Investigation of the kinetics of the development of the distribution

Dmitriy Marchenko

Volodymyr Dahl East-Ukrainian National University, Lugansk, Ukraine,

Summary. The article presents an analytical solution of the equations of Kolmogorov-Fokker-Planck, as well as the analysis of dynamics of development of cracks on the basis of the model of diffusion of defects.
Key words: stochastic process, diffusion of defects, the intensity of wear.

PROBLEM DEFINITION

The task of improving the efficiency of complex technical systems is directly connected with optimal for the design of the load conditions of materials, structural state of which optimized for the conditions of service, for which it is necessary the knowledge of the mechanism and of the evolution of damage accumulation of the material in the course of his service. Solution of the problem requires an interdisciplinary approach on the basis of the physics of strength and fracture mechanics and material science.

ANALYSIS OF THE PROBLEM

New approaches have arisen as a result of the development of non-equilibrium thermodynamics and information theory. Classical thermodynamics, as it is known, explores the irreversible processes, leading to an increase in energy, meaning the increase in the chaos and the destruction of the interior structural relations [Thursdays V.A., 2003]. On the contrary, nonequilibrium thermodynamics studies irreversible processes, leading to a decrease in entropy by self-organization ordered or dissipative structures, flows in open systems, the exchange of energy with the environment [Reshetov D.N., 1988]. Conclusions of non-equilibrium thermodynamics do not contradict the classical, as the assertion of a reduction in entropy refers to the local system, which is compensated by the increase in the entropy of the external environment, interacting with the system [Pakhomov E.A., 1985]. Nonequilibrium systems are inherent in bifurcation.

To Prigozhin, worked out the mathematical apparatus, describing the behavior of the system near the bifurcation points, the system due to random fluctuations selects one of several options for the future, both near these points fluctuations become strong. Reaching the stage of bifurcation, the system loses its stability. The alternation of stability and instability - a common phenomenon in the evolution of any open system, and the process is irreversible [Ostreikovskaya V.A., 2005]. This means that the system after the passage of the bifurcation cannot be returned to its original state [Terentyev V.F., 2003].

In the process of operation of any system is exposed to various influences, which eventually lead to the destruction of the system as a whole or any of its parts, which can also be regarded as a violation of the system, as it removes it from the system. However, begins the process of destruction with the appearance of defects in the system response to perturbations of the impact, its attempts to move to a stable state. For a description of this process, consider the parameter that defines the status of the defect [Marchenko D., Vetrov A., 2003]. For example, it may be the size of fatigue cracks [Klyuyev V.V,. 2003].

In some cases, the rate of change of the determining parameter can be described by the stochastic differential equation of the type:

$$dy = mdt + \sigma d\xi(t) \quad (1)$$

where: m – mathematical expectation of rate of formation of cracks; σ – the average quadratic deviation of the rate of formation of cracks; $\xi(t)$ – normal white noise with zero mathematical expectation and variance [Ivanova V.S., 1979]. Model (1) change of the determining parameter leads to the diffusion distribution of uptime, whose form is determined by the relevant provisions of the decisions of the equations of Kolmogorov-Fokker-Planck. The last equation is determined by the probability density of transition $p(t_0, y_0, t, y)$ of the Markov diffusion process from one state to another parts [Barenblatt G.I., 2003]. The equation of the Kolmogorov-Fokker-Planck has the form:

$$\frac{\partial p}{\partial t} + m\frac{\partial p}{\partial y} - \frac{\sigma^2}{2}\frac{\partial^2 p}{\partial y^2} = 0 . \quad (2)$$

View the solution of this equation is determined by the boundary conditions, which in the simplest case, you can establish, on the basis of the physical essence of the considered processes power [Gnedenko B.V., Belyaev Yu.K., Soloview A.D., 1965, Gadasin V.A., Ushakov I.A., 1975, Bortsova A.T., 1983, Barzilovich E.Yu., Kashtanov V.A., 1971]. In particular, if the implementation of the process (1) have the monotonous character, in the capacity of boundary conditions are accepted by the following:

$$p(y,t)\big|_{y=-\infty} = p(y,t)\big|_{y=\infty} = 0 \quad (3)$$

$$p(y,t)\big|_{y=0} = \delta(y), \quad (4)$$

where: $\delta(y)$ is a Delta - function.

The solution of equation (2) with the conditions (3), (4) is known as the fundamental solution and has the following form:

$$p = \frac{1}{\sigma\sqrt{2\pi t}} e^{\left(-\frac{(y-mt^2)}{2\sigma^2 t}\right)} . \quad (5)$$

You can, however, show that the equation of the Kolmogorov-Fokker-Planck under certain boundary conditions has an analytical solution.

The solution of diffusion equation allows us to offer analytical modeling of the process of education and development of cracks and methodology of the location of the uncertain parameters of the model [Baldin K.V., 2004]. To change the concentration of cracks in the process of operation of the system depending on the boundary conditions for an equation of the Kolmogorov-Fokker-Planck allows to receive the parametric solution either in an explicit form, or with the help of the numerical algorithm [Ayzinbud S.Y., 1990]..

THE AIM OF THE RESEARCH

Analytically find the General solution of the equations of Kolmogorov-Fokker-Planck, as well as the distribution functions of the time of the first achievements of the defining parameter of the defect a set limit, which will allow to determine the function of the intensity of wear and tear $r(t)$.

THE RESULTS OF THE STUDY

Let us consider the General case equation a random walk on the vertical axis, recorded in the form of the equation of the Kolmogorov-Fokker-Planck relatively concentrations

$$\frac{\partial p}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 p}{\partial y^2} + m\frac{\partial p}{\partial y} . \quad (6)$$

Here $p = p(t, y)$ is the fraction of defects (cracks) to the moment of time t in point y. Axis Oy believe aimed up; m – mathematical expectation of the rate of formation of cracks, м/с ; σ – average quadratic deviation of the rate of formation of cracks, $м^2/с$. In the General case, m and σ are not constant and change over time. However, these changes for a small period of time is insignificant. Defects (cracks) are not homogeneous in their properties .) [Dodonov A.G,. 1988, Golubenko A., 2008]. For each class of defects [Khaitun S.D., 1996] you can write the equation of a random walk in the working volume with their conditions of development of the defect y. Reaching the value $y = 0$, the defect with a probability $1-P$ is maintained, i.e., returns to the process of wandering, and continues to generate new defects, or with probability P disappears, i.e. exits process wandering. Thus, in the General case, the value y - is the value of the approximation to the point of the possible implementation phase transition, i.e. the restoration of the damaged defect of the surface area or the other phase transition associated with the achievement of the size of the defect some limiting value a. In the second case there is a break in the relations, which leads to a catastrophic failure of the system, or to its transition to a new state of equilibrium.

Reflection, return to the process of wandering, occurs with a probability $1-P=1-P(x)$. Therefore, the condition of full reflection can be written as equality to zero of a stream of particles (defects) through the screen. It has the form: $\frac{\sigma^2}{2}\frac{\partial p}{\partial y}+mp=0$. The condition of full recovery (disappearance of defect) corresponds to $p=0$. This event occurs with a probability P. With the probability of the particle (defect) enters a new phase state, becomes an attachment to the environment. It will not be further discussed in the process of wandering.

Therefore a weighted sum of these two equations gives the boundary condition for the differential equation (6) in partial derivatives provided $y=0$:

$$(1-P)\left(\frac{\sigma^2}{2}\frac{\partial p}{\partial y}+mp\right)-\mu mPp=0\ , \qquad (7)$$

where $\mu>0$ is the dimensionless ratio.

The solution of equation (6) is in the form of the product of two functions $H(y)$, $G(t)$:

$$p(t,y)=H(y)\cdot G(t). \qquad (8)$$

They have the form

$$G=G_0\cdot e^{-\lambda t},\ \lambda>0.$$
$$H=C_1e^{k_1y}+C_2e^{k_2y}$$

Where

$$k_1=\frac{-m-\sqrt{m^2-2\lambda\sigma^2}}{\sigma^2}$$
$$k_2=\frac{-m+\sqrt{m^2-2\lambda\sigma^2}}{\sigma^2}$$

So C_1, C_2 as two arbitrary constants, are to be determined, and the General solution is obtained by the substitution in (8) the obtained expressions for $H(y)$ and $G(t)$, then there is no need to introduce a third of the constant G_0. You can put $G_0=1$.

Therefore, the General solution of the equation (6) not taking into account the boundary conditions, we obtain the form of:

$$p(t,y)=\left(C_1e^{k_1y}+C_2e^{k_2y}\right)e^{-\lambda t}. \qquad (9)$$

Two of undefined constant C_1 and C_2 is found from the following considerations. In addition to the boundary condition (7), which will give one equation, it is necessary to use the initial condition $t=0$, the sum of all concentrations, that is, the integral $\int_0^\infty p(0,y)dy$ is equal to the derivative of the density of defects $\gamma(x)$. Proceeding from this,

$$C_1=\frac{\lambda\gamma}{\mu mP\sqrt{m^2-2\lambda\sigma^2}}\left[(1-P)\frac{m+\sqrt{m^2-2\lambda\sigma^2}}{2}-\mu mP\right],$$
$$C_2=-\frac{\lambda\gamma}{\mu mP\sqrt{m^2-2\lambda\sigma^2}}\left[(1-P)\frac{m-\sqrt{m^2-2\lambda\sigma^2}}{2}-\mu mP\right]$$

The equation of the Kolmogorov-Fokker-Planck allows you to determine the density of the distribution $f(t)$ of the time of the first achievements of the defining parameter of the defect a set limit. It has the look of

$$f(t)=\int_0^a\frac{\partial p}{\partial t}dy. \qquad (10)$$

With regard to (9) it will take the form of:

$$f(t)=\int_0^a\frac{\partial\left(C_1e^{k_1y}+C_2e^{k_2y}\right)e^{-\lambda t}}{\partial t}dy,$$

where:

$$k_1=\frac{-m-\sqrt{m^2-2\lambda\sigma^2}}{\sigma^2},$$

$$k_2=\frac{-m+\sqrt{m^2-2\lambda\sigma^2}}{\sigma^2},$$

$$C_1=\frac{\lambda\gamma}{\mu mP\sqrt{m^2-2\lambda\sigma^2}}\left[(1-P)\frac{m+\sqrt{m^2-2\lambda\sigma^2}}{2}-\mu mP\right]$$
$$C_2=-\frac{\lambda\gamma}{\mu mP\sqrt{m^2-2\lambda\sigma^2}}\left[(1-P)\frac{m-\sqrt{m^2-2\lambda\sigma^2}}{2}-\mu mP\right].$$

Продифференцировав the integrand the variable t, we obtain

$$\frac{\partial p(t,y)}{\partial t}=\frac{\partial\left(C_1e^{k_1y}+C_2e^{k_2y}\right)e^{-\lambda t}}{\partial t}=-\lambda\left(C_1e^{k_1y}+C_2e^{k_2y}\right)e^{-\lambda t}.$$

Than we obtain

$$f(t)=-\lambda\int_0^a\left(C_1e^{k_1y}+C_2e^{k_2y}\right)e^{-\lambda t}dy.$$

Thus, we finally obtain the

$$f(t)=-\lambda e^{-\lambda t}\int_0^a\left(C_1e^{k_1y}+C_2e^{k_2y}\right)dy=-\lambda e^{-\lambda t}\left(k_1C_1e^{k_1y}+k_2C_2e^{k_2y}\right)\Big|_0^a=$$
$$=-\lambda e^{-\lambda t}\left(\frac{C_1}{k_1}e^{ak_1}+\frac{C_2}{k_2}e^{ak_2}\right)+\lambda e^{-\lambda t}\left(\frac{C_1}{k_1}+\frac{C_2}{k_2}\right)=$$

$$=-\lambda e^{-\lambda t}\left(\frac{C_1}{k_1}e^{ak_1}+\frac{C_2}{k_2}e^{ak_2}-\frac{C_1}{k_1}-\frac{C_2}{k_2}\right)=$$

$$=-\lambda e^{-\lambda t}\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)=$$

$$=-\lambda\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)e^{-\lambda t}=f\left(t,a\right), \quad (11)$$

Having found it $f\left(t,a\right)$, you can find the distribution functions of the time of the first achievements of the defining parameter of the defect a set limit. It is enough to integrate (12) :

$$\int_0^t f\left(t,a\right)dt=\int_0^t\left(-\lambda\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)e^{-\lambda t}\right)dt=$$

$$=-\lambda\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)\int_0^t e^{-\lambda t}dt=$$

$$=\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)e^{-\lambda t}\Big|_0^t=\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)e^{-\lambda t}=F\left(t,a\right)$$

Knowing $F\left(t,a\right)$, you can define the intensity of wear of the system as:

$$r=\frac{F'}{1-F}=r\left(t\right), \quad (12)$$

or, substituting F' and F:

$$r=\frac{-\lambda\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)e^{-\lambda t}}{1-\left(\frac{C_1}{k_1}\left(e^{ak_1}-1\right)+\frac{C_2}{k_2}\left(e^{ak_2}-1\right)\right)e^{-\lambda t}},$$

where:

$$k_1=\frac{-m-\sqrt{m^2-2\lambda\sigma^2}}{\sigma^2},$$

$$k_2=\frac{-m+\sqrt{m^2-2\lambda\sigma^2}}{\sigma^2},$$

$$C_1=\frac{\lambda\gamma}{\mu mP\sqrt{m^2-2\lambda\sigma^2}}\left[\left(1-P\right)\frac{m+\sqrt{m^2-2\lambda\sigma^2}}{2}-\mu mP\right]$$

$$C_2=-\frac{\lambda\gamma}{\mu mP\sqrt{m^2-2\lambda\sigma^2}}\left[\left(1-P\right)\frac{m-\sqrt{m^2-2\lambda\sigma^2}}{2}-\mu mP\right]$$

CONCLUSIONS

Thus, analytically found the General solution of the equations of Kolmogorov-Fokker-Planck, as well as the distribution function of the time the first achievements of the defining parameter of the defect a set limit, which allowed to determine the function of the intensity of wear and tear $r\left(t\right)$.

The obtained dependences can be used for forecasting the processes of defect formation and choice of optimal design parameters tribosystems that will allow for a much more efficient use of complex technical systems. In practice, of interest is the determination of the time of trouble-free operation of the system. The reduction of formation of defects in the scheme of random walks allows you to easily implement the numerical solution of differential equations at any given boundary conditions. The possible implementation of the appropriate mathematical model on the computer.

REFERENCE

1. **Ayzinbud S.Y., 1990.:** Operation of locomotives. Transport. M., 261 p.
2. **Baldin K.V., 2004.:** Management Decisions: Theory and technology adoption. Design. M., 304 p.
3. **Barenblatt G.I., 2003.:** Nonlocal model of damage accumulation. Phys. Mesomech. 4, 6, 85-92.
4. **Barzilovich E.Yu., Kashtanov V.A., 1971.:** Some mathematical problems of the theory of service of complex systems. Sovetskoye radio. M., 180 p.
5. **Bortsova A.T., 1983.:** Electrorolling composition. Operation. Reliability. Repair. Transport. M., 350 p.
6. **Dodonov A.G,. 1988.:** Computing systems to solve problems quickly and organizational management. Naukova Dumka. Kiev, 213 p.
7. **Gadasin V.A., Ushakov I.A., 1975.:** Reliability of complex information-control system. Sovetskoye radio. M., 345 p.
8. **Gnedenko B.V., 1978.:** Mathematics and check of the production quality. Znanie. M., 265 p.
9. **Gnedenko B.V., Belyaev Yu.K., Soloview A.D., 1965.:** Mathematical methods in the theory of reliability. Nauka. M., 478 p.
10. **Golubenko A., 2008.:** Features of diagrams of phases and anomaly of structures of dynamic systems during degradation of their properties. TEKA. Commission of motorization and power industry in agriculture. Vol. VIII., 77-81.
11. **Ivanova V.S., 1979.:** Failure of metals. Metallurgy. M., 168 p.
12. **Khaitun S.D., 1996.:** Mechanics and irreversibility. Janus. M., 263 p.
13. **Klyuyev V.V,. 2003.:** Nondestructive testing and diagnostics. Mashinostroenie. M., 656 p.
14. **Koks D.R., Smith V.Z.,1967.:** The theory of restoration. Sovetskoye radio. M., 637 p.
15. **Marchenko D., Vetrov A., 2003.:** The increasing of wheejsets longevity Railway wheelsets. Silesian University of Technology. 59, 61-70.
16. **Nagovitsyn V.S., 2004.:** Diagnostic systems of rolling stock on the basis of information. VINITI. Moscow, 248 p.

17. **Ostreikovskaya V.A., 2005.:** Reliability Theory. High School. M., 463 p.
18. **Pakhomov E.A., 1974.:** Methods of diagnosis in the operation of locomotives. Transport. M., 40 p.
19. **Pakhomov E.A., 1985.:** Monitoring and evaluation of the technical condition of locomotives. Transport. M., 254 p.
20. **Panin V.E., 1985.:** Structural levels of deformation of solids. Nauka. M., 226 p.
21. **Reshetov D.N., 1988.:** Reliability of machines. High School. M., 235 p.
22. **Terentyev V.F., 2003.:** Fatigue of metallic materials. Nauka. M., 354 p.
23. **Thursdays V.A., 2003.:** Reliability of locomotives. Moscow route, 415 p.

ИССЛЕДОВАНИЕ КИНЕТИКИ РАЗВИТИЯ ПОВРЕЖДАЕМОСТИ

Дмитрий Марченко

Аннотация. Представлено аналитическое решение уравнения Колмогорова-Фоккера-Планка, а также анализ динамики развития трещин на основе модели диффузии дефектов.

Ключевые слова: случайный процесс, диффузия дефектов, интенсивность износа.