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SOME REMARKS
ABOUT A PAPER BY T. KACZOREK AND T. TRACZYK

In paper [5] the following two mathematical statements are proved:

STATEMENT I. *The partial derivatives of the Vandermonde determinant are different from zero.*

STATEMENT II. *If $M(s)$ and $M_0(s)$ are polynomial functions of degree n over the field of complex numbers and if*

$$M(s) = M_0(s) + \sum_{i=1}^{p} k_i s^{i-1} \quad (p \leqslant n)$$

for some coefficients $k_1, k_2, \ldots, k_p$, then the constants $k_1, k_2, \ldots, k_p$ are uniquely determined by any p roots of the polynomial $M(s)$.

Let $s_1, s_2, \ldots, s_k$ be some distinct roots of the polynomial $M(s)$, and let s_j be a root of order r_j $(j = 1, 2, \ldots, k)$, where $r_1 + r_2 + \ldots + r_k = p$. Then Statement II is equivalent to the following

STATEMENT III. *The system of linear equations with p unknowns $k_1, k_2, \ldots, k_p$, i.e.*

$$\frac{d^m}{ds^m} \sum_{i=1}^{p} k_i s^{i-1} \Big|_{s=s_j} = -\frac{d^m}{ds^m} M_0(s) \Big|_{s=s_j} \quad (j = 1, 2, \ldots, k), \tag{1}$$

where for every j the integer m varies from 0 to $r_j - 1$, has a uniquely determined solution.

Statement III is a special case of the following

STATEMENT IV. *The system of linear equations with p unknowns $k_1, k_2, \ldots, k_p$, i.e.*

$$\frac{d^m}{ds^m} \sum_{i=1}^{p} k_i s^{i-1} \Big|_{s=s_j} = b_{jm} \quad (j = 1, 2, \ldots, k), \tag{2}$$

where for every j the integer m varies from 0 to $r_j - 1$ and b_{jm} are arbitrary numbers, has a uniquely determined solution.

The problem of finding the solution of system (2) was formulated by Charles Hermite and is known as Hermite's interpolation problem (see, e.g., [1], [3] and [7]) or as the general interpolation problem (see [8]). There exist a number of proofs of Statement IV (see, e.g., [1], [6], [8] and [9]). In the synthesis of linear control systems the authors of paper [5] employ only Statement II, thus, for that purpose, it is not necessary to define and use the notion of partial derivatives of the Vandermonde determinant. On the other hand, the notion used in [5] is known in the mathematical literature (see, e.g., [2] and [4]) under the name of determinants of Hermite type confluent Vandermonde matrices. If the latter had been used, Statement I would have taken the form of the well-known statement (see, e.g., [4]) that Hermite type confluent Vandermonde matrices are non-singular.

References

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UWAGI DOTYCZĄCE PRACY T. KACZORKA I T. TRACZYKA

STRESZCZENIE

Autor wykazuje, że dwa twierdzenia z pracy Kaczorka i Traczyka [5] wynikają z ogólniejszych rezultatów teorii interpolacji Hermite'a.