WIEBE FUNCTION PARAMETER DETERMINATION FOR MASS FRACTION BURN CALCULATION IN AN ETHANOL-GASOLINE FUELLED SI ENGINE

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Abstract

The Mass Fraction Burn (MFB) and Heat Release Rate (HRR) reflect the amount of fuel burned and the rate of burning throughout the combustion process in an internal combustion engine. These parameters play a crucial role in research and development endeavours focused on engine efficiency, emissions, and overall operating performance. Analytically in a Spark-Ignition (SI) engine, these parameters are often modelled with the Wiebe function, a well known mass fraction burn formulation, which is a function of "a" (efficiency parameter), "m" (form factor), crank angle, and the duration of combustion. This function is a simple but powerful correlation model that is well suited for zero and one dimensional engine cycle simulations.

In this work, the Wiebe function parameters are determined over a range of fuel compositions and compression ratios by fitting the Wiebe function curve to the experimentally obtained MFB data from a single-zone HRR analysis. The Wiebe function parameters are determined using a curve fitting model by finding the minimum of a scalar function of several variables. This functionality has been built into the single-zone mass fraction burned model. Experiments with five ethanol-gasoline fuel blends: E0 (gasoline), E20, E40, E60, and E84 were conducted on a SI Cooperative Fuels Research (CFR) engine while holding a constant load of 330 kPa Net Indicated Mean Effective Pressure (Net IMEP). There were five methods introduced to fit the Wiebe function parameters, which utilized a combination of least square method and direct algebraic solution. This paper details the process used to determine the Wiebe function parameters, and compare the results obtained using these methods for the ethanol-gasoline mixture concentrations.

Keywords: *ethanol-gasoline blend, mass fraction burn, IC engine, Wiebe function*

1. Introduction

Single-Zone Model

Rassweiler and Withrow [1] developed an approximation of the mass fraction burned by calculating the ratio of the difference between the measured pressure and the polytrophic pressure to the total fuel energy. This method, known also as a single-zone heat release method, utilizes the in-cylinder pressure data to calculate the total heat release from the combustion of an air-fuel mixture in the combustion chamber, as the pressure rise over a given crank angle interval is proportional to the mass of fuel burned over that same interval. Heywood et al. [2] suggests the use of this method for SI engine simulation. Gatowski et al. [3] developed a single-zone heat release model including the crevice model, and later, Chun and Heywood [4] improved upon the single-zone model by introducing an accurate way to model the ratio of the specific heats (gamma). The method averages the gamma computed from two separate zones representing the burned and unburned masses. In the same field of study, Klein and Eriksson [5] discussed several ways to predict the value of gamma. Later Cheung and Heywood [6] concluded that the single-zone heat release model is remarkably robust, and any error most likely results from measurement errors in the pressure and mass flow rate data.

Previously a single-zone model was developed for comparison with the two-zone model using ethanol-gasoline fuel blends in a CFR engine [7]. Derived from the energy balance and the ideal gas equation, the single-zone model with two unknowns (temperature and mass fraction burn) has been proven to be as accurate as the two-zone model determination of combustion phasing. In this work, the mass fraction burn is calculated from experimental data using the single-zone model.

Wiebe Function

One approach used in engine simulation modelling is to estimate the mass fraction burned as a function of engine position using the Wiebe function [8]. The Wiebe function is expressed as:

$$x_b = \left\{ 1 - \exp\left[-a \left(\frac{\theta - \theta_o}{\Delta\theta} \right)^{m+1} \right] \right\}, \tag{1}$$

The Wiebe function curve has a characteristic S-shaped curve and is commonly used to characterize the combustion process. The mass fraction burned profile grows from zero, where zero mass fraction burn indicates the start of combustion, and then tends exponentially to one indicating the end of combustion. The difference between those two ends is known as the duration of combustion. Although the Wiebe function simple and robust in specifying the combustion process, there are inherent issues. These issues, along with a proposed solution, will be discussed in the remainder of this paper.

2. Experimental Setup

Cylinder pressure data for this research was taken from a single cylinder CFR engine manufactured by the Waukesha Motor Company. Several modifications had been made prior to this research. The modifications included relocating the sparkplug closer to the geometric center of the combustion chamber, and fabricating a custom piston which allows the compression ratio to be adjusted from 4.55:1 to 17.5:1, as opposed as the 4:1 to 10:1 with the original piston. The experiments were conducted by sweeping ethanol concentration, spark timing, and compression ratio at constant engine speed and a constant indicated load of 330 kPa Net IMEP. The cylinder pressure data was obtained with an AVL GH12D piezoelectric transducer. Data acquisition, including the measurement of cylinder pressure and various other critical pressures and temperatures, is accomplished using a combination of National Instruments (NI) hardware and software. A control system for this CFR engine had been previously developed with Mototron's Motohawk rapid engine control development environment [9]. Mototron's Mototune was used as the calibration tool and ECU interface. The calibration tool was also used to record engine control parameters such as intake manifold pressure, air flow rate, spark timing, fuel injection pressure, injection duration, equivalence ratio, etc.

3. Wiebe Function Fitting Methods

In looking closely at Equation (1), it is possible to see that the mass fraction burned never actually reaches one, but rather approaches a value of one as the exponential term asymptotically approaches zero. At a given crank angle such that equal to burn duration, the mass fraction burn is less than one, by factor of „exp (-a)". To account for this, the authors have introduced an „amplitude correction factor" „b" in the Wiebe function. Adding the amplitude correction factor, the modified Wiebe function is expressed as follows:

$$x_b = b\left\{1 - \exp\left[-a\left(\frac{\theta - \theta_o}{\Delta\theta}\right)^{m+1}\right]\right\}, \tag{2}$$

The effect of the amplitude correction factor "b" will be discussed later along with the brief description of each method.

Given the formulation in Equation (2), the three major parameters in the Wiebe function are, the combustion duration (Δθ), the start of combustion (θ_o), and the form factor (m). The „a" in the Wiebe function is directly related to the combustion duration and is not an independent parameter. For example, by defining the combustion duration as the 0% to 90% mass fraction burn duration (0-90 MFB), „a" has a fixed value of 2.3026. For a combustion duration corresponding to 0-99.9% MFB, „a" is 6.9078. For a combustion duration defined as the 0-90% MFB duration, the Wiebe function can then be written as:

$$x_b = b\left\{1 - \exp\left[-2.3026\left(\frac{\theta - \theta_o}{\Delta\theta_{0-90\%}}\right)^{m+1}\right]\right\}, \tag{3}$$

In this study, a number of possible methods were used to determine Wiebe function parameters by fitting the Wiebe function curve to the experimental data, and a summary of the five most promising methods are discuss here. The following is a brief discussion of each method.

Method 1

Using the least squares method, the Wiebe function is fitted to the mass fraction burn data with four independent variables ($\Delta\theta_{0\text{-}90\%}$, θ_o, m, and b). This method gives the best fit, as a comparison of the combustion phasing of the engine data and the Wiebe fitted curve shows a difference of less than 0.2^o crank angle for all fuel blends tested at the given operating condition. However, the start of the combustion (θ_o) is advanced by approximately 10 to 20 degrees before the actual spark timing. Fig. 1 shows the mass fraction burn curve overlaid with the Wiebe fitted curve for this method with gasoline and a spark advance of 10° BTDC. The blue line represents the MFB of the experimental data, the green line represents the Wiebe fitted curve treating the „b" as independent variable, and the red line represents the Wiebe fitted curve with „b" fixed equal to one. The two subplots at the right hand side of Fig. 1 shows the cross plots between the location of 0, 5%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 100% MFB as computed from experimental data, and the location of the same point as determined with the fitted Wiebe function. Further validation was performed by using the fitted Wiebe function to estimate the heat release, and with that information, compute the cylinder pressure during combustion. This computed cylinder pressure was then superimposed on the measured cylinder pressure, and the Net IMEP was calculated. The maximum difference in phasing, the difference in Net IMEP, and the Sum of Squared Error (SSE) of the MFB are given at the bottom of each subplot.

Method 2

Method 2 addresses the issue identified by method 1 (start of combustion advanced beyond the point of ignition) by fixing the start of combustion at the point of ignition. With the start of combustion fixed, the least squares method is again used to predict the remaining independent variables, in this case $\Delta\theta_{0\text{-}90\%}$, m, and b. Including „b" as an additional variable in the least square method improved the results, as shown in Fig. 2. The maximum difference in the combustion phasing (0-90%) between the model and the experimental data in this case does not exceed 1^o crank angle. The Net IMEP of the modelled data was within 0.02% of the Net IMEP as determined from the experimental data.

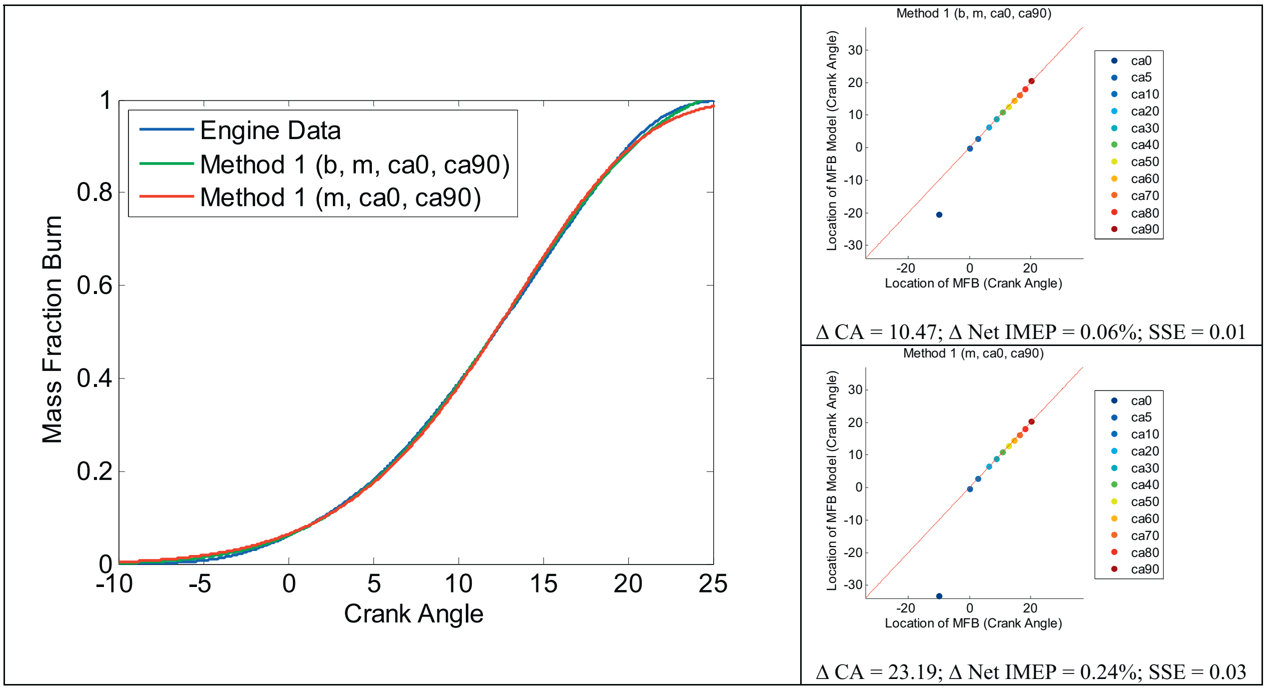

Fig. 1. The mass fraction burn and the Wiebe fitted curve using Method 1 (Gasoline, CR = 8.0:1, spark advanced = 10° BTDC, speed = 900 RPM, load = 330 kPa Net IMEP)

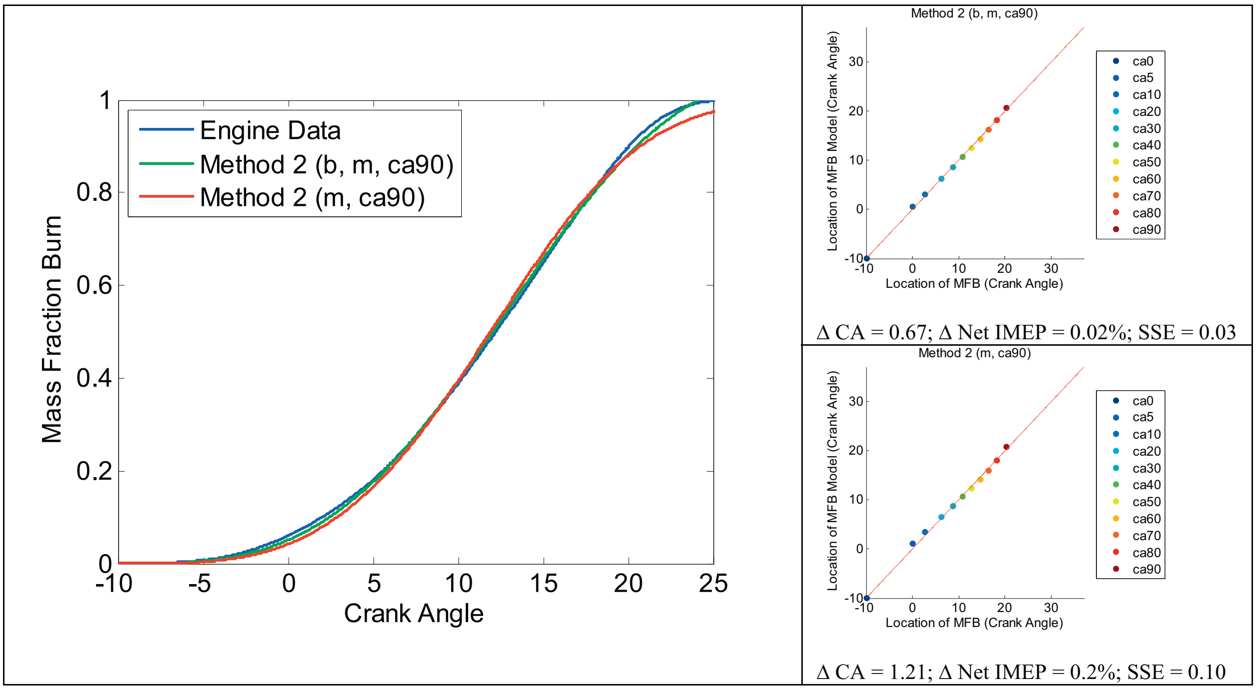

Fig. 2. The mass fraction burn and the Wiebe fitted curve using the Method 2 (Gasoline, CR = 8.0:1, spark advanced = 10° BTDC, speed = 900 RPM, load = 330 kPa Net IMEP)

Method 3

By fixing the start of combustion at the spark ignition and the duration of the combustion as the difference between the location of 90% MFB and the spark timing, the „m" and the „b" of the Wiebe function were once again determined using the least squares method. In this method, including the „b" term did not significantly change the results, as shown in Fig. 3. The modelled combustion phasing is slightly lower in the early phase (less than 1.5^o) and slightly higher during the second half of the combustion compared to the experimental data. Even though the modelled

Net IMEP did not change significantly, SSE of this method is higher than the other methods (approximately 0.3).

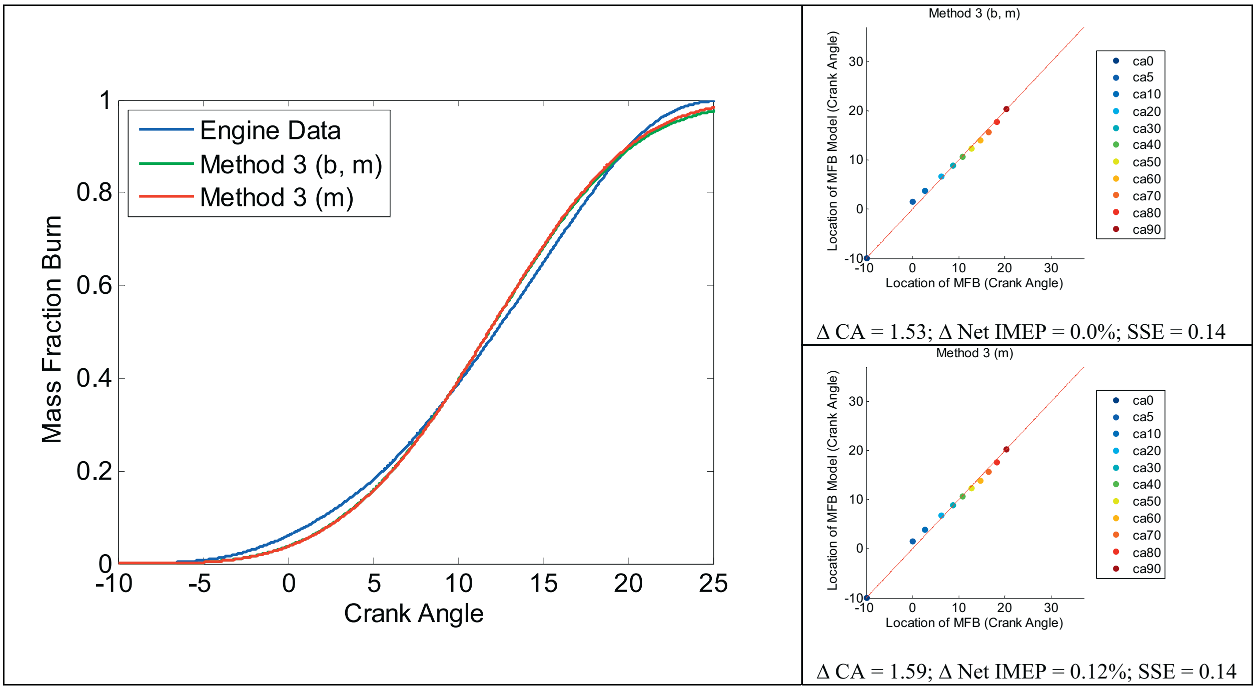

Fig. 3. The mass fraction burn and the Wiebe fitted curve using the method 3 (Gasoline, CR = 8.0:1, spark advanced = 10° BTDC, speed = 900 RPM, load = 330 kPa Net IMEP).

Method 4

This method is similar to Method 3. However, in this method, CA50 is used as a point of reference to define the duration of combustion using the analytical relation of „m" and „θ_o" as follows:

$$\Delta\theta_{0-90\%} = \frac{\theta_{50} - \theta_o}{\left(\frac{\ln(0.5)}{2.3026}\right)^{\frac{1}{m+1}}}, \tag{4}$$

Fig. 4 shows the Method 4 results, which by adding the „b" parameter on the least square method, refines the Wiebe fitted curve. The difference in the combustion phasing is approximately 0.5^o crank angle. TheSSE for this method is twice as much as for Method 2, but it is approximately one-third of the SSE obtained for Method 3.

Method 5

This method fit the mass fraction burn to the Wiebe function by defining the „m" as a function of two given points of reference. The Wiebe parameter „m" derived based on CA10 and CA90 is written as follow:

$$m = \frac{\ln\left(\frac{\ln(0.1)}{\ln(0.9)}\right)}{\ln\left(\frac{\theta_{90} - \theta_o}{\theta_{10} - \theta_o}\right)} - 1, \tag{5}$$

Fig. 5 shows the results obtained with this method by defining the „m" as a function of CA10 and CA90. The results obtained using this combined algebraic and least square method are not in a good agreement with the mass fraction burn engine data. The solution improves by using „b” as the parameter determined using the least square method and then algebraically to determine „m”.

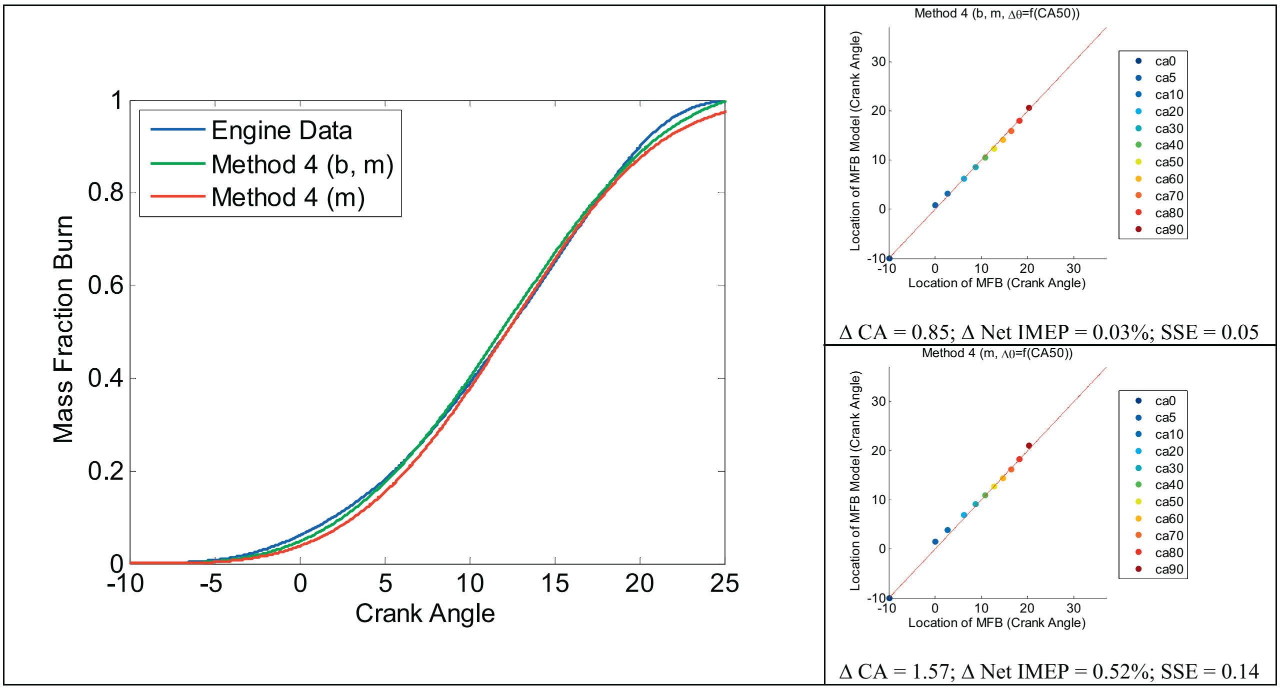

Fig. 4. The mass fraction burn and the Wiebe fitted curve using the method 4 (Gasoline, CR = 8.0:1, spark advanced = 10° BTDC, speed = 900 RPM, load = 330 kPa Net IMEP).

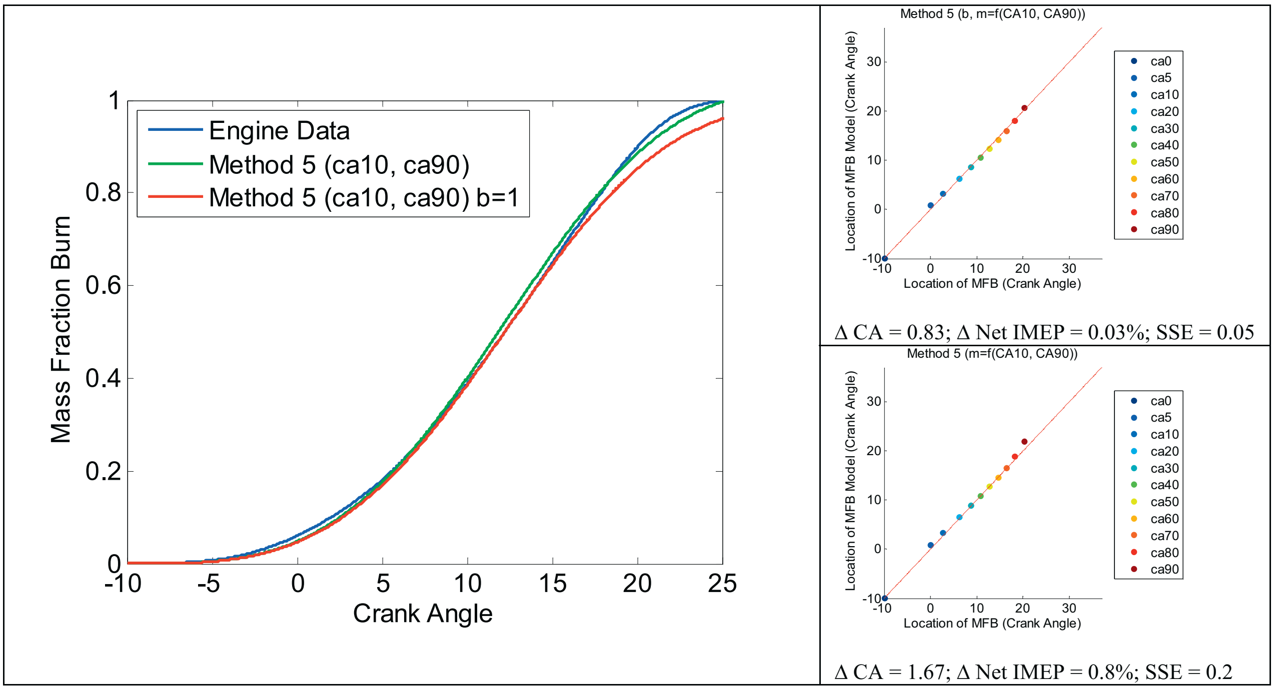

Fig. 5. The mass fraction burn and the Wiebe fitted curve using the method 5 (Gasoline, CR = 8.0:1, spark advanced = 10° BTDC, speed = 900 RPM, load = 330 kPa Net IMEP)

4. Conclusion

Five methods to fit the Wiebe function in the mass fraction burn profile have been discussed in this paper. Tab. 1 (appendix) presents a summary of the Wiebe function parameters obtained using the five methods that are briefly explain above for all the gasoline-ethanol blend data. Excluding Method 1, Method 2 using the least square method with 3 parameters (b, m, ca90) produces the best fit to the original mass fraction burn data followed by method 4 which have fixed „θ_o", and „θ_{50}", method 3 which have fixed „θ_o" and „θ_{90}", and Method 5 which algebraically solves for „m" using the „θ_o" and „θ_{90}".

The conclusions from this work:

- introducing the „b" parameter improved the fit in nearly all cases,
- the Net IMEP alone is not sufficient indicator of the fit produced by the model. The main reason for this is that the area under the mass fraction burn curve can remain the same even though the path is different. The addition of the SSE provides an addition important metric on how well the MFB curve fits the data,
- Method 2, using three parameters (b, m, ca90) of the Wiebe function, resulted in the best fit for this operating condition while matching the $\Delta\theta_{0\text{-}90\%}$.

5. Nomenclature

a the efficiency parameter of the Wiebe function,
b the amplitude correction factor of the Wiebe function,
m the form factor of the Wiebe function,
x the mass fraction burned,
θ the crank angle,
$\Delta\theta$ the combustion duration,
CA the location of crank angle.

Subscripts
b burned,
o start of combustion.

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Appendix

Tab. 1. The Wiebe function parameters using five different methods of five different ethanol concentration fuels (CR = 8.0:1, spark advanced = 10° BTDC, speed = 900 RPM, load = 330 kPa Net IMEP)

			Method 1		Method 2		Method 3		Method 4		Method 5	
	Wiebe Fitted Params.	MFB of Engine Data	LSM (b, m, ca0, ca90)	LSM (m, ca0, ca90)	LSM (b, m, ca90)	LSM (m, ca90)	LSM (b, m)	LSM (m)	LSM (b, m) & $\Delta\theta_{0-90\%}$ = f(ca50)	LSM (m) & $\Delta\theta_{0-90\%}$ = f(ca50)	m constant (ca10, ca90)	m constant (ca10, ca90) b=1
E0	a	-	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30
	m	-	4.29	6.73	2.43	2.75	2.94	2.96	2.57	2.84	2.56	2.56
	θ_0	-10.00	-20.47	-33.19	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00
	$\Delta\theta_{0-90\%}$	30.23	42.04	53.61	33.10	30.90	30.23	30.23	31.86	31.10	30.23	31.90
	b	-	1.04	1.00	1.08	1.00	0.99	1.00	1.04	1.00	1.04	1.00
	SSE	-	0.01	0.03	0.03	0.10	0.14	0.14	0.05	0.14	0.05	0.20
	Net IMEP (%)	-	-0.06	-0.24	0.02	-0.20	0.00	0.12	0.03	-0.52	0.03	-0.80
	Max $\Delta\theta$ Diff	-	10.47	23.19	0.67	1.21	1.53	1.59	0.85	1.57	0.83	1.67
E20	a	-	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30
	m	-	5.41	9.00	2.57	2.95	3.15	3.19	2.74	3.05	2.70	2.70
	θ_0	-10.00	-24.80	-42.53	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00
	$\Delta\theta_{0-90\%}$	28.82	44.79	61.51	31.77	29.47	28.82	28.82	30.41	29.67	28.82	30.51
	b	-	1.04	1.00	1.10	1.00	0.99	1.00	1.04	1.00	1.05	1.00
	SSE	-	0.01	0.02	0.03	0.11	0.15	0.15	0.05	0.15	0.06	0.22
	Net IMEP (%)	-	-0.07	-0.21	0.02	-0.18	0.00	0.13	0.03	-0.49	0.03	-0.77
	Max $\Delta\theta$ Diff	-	14.80	32.53	0.90	1.48	1.80	1.86	1.12	1.85	1.03	1.69
E40	a	-	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30
	m	-	4.75	7.15	2.59	2.90	3.08	3.10	2.72	2.99	2.69	2.69
	θ_0	-10.00	-21.18	-32.70	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00
	$\Delta\theta_{0-90\%}$	28.53	40.73	51.36	30.91	29.08	28.53	28.53	29.90	29.25	28.53	29.96
	b	-	1.03	1.00	1.08	1.00	0.99	1.00	1.04	1.00	1.04	1.00
	SSE	-	0.01	0.02	0.03	0.08	0.11	0.11	0.04	0.11	0.04	0.17
	Net IMEP (%)	-	-0.05	-0.18	0.02	-0.17	0.00	0.09	0.03	-0.42	0.03	-0.67
	Max $\Delta\theta$ Diff	-	11.18	22.70	0.79	1.27	1.54	1.58	0.94	1.57	0.89	1.43
E60	a	-	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30
	m	-	4.79	7.60	2.57	2.91	3.10	3.13	2.72	3.00	2.70	2.70
	θ_0	-10.00	-21.36	-34.81	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00
	$\Delta\theta_{0-90\%}$	28.26	40.77	53.23	30.88	28.86	28.26	28.26	29.71	29.04	28.26	29.76
	b	-	1.04	1.00	1.09	1.00	0.99	1.00	1.04	1.00	1.04	1.00
	SSE	-	0.01	0.02	0.03	0.09	0.12	0.13	0.04	0.12	0.04	0.18
	Net IMEP (%)	-	-0.05	-0.19	0.02	-0.17	0.00	0.12	0.03	-0.43	0.03	-0.68
	Max $\Delta\theta$ Diff	-	11.36	24.81	0.74	1.26	1.55	1.61	0.93	1.58	0.89	1.50
E84	a	-	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30
	m	-	5.05	8.51	2.46	2.84	3.04	3.08	2.65	2.94	2.60	2.60
	θ_0	-10.00	-23.38	-40.20	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00
	$\Delta\theta_{0-90\%}$	28.03	42.65	58.38	31.09	28.68	28.03	28.03	29.60	28.87	28.03	29.71
	b	-	1.04	1.00	1.10	1.00	0.99	1.00	1.04	1.00	1.04	1.00
	SSE	-	0.01	0.02	0.03	0.11	0.15	0.15	0.06	0.15	0.06	0.22
	Net IMEP (%)	-	-0.07	-0.22	0.03	-0.17	0.01	0.13	0.03	-0.45	0.04	-0.73
	Max $\Delta\theta$ Diff	-	13.38	30.20	0.93	1.51	1.82	1.89	1.16	1.86	1.07	1.68