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Marta Małecka[*]

GARCH PROCESS APPLICATION IN RISK VALUATION FOR WIG20 INDEX

Abstract. The recent economic crisis of 2008/2009 boosted a discussion about effectiveness of popular methods of controlling risk in financial markets, with value-at-risk approach being a topical issue. The paper contrasted a *GARCH* model for 1% VaR estimation for WIG20 with five basic approaches: variance-covariance, historical simulation, Risk Metrics™, Monte Carlo simulation and bootstrap method. A comprehensive study was supplied, with the focus on sample choice, to emphasize the influence of extraordinary price movements during the crisis. The study showed that nonparametric methods prevail over other models in the sense that the probability of exceeding the assumed loss level is the lowest. Further enquiry supported the view that *GARCH* model outperforms all techniques based on the assumption of a specific probability distribution of log returns. The problem of attaining the required level of tolerance in conditions of high instability of prices was evident from Kupiec tests results. A complementary analysis of capital requirements in relation to VaR estimation technique, gave the additional argument for *GARCH* model superiority over other risk valuation methods.
Keywords: VaR estimation, GARCH.

I. INTRODUCTION

The concept of value at risk (VaR) plays a vital role in risk management in today's financial market. Clear interpretation and the ability to express risk exposure related to many assets of different classes with one figure, with the account of diversification effect in portfolios, have decided on a prolific use of VaR. The scope of applications within controlling exposure to risk factors involves calculation of a potential loss of a institution at a given probability level, setting exposure limits and comparing risk entailed by different classes of assets. VaR popularity has been boosted substantially by the banking supervisory institutions that, since 1996, formulate their standards, recommendations or requirements in terms of VaR. Jorion [1996] emphasizes the fact of improving transparency and stability in financial markets and recons that financial institutions that go through the process of computing their VaR are forced to confront their exposure to financial risks and set up a risk management function to supervise the

[*] Assistant, Department of Statistical Methods, University of Łódź.

front and back offices. Best [2000] draws attention to the fact that VaR underlines the concept of risk adjusted performance measures.

On the other hand, criticism of VaR approach presented in literature is mainly connected with the fact of associating potential risk with one figure instead of presenting the whole cumulative density function, which initiates a discussion on a proper quantile order to present. Criticized oversimplification relates also to the fact that no information is reported as to the potential loss in case of VaR exception. Thus the use of complimentary risk measures is commonly recommended. The important shortcoming of VaR approach results from the lack of specific recommendations according estimation method, which translates into lack of comparability of VaR estimates reported by different institutions. In context of the measure theory VaR is criticized for not fulfilling the subadditivity postulate, which might be used to manipulate risk estimates by market agents.

The paper addressed the question of the recommended estimation method. The aim of the study was to provide a comparative empirical study of GARCH process application in VaR estimation for WIG20 index. The *GARCH* model was contrasted with five approaches to 1% VaR estimation: variance-covariance, historical simulation, RiskMetrics™, Monte Carlo simulation and bootstrap method. The quality of VaR predictions corresponding to different estimation methods was assessed on the basis of the failure rate with relevant testing procedure. Capital requirements calculation was presented as a complementary analysis.

II. VAR DEFINITION

Let $\left(\Omega,\mathrm{F},\mathrm{P}\right)$ be the probability space, where Ω is the space of all possible outcomes related to risk factors, F is the Borel σ-algebra of all subsets of Ω and P is the probability. The class of all subsets of risk factors values at time t generates the filtration F_t. Let $v:\mathbf{R}^k \mapsto \mathbf{R}$ be the payoff function defined on the space of risk factors. Let V_t denote the value of the payoff function: $V_t = v(P_t)$, where $P_t = \left[P_{t1}, P_{t2},\ldots,P_{tk}\right]'$ is the vector of risk factors. Let $L:\mathbf{R}^k \mapsto \mathbf{R}$ denote the loss function defined as

$$L(P_t, X_t) = v(P_t) - v(P_{t+1}), \tag{1}$$

where $P_{t+1} = P_t \cdot \exp(X_t)$, $X_t = \left[\ln\frac{P_{t+1,1}}{P_{t1}}, \ln\frac{P_{t+1,2}}{P_{t2}}, ..., \ln\frac{P_{t+1,k}}{P_{tk}} \right]'$. VaR is defined as:

$$VaR_\alpha = F_t^{-1}(1-\alpha) \tag{2}$$

where F_t is the cdf of the random variable L_t, where $L_t = L(P_t, X_t)$. The cdf F_t is often referred to as P&L cdf.

VaR is often defined as the $(1-\alpha)$ quantile of the distribution of the random variable L_t:

$$P(L_t < VaR_\alpha) = 1-\alpha \, . \tag{3}$$

If we denote by L_t^* the L_t quantile of order $(1-\alpha)$, we get:

$$VaR_\alpha = L_t^* . \tag{4}$$

By simple transformations we can express VaR in terms of the portfolio value V_t as:

$$VaR_\alpha = EV_{t+1} - V_t^* , \tag{5}$$

where V_t^* is the V_t quantile of order α, or in terms of R_t quantile of order α, where R_t is the random variable denoting log returns from V_t, with the expected value μ_t:

$$VaR_\alpha = V_t \left(\exp(\mu_t) - \exp(R_t^*) \right) . \tag{6}$$

III. DATA AND METHODOLOGY

In the paper we calculated daily VaR for long and short positions in WIG20 index. Time series of log returns from WIG20 index used in the empirical part dated form January, 1995 to September, 2011. The on-day-ahead VaR predictions were based on a rolling window out-of-sample procedure. The paper contrasted a *GARCH* model for VaR estimation with five basic approaches: variance-covariance, historical simulation, RiskMetrics™, Monte Carlo simulation and bootstrap method. The window length was set to 250 observations for all methods, with the exceptions of RiskMetrics™ and GARCH approach. The choice of the number of observations was made with the view on comparability with other studies presented in literature and according to the Basel Committee recommendations. In the RiskMetrics™ technique exponential weights decide on the influence of newest data in relation to the older market information and the effective window length depends on the choice of a decay factor λ. For the sake of comparability with other studies, the λ parameter was set to 0,94, which means that the effective number of used observations equaled 30 [Fiszeder, 2009]. In GARCH models estimation the window length was fixed at 1000 with the account of the quasi maximum likelihood estimation technique, which requires a large number of observations to obtain the satisfactory statistical properties of estimators as well as to guarantee the algorithm convergence.[1]

The level of tolerance in the study was set to 1%. In the previous literature on VaR, some authors presented the view that, due to the fat-tailed probability distribution, large mistakes are inherent in all attempts to estimate 1% quantile of financial returns processes and, in consequence, a more reasonable approach would be to rely on 5% quantile estimates. In order to guarantee greater safety of financial investment, capital requirements could be multiplied by a constant [Best, 2000]. The idea to multiply VaR estimates by a constant has already been subject to criticism for inclusion of an arbitrary factor. Moreover, for the lack of linearity in probability distribution function, VaR estimate multiplied by a constant does not correspond to any probability level. In consequence the resulting figure does not have any statistical interpretation. Finally 5% VaR estimates, while obtained with better precision, with the use of standard normality assumption, do not reflect the fat tails property of financial processes, thus their sole use in risk management may bee regarded as disputable.

According to the literature on VaR models performance, criteria used in the context of estimates assessment are still subject to a discussion. In this paper we decided to build our conclusions mainly on the basis of the failure rate, treating too high and too low VaR estimates as equally unsatisfactory. Capital requirements calculation was presented as a complementary analysis.

[1] All parametric methods were based on the normality assumption. The normality assumption was chosen as so far no consensus has been reached in literature according to the proper distribution to describe financial processes. Most common propositions are t-Student, skewed t-Student, GED, skewed generalized t-Student. The size of the paper does not allow however for a comprehensive study of all methods with all above distributions.

IV. EMPIRICAL RESULTS

The adopted rolling estimation procedure resulted in an empirical failure process for each method. The lowest number of failures was obtained with the use of historical simulation and bootstrap method, for both long and short positions. The two approaches are nonparametric in the sense that they do not adopt any assumptions about the parametric form of underlying probability distributions. That confirms the hypothesis that normal distribution does not give a good approximation of the financial time series properties in terms of the process behavior in its 1% tails. Nonparametric techniques turned out to be more effective in forecasting 1% VaR than all methods based on the normality assumption, independent of a specific model. Moreover the results showed that parametric methods systematically underestimate risk incurred by financial investment.[2] A possible solution to the problem of normality assumption failure would be adoption of a different probability distribution, which is often postulated in the literature [Cheng, Hung 2010, Łach, Weron 2000, Pipień 2006, Piontek 2002].[3] On the other hand some authors pointed out the problem of greater variance of VaR estimators corresponding to other than normal probability density function, which reflects the common problem of trade-off between variance and bias [Jorion 1996].

Among all parametric methods the lowest number of failures was achieved for the GARCH model, with the failure rate for long position being similar to that rate in variance-covariance approach. Monte Carlo and RiskMetrics™ methods turned out to be the least effective in terms of the empirical number of failures. The normality assumption adopted for the Monte Carlo study is consistent with the popular assumption that the price process in generated by a geometric, heteroskedastic Brownian motion. The same fact forms the basics of the RiskMetrics™ method, which puts the Brown process assumption in question, in context of its performance in risk valuation problem.

[2] The same conclusion was presented in the study of Bałamut [2002], who compared variance-covariance, RiskMetrics and historical simulation methods on the examples of portfolios build of instruments form Polish and American capital markets.

[3] Piontek [2002] used longest possible time series of WIG, S&P500 and DJIA indexes to show that, in contrast to 5% VaR, normality assumption in VaR estimation at 1% tolerance level produces larger failure rate than the assumed one. T-Student and GED distributions gave better results, however at 1% tolerance level even these distributions did not fully reflect fat tails effect, producing higher failure rate than the expected one.

Table 1. VaR predictions assessment, long positions

Estimation method	N	$\frac{N}{T}$	LR_{uc}	$p(LR_{uc})$	LR_{ind}	$p(LR_{ind})$
Variance-covariance	51	1,60%	9,837	0,002	176,118	0,000
Historical simulation	44	1,38%	4,184	0,041	128,442	0,000
GARCH	51	1,60%	9,837	0,002	109,035	0,000
Monte Carlo	55	1,73%	13,963	0,000	190,128	0,000
RiskMetrics™	56	1,76%	15,089	0,000	105,880	0,000
Bootstrap	45	1,41%	4,861	0,027	145,731	0,000

Source: own calculations.

Table 2. VaR predictions assessment, short positions

Estimation method	N	$\frac{N}{T}$	LR_{uc}	$p(LR_{uc})$	LR_{ind}	$p(LR_{ind})$
Variance-covariance	55	1,73%	13,963	0,000	94,929	0,001
Historical simulation	41	1,29%	2,434	0,119	101,599	0,000
GARCH	52	1,63%	10,811	0,001	52,774	0,444
Monte Carlo	58	1,82%	17,449	0,000	108,008	0,000
RiskMetrics™	65	2,04%	26,785	0,000	86,284	0,040
Bootstrap	47	1,48%	6,349	0,012	113,109	0,000

Source: Author's calculations.

Table 3. Capital requirements, long positions

Estimation method	MRC_t
Variance-covariance	0,3790
Historical simulation	0,4136
GARCH	0,3731
Monte Carlo	0,3839
RiskMetrics™	0,3647
Bootstrap	0,4116

Source: own calculations.

Table 4. Capital requirements, short positions

Estimation method	MRC_t
Variance-covariance	0,3877
Historical simulation	0,4128
GARCH	0,3680
Monte Carlo	0,3892
RiskMetrics™	0,3792
Bootstrap	0,4145

Source: own calculations.

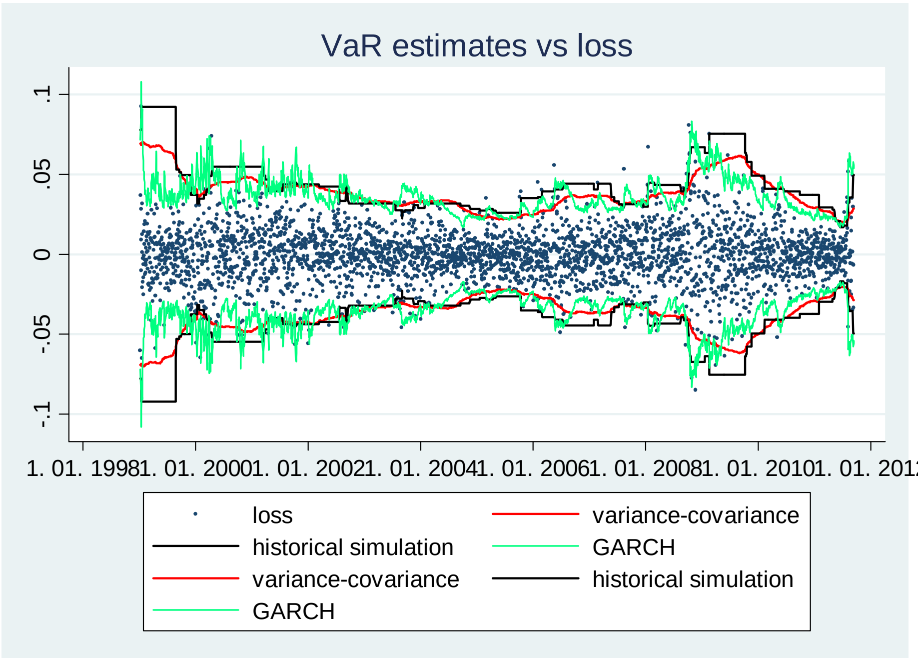

Figure 1. VaR estimates corresponding to variance-covariance, historical simulation and GARCH methods

Source: own work.

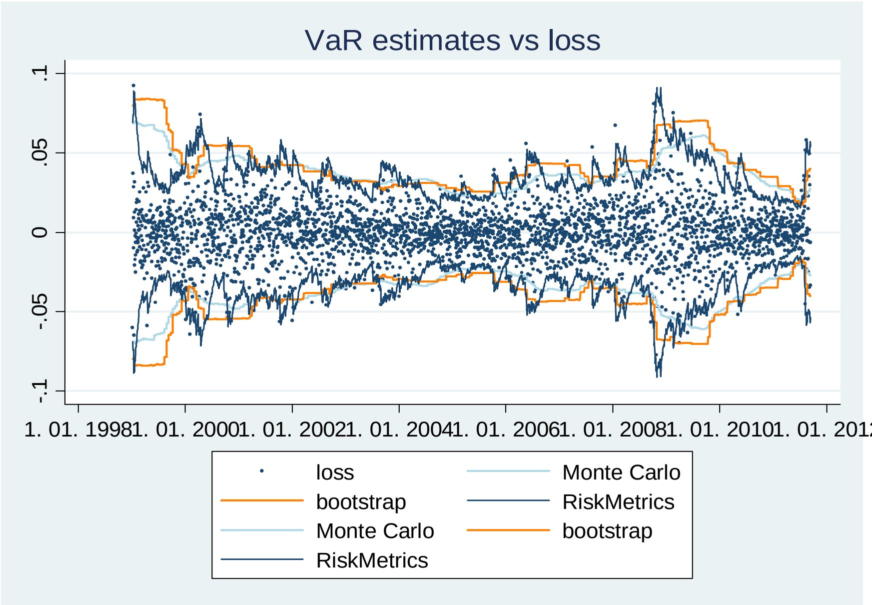

Figure 2. VaR estimates corresponding to bootstrap, MonteCarlo and RiskMetrick^{TM} methods
Source: own work.

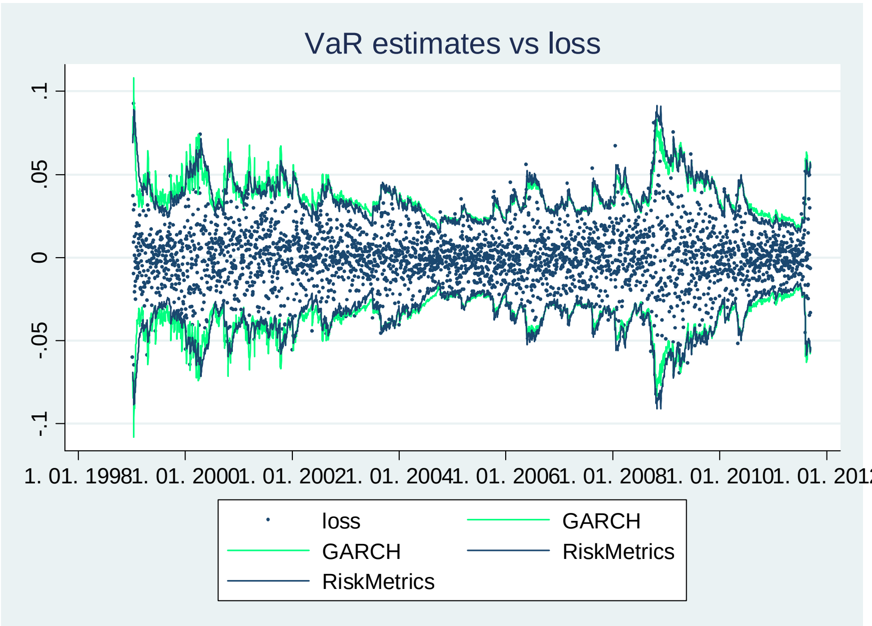

Figure 3. Comparison of VaR estimates corresponding to GARCH and RiskMetrick^{TM} methods
Source: own work.

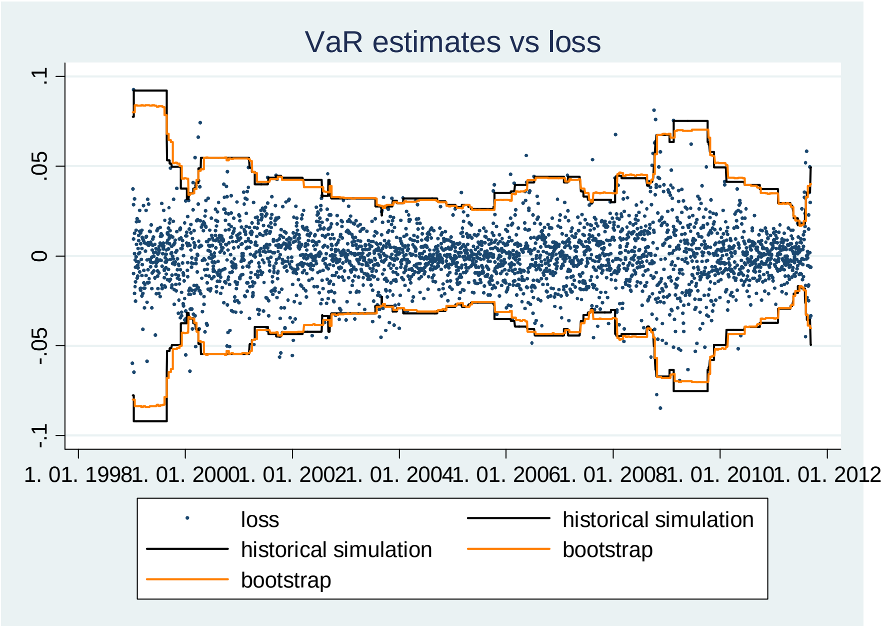

Figure 4. Comparison of VaR estimates corresponding to historical simulation and bootstrap methods

Source: own work.

The analysis of graphical presentation of quantile estimates corresponding to different estimation methods showed that historical simulation and bootstrap VaR predictions change less frequently in time but the drops and rises are sharper, whereas other methods exhibit smooth adoption to market conditions. The graphical interpretation of the outcomes justified also the conclusion that GARCH and RiskMetrics™ estimates exhibit highest dynamics in incorporating market news, which might decide on their prevalence in terms of predicting the size of potential losses in case of VaR exceptions.

In the light of the Kupiec test [1995] there was evidence that the rate of days when the real loss exceeded the estimated VaR was significantly different than 1%, independent of an adopted estimation technique, and in all cases the difference was positive. However, the unconditional coverage test showed that Kupiec test results may be distorted due to the lack of independence of VaR exceptions over time. Analysis of the results with the view on other similar studies gave the observation that conditional and unconditional tests outcomes may be attributable to the inclusion of the crisis period in the sample.

Test results showing VaR exceptions dependence in time and a positive difference between the failure rate and the assumed tolerance level are in line with similar studies in which VaR methods were examined on samples including both low and high volatility periods [Bałamut 2002, Łach and Weron 2000, Pipień 2006, Rokita 2003]. Moreover it is the common observation that RiskMetrics™ outperforms other methods in terms of unconditional coverage test [e.g. Bałamut 2002]. In the light of studies on VaR methods application to other markets than capital, the above test results do not describe general characteristics of VaR estimates, but rather present specific features of financial processes [e.g. Ganczarek, 2007, Cheng, Hung 2010].

The final stage of the enquiry was conducted with the focus on capital requirements in relation to the VaR estimation method. Lowest capital requirements were obtained with the use of GARCH and RiskMetrics™ models. Worst results in terms of minimizing capital requirements corresponded to nonparametric methods of historical simulation and bootstrapping. That supported the view that volatility clustering information is not incorporated effectively in these methods, not allowing to take advantage of low volatility periods to diminish risk estimates, which translates into capital requirements. With window length of 250, adding a new observation to the time series has little influence on the underlying empirical distribution. On the other hand the number of included observations in not subject to any estimation procedure which reduces the possibility to use all information contained in the history of the process. The results showing low failure rate at the cost of high capital requirements in historical simulation method are in line with the study presented by Bałamut [2002] with the use of portfolios built of financial instruments from Polish and American market. Among considered methods, the postulate to incorporate volatility clustering information in the model with all parameters subject to estimation process is fulfilled only in case of GARCH approach. This may determine its superiority over other methods in terms of capital requirements minimization problem and relatively good performance with reference to the failure rate.

V. CONCLUSION

The paper provided a comparative study of GARCH process application in VaR estimation for WIG20 index. The *GARCH* model for 1% VaR estimation was contrasted with five basic approaches: variance-covariance, historical simulation, Risk Metrics™, Monte Carlo simulation and bootstrap method. VaR predictions quality was assessed on the basis of the failure rate together with unconditional coverage and failure independence tests. Capital requirements calculation was presented as a complementary analysis.

The lowest number of failures was obtained with the use of historical simulation and bootstrap method, for both long and short positions. Thus nonparametric techniques turned out to be more effective in forecasting 1% VaR than all methods based on the normality assumption, independent of a specific model. The results showed that parametric methods systematically underestimate risk incurred by financial investment.

Among all parametric methods the lowest number of failures was achieved for the GARCH model, with the failure rate for long position being similar to that rate in variance-covariance approach. Monte Carlo and RiskMetrics™ methods turned out to be the least effective in terms of the empirical number of failures. The graphical interpretation of the outcomes pointed to the conclusion that GARCH and RiskMetrics™ estimates exhibit highest dynamics in incorporating market news.

The final stage of the enquiry was conducted with the focus on capital requirements in relation to the VaR estimation method. Lowest capital requirements were obtained with the use of GARCH and RiskMetrics™ models. Comparing GARCH approach to RiskMetrics™ it should be emphasized that in the latter method the number of observations in not subject to any estimation procedure which reduces the possibility to use all information contained in the history of the process. Worst results in terms of minimizing capital requirements corresponded to nonparametric methods of historical simulation and bootstrapping, which supported the conclusion that volatility clustering information is not incorporated effectively in these methods. In consequence, nonparametric methods do not allow taking advantage of low volatility periods to diminish risk estimates. That results in higher capital requirements.

Putting together all results, GARCH model outperformed all other methods in terms of capital requirements minimization problem and all parametric models in terms of failure rate. Nonparametric models, which gave lower failure rates, showed unsatisfactory dynamics in adoption to market changes. On account of high cost of capital requirements in return for low failure rate in case of nonparametric methods, GARCH model seemed superior to all presented estimation approaches. In was also the only method satisfying the postulate to incorporate volatility clustering information in the model with all parameters subject to estimation process.

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Marta Małecka

ZASTOSOWANIE PROCESÓW GARCH DO OCENY RYZYKA DLA INDEKSU WIG20

Kryzys przełomu lat 2008/2009 wywołał dyskusję dotyczącą efektywności popularnie stosowanych metod kontroli ryzyka na rynku finansowym, co w szczególności spowodowało wzrost zainteresowania metodologią VaR. W niniejszym opracowaniu przedstawione zostało porównanie metody VaR-*GARCH* do szacowania 1% VaR dla indeksu WIG20 z pięcioma innymi popularnymi podejściami: wariancji-kowariancji, symulacji historycznej, Risk Metrics™, Monte Carlo, metodą symulacyją i bootstrapową. Szczególną uwagę zwrócono na wybór próby, w celu podkreślenia wniosków specyficznych dla okresu kryzysu finansowego. Pokazano, że nieparametryczne metody przeważają nad pozostałymi w kontekście prawdopodobieństwa przekroczenia przewidywanego poziomu straty. Badanie potwierdziło hipotezę, że model *GARCH* daje lepsze rezultaty niż metody oparte na założeniu niezmiennego w czasie rozkładu logarytmicznych stóp zwrotu. Wyniki testu Kupca pokazały problem przekraczania założonego poziomu tolerancji w warunkach kryzysu. Badanie uzupełniono analizą wymogów kapitałowych w zależności od techniki estymacji VaR, co dodatkowo potwierdziło przewagę modelu *GARCH* nad innymi sposobami szacowania ryzyka.