Research of trajectory design error of coil piling of geodesic winding

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Summary. In the article research of a geodesic trajectory design error of reinforcing material piling is considered. For the estimation of error the comparison of analytical data of different trajectories on the cylinder and cone surfaces with the data got in a mathematical model for the corresponding surfaces.is made. The analysis of reasons and character of error is done, the criterion of breaking up is offered.

Key words: geodesic winding, research of error, Ansys, setting trajectory.

INTRODUCTION

In many industries technological process of winding perfectly proved as an effective method of a morphogenesis of the shell constructions made with compositional materials (CM). Winding of the CM reinforced by fibers, allows to make not only large products practically any sizes, but also to implement the maximum indexes of physicomechanical properties of polymeric aggregates. Industrial manufacturing of products by winding is also characterized by good productivity, high automatisation and low waste.

The first operations on process modeling of winding and development of systems of controlling programs (CP) automated preparation began with modeling of a mandrel surface on an internal theoretical circuit of the product, one round on a mandrel surface, being the geodesic line, i.e., the line of the equal deviation. These issues where studied in the works of A. F. Parnyakov [1-3], M. V. Orlov [4, 5], A. K. Dobrovolsky, V. I. Kostrov [6], G. B. Evgenev, V. M. Morozov, A. N. Petukhov, J. M. Pidgayny, V. A. Dudko, D. Struve [7-8], Y. Isakov [9], Y. Y. Chikildin, V. E. Shukshunov, J. M. Alpatov [10], V. I. Zborzhevskiy [11], V. A. Grechishkin [12].

The works dealt mainly with shell-shaped body of revolution. Despite the fact that such shells are fairly common in industry, they do not exhaust the whole variety of shell elements of modern constructions.

Trajectory modeling of reinforcement material (RM) spiral winding is one of the main objectives of the preparation manufacturing of products made by winding the RM [13]. Issues of forming surface models and setting the curves for them are considered in the work of the following authors: I. Kotov [13,14], H. H. Ryzhov [15,16], Aushev [17] and others. Currently, there are software packages that allow to calculate the trajectory of the winding of complex shape products: Composite Software from the world leader MC Laren Anderson, CADWIND from company MATERIAL [18,19], as well as the software systems allowing to model the strength properties of composite products, for example Ansys Composite Prepost from company Ansys [20, 22-23]. However, the problem of modelling and research of the winding process during technical preparation of manufacturing products made of CM is relevant because of the lack of the complex approach which allows to combine the tasks of building separate RM winding turns, products' thickness design and their strength determination.

Geodesic winding - one of the most efficient and technologically advanced ways to create simple simulated products of CM by the method of winding. Geodesic winding suggest laying RM path without friction, i.e., the wind is in the position of equilibrium on the entire trajectory. One method of representation of complex bodies surfaces is the method of partitioning into finite elements (FE), which is widely used in CAD/CAM/CAE systems. In the work [Mogilniy G.A. 2012] a mathematical RM laying model on the surface of the complex bodies divided into triangular finite elements with software implementation in the package Ansys was developed. An important component of wind laying trajectory modeling is to obtain the results of calculation with the accuracy which is sufficient to implement the process of winding. The purpose of this work is to investigate the error of the mathematical model of the RM wind on the surface of the complex bodies divided into triangular finite elements.

RESEARCH OBJECT

The research of the error is suggested to be held on the example of a cylindrical body and a conical body by comparing the output of the models exit data implemented in APDL (system Ansys) with analytical calculations for the selected bodies.

To set the parameters in the model [Mogilniy G.A. 2012], the following assumptions are suggested: the mandrel is divided into the triangular FE, RM is inextensible and has infinitely small width. Any smooth body is a polyhedron. Base point for the calculation is the point belonging to wind placement in which it is convenient to specify the angle of reinforcement. The basic point is one of the key points of some arbitrary finite element, which belongs to wound surfaces.

It is known that for determining the angle of reinforcement RM wind laying on the cylinder surface the theoretical identity is true:

$$\lambda = const. \quad (1)$$

For conical bodies in any basic point it can be written as:

$$R \sin \lambda = const, \quad (2)$$

where: R – radius of the cone by a plane perpendicular to the axis of rotation at some point;

λ – the angle of reinforcement.

We assume that the angle of deviation from the theoretical reinforcement at a given point is an error of mathematical model of wind laying δ for that body.

An important stage of the study is to determine the accuracy of modelling conditions affecting its value. Therefore, besides "the fineness of the partition" (parameter of Ansys) bodies in the FE, it is necessary to study the influence of the number of partition elements, to explore the possibility of error accumulation in the modeling process of winding RM on the surface of the wound body.

Firstly, let us consider the cause of accumulated error δ.

It is known that on a cylinder the angle of reinforcement (i.e. the angle between the projection of the rotation axis and the direction which is calculated in the tangent plane) has a constant value.

However, even for a cylindrical surface for random arbitrary partitions there will be normal fluctuations in the FE, and, therefore, there will be projection axis swing. Consequently, the rotation axis projections on the FE plane will not be parallel. This will cause the oscillation of the reinforcement angle and the error of the mathematical model. Similar effects will be observed on all the surfaces of complex shape.

Secondly. There is a difference of areas of the polyhedron and the real surface, therefore, the difference of the lengths of the geodesic curve, and the broken line built on the corresponding surfaces. For example, let us consider a circular cross-section (fig. 1).

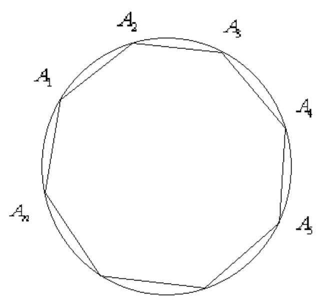

Fig.1. Section of the RM wind trajectory
$\{A_1, A_2, \ldots, A_n\}$ – Nodal points of finite elements.

We suggest the same distance between FE nodal points. Then, the ratio of the geodesic curve to the broken line will be as follows:

$$\frac{P}{C} = \frac{n \sin(\frac{180^\circ}{n})}{\pi}, \quad (3)$$

here: C - the length of a geodesic curve

P - the length of a broken line

n - the number of broken links.

The modeling process of winding includes several stages. Phase trajectory modeling is the first. Therefore, at this stage, the error should be less than 0.5%. At 36 polyhedron, the dihedral angle between the normals of two adjacent FE is 10 degrees and the length error should be less than 0.2%, which satisfies the accuracy of modelling stage.

Thirdly, we examine the nature of the error δ.

Let us consider two pairs of adjacent finite elements {(ABC); (ADB)} and {(ABC); (ACE)}, through which the wind is going (fig. 2). Element (ADB) is rotated clockwise relating the (ABC), therefore, it introduces an error with the "+" and the element (ACE) is rotated counter-clockwise relating to the (ABC), therefore, if introduces an error with the sign "-".

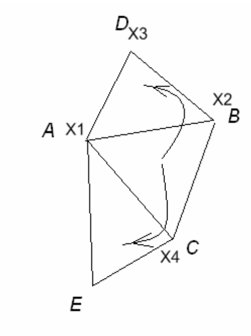

Fig. 2. Adjacent finite elements

Since the wind has a cyclic trajectory, we can assume that the wind laying error δ passing i-th element has equally probable, both the rising and descending characters. Probability of changing the angle is determined by the direction of reinforcement wind and design of finite-element partitioning. Hence, during the change of the mathematical model error δ has alternating character in case of winding the surface of complex bodies.

RESULTS OF RESEARCH

To test this model (in the Ansys) numerical experiment was performed. Cylinders with diameters of 10, 20 cm and a length of 36 cm and 180 cm were selected as the wound surfaces. The starting points are arbitrary points on the left edge of the cylinder, the end points are the points at the right edge of the cylinder. Figure 3 shows the example of one of the wind's modelling.

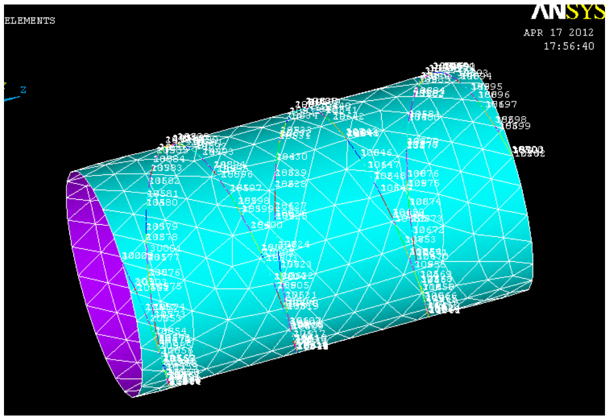

Fig. 3. RM wind laying modeling on the cylinder in Ansys

Each wind is modelled with reinforcement angles at the starting point of 30, 45, 70, 85 degrees. Large initial reinforcement angles experimentally demonstrated the greatest error of the mathematical model of wind laying δ.

For each cylinder different models of FE partitioning provided by the package Ansys were chosen. Among the main characteristics of the partition we can identify "fineness of the partition" and "partition type." Experiment was conducted on the "fineness of the partition" in the range from 0.6 cm to 0.05 cm, and various types of partitions: Smart-sized, Mapped, Sized. The lower boundary of fineness partitioning is caused by its unreasonable calculating resource taking. At each point of the wind the program stores the data on the value of the angle of reinforcement, which are compared with the theoretical ones.

Based on the data the graphs of the error of the mathematical model of the δ fineness partitioning are obtained. For example, for the fineness of the partition of the cylinder 0.3 cm long and 36cm radius of 10 cm, the starting point of 24 will get the chart in figure 4.

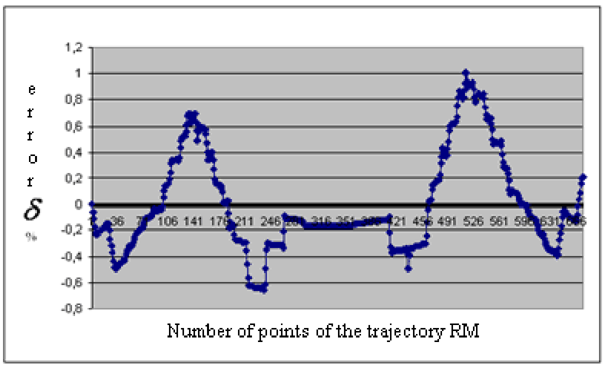

Fig. 4. Graph of error size dependence on the point RM wind trajectory on the cylinder (for δ error percentage of the reinforcement angle is taken).

The main parameter that affects the accuracy of the mathematical model is the angle between the normal's of adjacent finite elements. Therefore, for different products and partitioning models the average angles between the normals of neighboring finite elements are obtained and the values of the standard deviations of angles relating to their average values.

On the basis of all the results we get the graph of the mathematical model error of the average angle between the FE normals - figure 5.

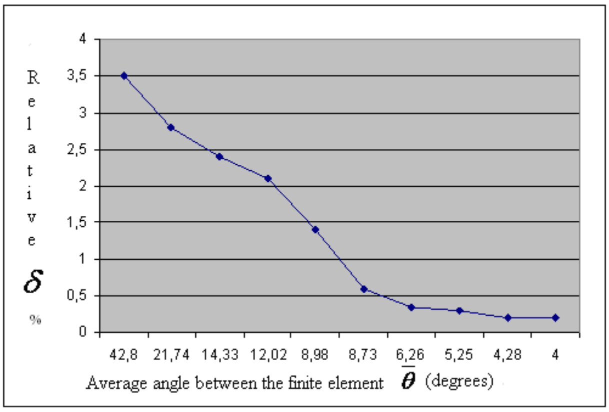

Fig. 5. Error as a function of the average angle of the finite element partitioning of the cylinder (for δ it is accepted percentage error of the angle of reinforcement).

Figure 5 shows that in order to achieve a relative error of the mathematical model of no more than 0.25% it is necessary to split cylindrical body with an average angle between the normals to FE of no more than 4 degrees. As experimental conical bodies, turned one to one cones with an apex angle of 32 degrees, a height of 35 cm and a cone with an apex angle of 67 degrees, a height of 35 cm were chosen (figure 6).

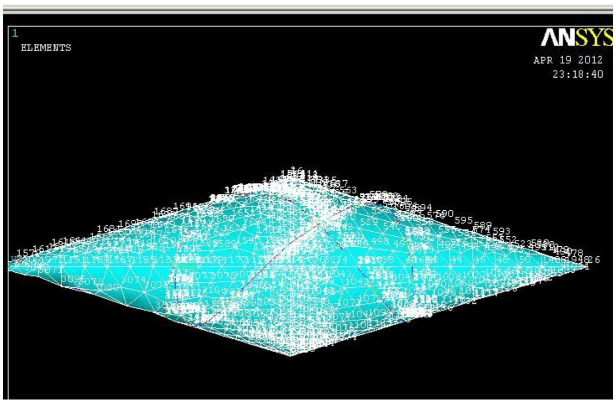

Fig. 6. Modeling of RM wind laying on the cone in Ansys

Similarly, for these conical bodies different starting points with the angle of 90 degrees of reinforcement are chosen. As a result we get the graphs of mathematical model's error δ. For example, for the "fineness partitions" 0.03 cm to 35 cm and the height of the cone apex angle of 32 degrees, the starting point of 14 will receive the chart in Figure 6. Also, for different products and models of decomposition average angle between the normals of neighboring FE and the values of the standard deviation are obtained.

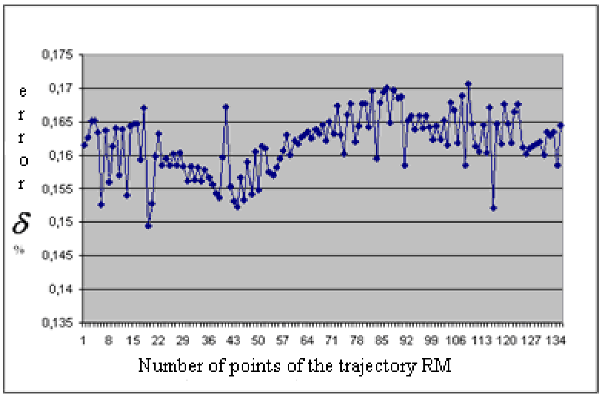

Fig. 7. Graph of dependence the error size on the wind track points of the cone (for δ it is accepted a percentage error of the angle of reinforcement).

On the basis of the received data we get the graph of dependence the mathematical model error of wind laying on the average angle between the normals of finite elements - figure 7.

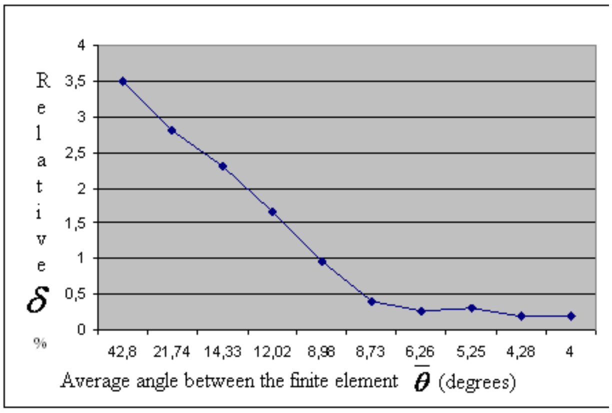

Fig. 8. Error as a function of the mean angle of the finite element partition of the cone (for δ it is accepted percentage error of the angle of reinforcement).

Figure 8 shows that, to achieve a relative error of the mathematical model of no more than 0.25% it is necessary to split conical body with an average angle between the normals of finite elements no more than 4 degrees, which corresponds to the data for the cylinder.

The data of figures 5 and 8, for the bodies of the conical and cylindrical shape are identical, they prove our theoretical assumptions and give numerical error meanings of the finite element model of the trajectory of laying RM. General view of the relationships suggests that the error of the finite element model does not depend on the shape of the simulated body. In addition, the nature of the dependences obtained suggests that a further decrease in the angle between the normals FE would increase processing power without significantly increasing the accuracy of the model.

CONCLUSIONS

Based on the experiment it is suggested regardless the form of the product for partitioning the criterion for FE (in the Ansys) on surfaces of any bodies - the angle between any two adjacent elements of the partition: $(\vec{N}_1 \hat{;} \vec{N}_2) \le 4^\circ$, where $\vec{N}_1, \vec{N}_2$ - normal corresponding elements.

Let two adjacent finite elements ABC and DBC be presented by the points A (x_1, y_1, z_1), B (x_2, y_2, z_2), C (x_3, y_3, z_3), D (x_4, y_4, z_4), then:

$$\arccos\left(\frac{|E_1E_2 + K_1K_2 + F_1F_2|}{\sqrt{E_1^2 + K_1^2 + F_1^2} * \sqrt{E_2^2 + K_2^2 + F_2^2}}\right) \le 4^\circ, \quad (8)$$

Where are $E_1, E_2, K_1, K_2, F_1, F_2$ calculated on formulas:

$$E_1 = (y_2 - y_1)*(z_3 - z_1) - (y_3 - y_1)*(z_2 - z_1)$$
$$K_1 = (x_3 - x_1)*(z_2 - z_1) - (x_2 - x_1)*(z_3 - z_1)$$
$$F_1 = (x_2 - x_1)*(y_3 - y_1) - (x_3 - x_1)*(y_2 - y_1)$$
$$E_2 = (y_2 - y_1)*(z_4 - z_1) - (y_4 - y_1)*(z_2 - z_1)$$
$$K_2 = (x_4 - x_1)*(z_2 - z_1) - (x_2 - x_1)*(z_4 - z_1)$$
$$F_2 = (x_2 - x_1)*(y_4 - y_1) - (x_4 - x_1)*(y_2 - y_1)$$

With this partitioning it is possible to simulate geodesic RM wind laying with accuracy sufficient for practical implementation of the winding process.

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ИССЛЕДОВАНИЕ ПОГРЕШНОСТИ МОДЕЛИРОВАНИЯ ТРАЕКТОРИИ УКЛАДКИ ВИТКА ГЕОДЕЗИЧЕСКОЙ НАМОТКИ

Геннадий Могильный, Владимир Донченко

Аннотация. В статье рассматривается исследование погрешности моделирования траектории намотки армирующего материала по геодезической траектории. Выполняется сравнение аналитических данных различных траекторий на поверхности цилиндра и конуса с данными полученными в математической модели для соответствующих поверхностей. Проводится анализ причин и характера погрешности, вырабатывается критерий точности.

Ключевые слова: геодезическая намотка, исследование погрешности, Ansys, траектория укладки.