h:=k1;
  i:=n1;
  k1:=k
 end k≤n1
 else
 begin
  h:=n1-1;
  i:=k;
  k1:=n1
 end k>n1;
w:=s:=c:=.0;
n1:=n+1;
a:=y[1];
for j:=2 step 1 until n do
 begin
  b:=y[j]/(if j≤h then j else if j≤i then k1 else n1-j);
  c:=c+1.0/(n1-j);
  a:=b-a;
  R:=c×a;
  w:=w+R;
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    s:=s+a×R;
    a:=b
   end j;
  w:=w/(n-1);
  s:=sqrt(s/(n-1)-w×w);
  R:=s/(n×sqrt(1.0-alfa));
  a:=c:=c+1.0/n;
  y[0]:=y[n];
  r[0]:=a×R;
  for j:=1 step 1 until p do
   begin
    c:=c-1.0/j;
    a:=a+c;
    y[j]:=y[j-1]+w;
    r[j]:=a×R
   end j
 end Prognoza
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$x_i(t) = a_i t + b_i$ are calculated using least-squares and the trend increments $w_2, w_3, \ldots, w_n$, where $w_j = \bar{x}_j - \bar{x}_{j-1}$ and

$$\bar{x}_j = \begin{cases} \dfrac{1}{j} \sum\limits_{i=1}^{j} x_i(j) & \text{for } j = 1, 2, \ldots, m-1, \\ \dfrac{1}{m} \sum\limits_{i=\max(1, j-k+1)}^{\min(j, n-k+1)} x_i(j) & \text{for } j = m, m+1, \ldots, M, \\ \dfrac{1}{n-j+1} \sum\limits_{i=j-k+1}^{n-k+1} x_i(j) & \text{for } j = M+1, M+2, \ldots, n, \end{cases}$$

with the notation $m = \min(k, n-k+1)$ and $M = \max(k, n-k+1)$, are determined. Having calculated

$$\bar{w} = \frac{1}{n-1} \sum_{i=2}^{n} c_i w_i, \qquad s = \sqrt{\frac{1}{n-1} \sum_{i=2}^{n} c_i (w_i - \bar{w})^2},$$

where $c_1 = 0$, $c_i = c_{i-1}+1/(n-i+1)$ $(i = 2, 3, \ldots, n)$, and

$$a_0 = c_n+1/n, \quad b_0 = a_0,$$

$$a_j = a_{j-1}-1/j, \quad b_j = b_{j-1}+a_j \quad (j = 1, 2, \ldots, p),$$

one obtains finally

$$y_j = \bar{x}_n+j\bar{w}, \quad r_j = sb_j/n\sqrt{1-a} \quad (j = 1, 2, \ldots, p).$$

3. Verification. Procedure *Prognoza* has been translated on the Odra 1204 computer. The example from [1], as well as other ones, have been tested and correct results obtained.

Reference

[1] Z. Hellwig, *Schemat budowy prognozy statystycznej metodą wag harmonicznych*, Przegl. Statyst. 14 (1967), p. 133-153.

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WYZNACZENIE PROGNOZY STATYSTYCZNEJ METODĄ WAG HARMONICZNYCH

STRESZCZENIE

Procedura *Prognoza* oblicza, metodą opisaną w [1], wartości przewidywane $y_1, y_2, \ldots, y_p$ $(t = n+1, n+2, \ldots, n+p)$ dla szeregu czasowego $x_1, x_2, \ldots, x_n$ oraz przedziały ufności dla wartości przewidywanych na poziomie ufności a.

Dane:
n – liczba wyrazów szeregu czasowego,
k – liczba punktów w segmencie, gdzie $1 < k \leqslant n$,
p – horyzont prognozy, gdzie $0 < p < n$,
alfa – poziom ufności,
$x[1:n]$ – wartości szeregu czasowego.

Wyniki:

w — średnia przyrostów trendu,

s — odchylenie standardowe przyrostów trendu,

$r[0:p]$ — promienie przedziałów ufności,

$y[0:p]$ — gdzie $y[0]$ — n-ta wartość szeregu czasowego obliczona z trendu, $y[1:p]$ — wartości przewidywane; tablica y musi mieć wymiar $y[0:n]$.

Obliczenia wykonane na maszynie cyfrowej Odra 1204 wykazały poprawność procedury.