Problems Column

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ON DISTANCE EDGE COLOURINGS OF A CYCLIC MULTIGRAPH

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We shall use the distance chromatic index defined by the present author in early nineties, cf. [5] or [4] of 1993. The *edge distance* of two edges in a multigraph M is defined to be their distance in the line graph $L(M)$ of M. Given a positive integer d, define the d^+-*chromatic index* of the multigraph M, denoted by $q^{(d)}(M)$, to be equal to the chromatic number χ of the dth power of the line graph $L(M)$,

$$q^{(d)}(M) = \chi(L(M)^d).$$

Then the colour classes are matchings in M with edges at edge distance larger than d apart.

Call C to be a *cyclic multigraph* if C consists of a cycle on n vertices with possibly more than one edge between two consecutive vertices.

The following problem was presented in [6].

Problem. Given an integer $d \geq 2$ and a cyclic multigraph C, find (or estimate) $q^{(d)}(C)$, the d^+-chromatic index of C.

In other words, generalize the following formula due to Berge [1] for the ordinary chromatic index ($q = q^1$)

$$q(C) = \begin{cases} \max\left\{\Delta(C), \left\lceil \frac{e(C)}{\lfloor \frac{n}{2} \rfloor} \right\rceil\right\} & \text{for odd } n, \\ \Delta(C) & \text{for even } n, \end{cases}$$

where $\Delta(C)$ and $e(C)$ are the maximum degree among vertices and the size of C, respectively.

Remarks 1. 2^+-chromatic index $q^{(2)}$ is known under the name *strong chromatic index*, estimations of $q^{(2)}(C)$ being studied in [2, 3].

2. In [5] it is proved that

$$q^{(d)}({}^pC_n) = \begin{cases} pn & \text{if } n \leq 2d+1, \\ \left\lceil \frac{pn}{\lfloor \frac{n}{d+1} \rfloor} \right\rceil & \text{if } n \geq d+1 \end{cases}$$

where ${}^pC_n$ is the cyclic multigraph C with all edge multiplicities equal to p.

3. Let M be a loopless multigraph whose underlying graph is a forest. Then $q^{(d)}(M)$, the d^+-chromatic index of M, can be seen to be equal to the diameter-d cluster (or diameter-d edge-clique) number of M (i.e., the density of the dth power, $L(M)^d$, of the line graph of M). This extends the known corresponding results on a tree [5] and on $q^{(2)}(M)$ in [2].

References

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[3] P. Gvozdjak, P. Horák, M. Meszka and Z. Skupień, *On the strong chromatic index of cyclic multigraphs*, Discrete Appl. Math., to appear.

[4] Z. Skupień, *Some maximum multigraphs and chromatic d-index*, in: U. Faigle and C. Hoede, eds., 3rd Twente Workshop on Graphs and Combinatorial Optimization, (Fac. Appl. Math. Univ. Twente) Memorandum No. **1132** (1993) 173–175.

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