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ALGORITHM 15

S. GRZEGÓRSKI (Lublin)

EVALUATION OF PROBABILITY FOR THE *F*-SNEDECOR DISTRIBUTION

1. Function declaration.

```
real procedure PFSnedecor(n, m, FO, error);
  value n, m, FO;
  integer n, m;
  real FO;
  label error;
  comment PFSnedecor calculates the probability P(F > FO),
     where FO is a given value of the F-Snedecor function.
     Data:
        FO  – value of F-Snedecor test function,
        n   – number of degrees of freedom for the first random
              variable (in the numerator of the F-formula),
        m   – number of degrees of freedom for the second random
              variable (in the denominator of the F-formula).
     Other parameters:
     error  – label indicating error exit (n < 1 or m < 1 or FO < 0);
  begin
    integer i, k;
    real a, PF, RF;
    Boolean B;
    switch E: = pp, pn, np, np;
    if n < 1 ∨ m < 1 ∨ FO < .0 then go to error;
    B: = true;
    PF: = .0;
    go to if 1.0 + FO = 1.0 then FIN
                         else E[2 × (n − n ÷ 2 × 2 − m ÷ 2) + m + 1];
pp: if m ⩽ n then go to np;
pn: k: = n;
    n: = m;
    m: = k;
```

```
      FO: = 1.0/FO;
      B: = false;
np:   FO: = m/(m+n×FO);
      if m = 1 then go to mone;
      i: = m÷2;
      k: = i+i;
      a: = RF: = PF: = (1.0−FO) ↑ (n/k);
      n: = n−2;
      m: = m÷2;
      for k: = m−k+4 step 2 until m do
        begin
Lnd:      if RF > 2.0 then
            begin
              PF: = PF×a;
              RF: = RF×a;
              i: = i−1;
              go to Lnd
            end RF > 2.0;
          RF: = (n+k)×FO×RF/k;
          PF: = PF+RF
        end k;
      PF: = PF×a ↑ (i−1);
      if m÷2×2 = m then go to FIN;
      n: = n+2;
      PF: = n×PF×sqrt(FO);
mone: FO: = 1.0−FO;
      RF: = .0;
      a: = sqrt(FO−FO×FO);
      for k: = n step −2 until 3 do
        begin
          RF: = (FO−FO/k)×RF+a;
          PF: = PF−PF/k
        end k;
      PF: = .5−(2.0×(RF−PF)−arctan((FO−.5)/a))×.31830988618;
      comment .31830988618379... = 1.0/π;
FIN:  if PF > 1.0 then PF: = 1.0;
      PFSnedecor: = if B then 1.0−PF else PF
    end PFSnedecor
```

2. Method used. The well-known formula for the probability of the F-Snedecor distribution has the form

$$\mathrm{P}(F > F_0) = n^{n/2} m^{m/2} \int_{F_0}^{\infty} F^{n/2-1}(nF+m)^{-(n+m)/2} dF/B(n/2, m/2),$$

where

$$B(p, q) = \int_0^1 t^{p-1}(1-t)^{q-1}\,dt.$$

Substituting $x = nF/(nF+m)$, $x_0 = nF_0/(nF_0+m)$, and writing

$$Q(n, m, x) = \int_0^x t^{n/2-1}(1-t)^{m/2-1}\,dt,$$

we have $B(n/2, m/2) = Q(n, m, 1)$ and

(1) $$\mathrm{P}(F > F_0) = 1 - Q(n, m, x_0)/Q(n, m, 1).$$

Let us write

$$P(n, m, x) = x^{n/2} \sum_{k=0}^{m/2-1} \frac{(n+2k-2)!!}{(n-2k)!!(2k)!!}(1-x)^k,$$

$$R(n, m, x) = 2\sqrt{x(1-x)} \sum_{k=0}^{n/2-1} \frac{(2k)!!}{(2k+1)!!} x^k + \arctan \frac{0.5-x}{\sqrt{x(1-x)}}$$

and

$$H(n, m, x) = \begin{cases} P(n, m, x) & \text{for } m \text{ even}, \\ 1 - P(m, n, 1-x) & \text{for } n \text{ even}, \\ 0.5 + [R(n, m, x) - 2\dfrac{(n-1)!!}{(n-2)!!}\sqrt{1-x}\,P(n, m, x)]/\pi & \\ \qquad\qquad \text{for } n, m \text{ odd}. \end{cases}$$

With the recurrent formulas

$$Q(n, 2, x) = 2x^{n/2}/n \quad \text{for } n = 1, 2, \ldots,$$

$$Q(1, 1, x) = \arctan \frac{0.5-x}{\sqrt{x(1-x)}},$$

$$Q(k, l, x) = \begin{cases} [x^{k/2-1}(1-x)^{l/2} + (k/2-1)Q(k-2, l, x)]/(k/2+l/2-1) \\ \qquad\qquad \text{for } k = 3, 4, \ldots, \\ [x^{k/2}(1-x)^{l/2-1} + (l/2-1)Q(k, l-2, x)]/(k/2+l/2-1) \\ \qquad\qquad \text{for } l = 3, 4, \ldots \end{cases}$$

we may prove that

(2) $$Q(n, m, x) = Q(n, m, 1)^* H(n, m, x).$$

Finally, from (1) and (2) it follows that

$$\mathrm{P}(F > F_0) = 1 - H(n, m, x_0).$$

3. Certification. *PFSnedecor* calculates the same value as *Ftest* [3] or *Fisher* [1]. All three procedures were tested and compared on the Odra 1013 computer with 31-bit floating-point mantissa, in the FALA-69 autocode. It appears that

1° *PFSnedecor* is much faster than *Fisher* and in some cases faster than *Ftest.*

2° When one of the degrees of freedom is greater than 120, *Fisher* gives floating-point overflow. When one of the degrees of freedom is greater than 128, *Ftest* uses some approximative formulae, giving a lesser accuracy. *PFSnedecor* uses exact formulae for all n and m.

3° *Ftest* is 2 times longer than *PFSnedecor*, uses 2 times more variables, and must be complemented by a procedure *Gauss* [2]. Moreover, *Ftest* exploits the functions *sin* and *cos*.

References

[1] E. Dorrer, *Algorithm 322: F-Distribution* Comm. ACM 11 (1968), p. 116-117.
[2] D. Ibbetson, *Algorithm 209: Gauss,* ibidem 6 (1963), p. 616.
[3] J. Morris, *Algorithm 346: Ftest,* ibidem 3 (1969), p. 184.
[4] W. Oktaba, *Metody statystyki matematycznej w doświadczalnictwie,* Warszawa 1967.
[5] W. Sadowski, *Statystyka matematyczna,* Warszawa 1965.
[6] G. W. Snedecor, *Statistical methods,* Iowa State Univ. Press, Ames, Iowa, 1956.

INSTITUTE OF MATHEMATICS
MARIA CURIE-SKŁODOWSKA UNIVERSITY
LUBLIN

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ALGORYTM 15

S. GRZEGÓRSKI (Lublin)

OBLICZANIE PRAWDOPODOBIEŃSTWA DLA ROZKŁADU F-SNEDECORA

STRESZCZENIE

Procedura *PFSnedecor* oblicza prawdopodobieństwo $P(F > F0)$ dla dowolnych liczb stopni swobody, wykorzystując wzory dokładne.

Dane:

F0 – wartość funkcji testowej F-Snedecora,
n – liczba stopni swobody zmiennej występującej w liczniku,
m – liczba stopni swobody zmiennej występującej w mianowniku.

Przez zastosowanie tej procedury, proces weryfikowania hipotez wykorzystujących rozkład F-Snedecora sprowadza się do obliczenia prawdopodobieństwa i ewentualnego porównania go z ustalonym poziomem ufności. Nie ma zatem potrzeby korzystania z tablic rozkładu F-Snedecora.
