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ALGORITHM 93

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NON-CENTRAL F OR BETA DISTRIBUTION

1. Procedure declaration. The non-central beta distribution is defined as

(1)
$$B(t: p, q; \lambda) = e^{-\lambda} \sum_{k=0}^{\infty} (\lambda^k/k!) B(t: p+k, q),$$

where $0 \leqslant t \leqslant 1$, $p > 0$, $q > 0$, and $B(t: p, q)$ is the incomplete beta ratio,

(2)
$$B(t: p, q) = \int_0^t s^{p-1}(1-s)^{q-1}\, ds/B(p, q).$$

The non-central F and non-central beta distributions are related by

(3)
$$F(w: n_1, n_2; \lambda) = B(t: p, q; \lambda),$$

where $p = n_1/0.5$, $q = n_2/0.5$, and $t = pw/(pw+q)$. Throughout this note we assume that $\lambda \geqslant 0$.

For given values x, a, b and h ($a > 0$, $b > 0$, $h \geqslant 0$), the real procedure *nonfob* computes the value of the distribution function of the non-central beta distribution (1) or of the non-central F distribution (3).

Data:

x – the value of t in (1) or of w in (3);
a – the value of p in (1) or of n_1 in (3), $a > 0$;
b – the value of q in (1) or of n_2 in (3), $b > 0$;
h – the value of λ in (1) and (3), $h \geqslant 0$;
ind – Boolean variable; if *ind* $\equiv$ **true**, then the real procedure *nonfob* computes the value of (1), and if *ind* $\equiv$ **false**, then the real procedure *nonfob* computes the value of (3);
eps – the accuracy of the method.

Result:

nonfob – the value of the distribution function of the non-central F or non-central beta distribution.

Other parameters:

loggamma – real procedure with the following head: **real procedure** *loggamma* (x); **value** x; **real** x; this procedure is published in [5] and

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real procedure betain(x,p,q,beta,acu);
  value x,p,q,beta,acu;
  real x,p,q,beta,acu;
  begin
    integer ns;
    real psq,cx,xx,pp,qq,term,ai,rx,temp,be;
    Boolean index;
    psq:=p+q;  cx:=1.0-x;
    if p<psq*x
      then
      begin
        xx:=cx;  cx:=x;  pp:=q;  qq:=p;  index:=true
      end p<psq*x
      else
      begin
        xx:=x;  pp:=p;  qq:=q;  index:=false
      end p>=psqx;
    term:=ai:=be:=1.0;  ns:=qq+cx*psq;  rx:=xx/cx;
E1: temp:=qq-ai;
    if ns=0 then rx:=xx;
E2: term:=term*temp*rx/(pp+ai);  be:=be+term;  temp:=abs(term);
    if temp>acu or temp>acu*be
      then
      begin
        ai:=ai+1.0;  ns:=ns-1;
        if ns>=0
          then goto E1
          else
          begin
            temp:=psq;  psq:=psq+1.0;  goto E2
          end ns<0
      end temp>acu or temp>acu*be;
    betain:=be:=be*exp(pp*ln(xx)+(qq-1.0)*ln(cx)-beta)/pp;
    if index then betain:=1.0-be
  end betain
```

```
real procedure nonfob(x,a,b,h,ind,eps,loggamma,betain,jump);
  value x,a,b,h,eps;
  real x,a,b,h,eps;
  Boolean ind;
  label jump;
  real procedure loggamma,betain;
  begin
    integer n;
    real c,e,s,r,p,d;
    if x<.0 or (x>1.0 and ind) or a<=.0 or b<=.0 or h<.0
      then goto jump;
    if x=.0 or (x=1.0 and ind)
      then nonfob:=x
      else
      begin
        if not ind
          then
          begin
            a:=a*.5;   b:=b*.5;   c:=a*x;   x:=c/(c+b)
          end not ind;
        c:=exp(-h);
        s:=betain(x,a,b,loggamma(a)+loggamma(b)-loggamma(a+b),
           eps);
        if h≠.0
          then
          begin
            n:=0;   r:=h;
lab:        n:=n+1;
            p:=betain(x,a+n,b,loggamma(a+n)+loggamma(b)
               -loggamma(a+n+b),eps);
            s:=s+r*p;   r:=r*h/(n+1);   d:=c+h/(n+2);
            if d>1.0 then d:=1.0;
            d:=r*p*d;
            if d>eps then goto lab
          end h≠.0;
        nonfob:=c*s
      end not (x=.0 or (x=1.0 and ind))
  end nonfob
```

computes the value of the natural logarithm of the gamma function for the given argument x;

betain – real procedure with the following head: **real procedure** *betain* (*x*, *p*, *q*, *beta*, *acu*); **value** *x*, *p*, *q*, *beta*, *acu*; **real** *x*, *p*, *q*, *beta*, *acu*; this procedure is a combination of those in [1] and [3] and computes the value of the incomplete beta ratio (2), where x is the value of t, p and q are the values of p and q in (2), respectively, *beta* is the value of the beta function equal to $B(p, q)$ in (2), and *acu* is the accuracy of the method;

jump – label outside of the procedure body to which a jump is made if $a \leqslant 0$, or $b \leqslant 0$, or $h < 0$, or $x < 0$, or $x > 1$ for the non-central beta distribution.

2. Method used. The algorithm is described in [4].

3. Certification. The procedure *nonfob* has been verified on the Odra 1204 computer for many examples described in [2] and [6]. Here are some of them presented as examples (*ind* $\equiv$ **false**, $eps = {}_{10}{-7}$):

$$F(7.778\colon\ 14.0,\ 6.0;\ 7.0) = .950\,003\,6135,$$

$$F(6.811\colon\ 2.0,\ 15.0;\ 1.0) = .950\,005\,0110,$$

$$F(497.973\colon\ 18.0,\ 1.0;\ 9.0) = .949\,998\,3166,$$

$$F(3.297\colon\ 12.0, 1000.0;\ 6.0) = .949\,988\,9998,$$

$$F(446.357\colon\ 3.0, 1.0;\ 1.5) = .950\,003\,2681.$$

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