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Erratum and addendum to the paper
 ”Compactness and essential norms of composition
 operators on Orlicz-Lorentz spaces” [Applied
 Mathematics Letters 25 (2012) 1778–1783]

- Page 1778 (=page 1 in the paper), 5th line of the Introduction: instead of ” μ a.e.” there should be ” μ -a.e.”.
- Page 1781 (=page 4 in the paper): instead of ” $\chi_{A_n} \rightarrow \emptyset$ ” there should be ” $A_n \downarrow \emptyset$ ” which means that $A_{n+1} \subset A_n$ for any $n \in \mathbb{N}$ and $\bigcap_{n=1}^{\infty} A_n = \emptyset$.
- Page 1781, line 13 of the proof of Theorem 2.3: instead of ” $\|\chi_{A_n}\|_{\varphi,\omega} \|g\chi_{A_n}\|_{X^*}$ ” there should be ” $\|g\chi_{A_n}\|_{X^*}$ ”.
- Page 1781, line 12 from below: instead of ”and it follows” there should be ”and we have”.
- In Theorem 2.3, page 1781 instead of ”Suppose $\varphi \in \Delta_2(\mathbb{R})$ or $\varphi \in \nabla(\mathbb{R})$ ” there should be ”Suppose $\varphi \in \nabla_2(\mathbb{R})$ ”.

The last mistake was caused by wrong inequality: $g(f_n) \leq \|\chi_{A_n}\|_{\varphi,\omega} \|g\chi_{A_n}\|_{X^*}$. If this inequality was true then condition ” $\varphi \in \Delta_2(\mathbb{R})$ or $\varphi \in \nabla_2(\mathbb{R})$ ” would be enough to get $g(f_n) \rightarrow 0$ as $n \rightarrow \infty$. However, this inequality should be changed into $g(f_n) \leq \|g\chi_{A_n}\|_{X^*}$ and we can get that $g(f_n) \rightarrow 0$ whenever $\|g\chi_{A_n}\|_{X^*} \rightarrow 0$, which is true if $\varphi \in \nabla_2(\mathbb{R})$.

Since some details in the proof of Theorem 2.3 were omitted, we explain here some facts additionally.

Let us note that the assumption that $\varphi \in \nabla_2(\mathbb{R})$ implies the equality $b(\varphi) = +\infty$. This, in turn, implies that $(\Lambda^{\varphi,\omega})_a \neq \{0\}$, where $(\Lambda^{\varphi,\omega})_a$ is the subspace of $\Lambda^{\varphi,\omega}$ that consists of all order continuous elements in $\Lambda^{\varphi,\omega}$. Since $\text{supp}[(\Lambda^{\varphi,\omega})_a] = \text{supp}(\Lambda^{\varphi,\omega}) = \Omega$, so from the general knowledge on the duality of Köthe spaces, we have

$$[\Lambda^{\varphi,\omega}]^* = [\Lambda^{\varphi,\omega}]' \oplus S,$$

where $[\Lambda^{\varphi,\omega}]'$ is the Köthe dual of $\Lambda^{\varphi,\omega}$ (which coincides with the Köthe dual of $(\Lambda^{\varphi,\omega})_a$) and S is the space of all singular functionals from $[\Lambda^{\varphi,\omega}]^*$, that is,

$x^* \in [\Lambda^{\varphi, \omega}]^*$ belongs to S if and only if $x^*(f) = 0$ for any $f \in [\Lambda^{\varphi, \omega}]_a$. Since $f_n \in (\Lambda^{\varphi, \omega})_a$ for any $n \in \mathbb{N}$, where f_n are the functions defined in the 5th line of the proof of Theorem 2.3, so $x^*(f_n) = 0$ for any $n \in \mathbb{N}$ and any $x^* \in S$. Therefore, the proof that $f_n \rightarrow 0$ weakly as $n \rightarrow \infty$, reduces to the proof that $x^*(f_n) \rightarrow 0$ as $n \rightarrow \infty$ for any $x^* \in [\Lambda^{\varphi, \omega}]'$, and this can be proceeded in the same way as it was done in the proof of Theorem 2.3. Moreover, it is worth adding that we can easily reduce this proof to the case of a finite measure, because for any $g \in [\Lambda^{\varphi, \omega}]' = [(\Lambda^{\varphi, \omega})_a]'$, by $\varphi \in \nabla_2(\mathbb{R})$ and the results of the article [7] from the references of the original paper, where the Köthe dual $[\Lambda^{\varphi, \omega}]'$ was described, we have that the function g is order continuous. Therefore, for any $\varepsilon > 0$ there is $A \in \Sigma$ with $\mu(A) < \infty$ such that $\|g\|_{[\Lambda^{\varphi, \omega}]'} < \varepsilon$ and $\|g\chi_{A_n}\|_{[\Lambda^{\varphi, \omega}]'} \rightarrow 0$, as $n \rightarrow \infty$, for any sequence $\{A_n\}_{n=1}^\infty$ in Σ such that $\mu(A \cap A_n) \rightarrow 0$ as $n \rightarrow \infty$.

Note. It would be natural to publish this erratum and addendum in Applied Mathematics Letters, where the original paper was published. However, we did not get permission for publishing it there. We were informed that the journal received too many erratums to the papers published in its last volumes to publish them.

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