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MODELLING AND SIMULATION OF POWER TRANSMISSION SYSTEM ORIENTED ON DIAGNOSIS OF FAILURES IN TOOTHED GEAR

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Abstract

In the article are presented chosen research results of influence of local defects in toothed gear on the vibroacoustic signal. Vibroacoustic methods of diagnostics are perfectly suitable to the task of monitoring the status of devices during their normal work. Monitoring of vibrations during operation is particular importance for industrial toothed gear as well gears working in the power transmission systems of aircrafts. Taken into account negative consequences of damages in toothed gears, determination of changes in vibroacoustic signal can prevent possible failures, and is therefore very important.

Researches were performed with use residual signals created from synchronously averaged time signals generated by identified dynamic model of toothed gear working in circulating power system. Superior purpose of work is development of effectively methods of damage detection, especially in its early stages, what has great importance during operating aircrafts. The study shows that local damage of the wheels affects changes in the frequency distribution of vibroacoustic signals energy. The degree of damage significantly impacts to the possibility of detecting it on the basis of time signals analyses.

Keywords: *power transmission system, toothed gear, dynamic model, local damages*

1. Introduction

Toothed gears find wide application in aircraft and serve many purposes in aircraft construction. They are present not only in power transmission systems (for example in helicopters), where perform very significant function, but also in many assemblies. In each application, especially which have influence to the safety during flight, for prevention of unexpected air events, important meaning has correct determination of technical state of toothed gear.

The power transmission system with toothed gear from dynamics point of view is a set of inertial and elastic elements, which have a tendency to vibration. During work of gear, for load of the wheel teeth, besides the steady transmitted power, highly influences have dynamic surpluses. They result mainly from the effect of factors related to the same gear, as well as factors related with the rest of the power transmission system. Among the first, it should be noted inaccuracy of teeth, its elastic deformation, deformation of shafts etc. The second include extortion derived mainly from the driven equipment and the nature of the work and from the driving equipment – usually the drive motor.

Calculation of load from the steady transmitted power is not complicated, but many troubles make calculations of dynamic surpluses. In dynamics of gears, internal factors of dynamic surpluses are very important but more often are taken into account also external sources, which generate low frequency vibrations and thus having less importance.

To determine the effect of each factor on the generation of mechanical vibrations, and therefore the disturbance of torque, growth of noise and wear intensity of the system, they are often carried out for the same purpose, computer simulations and experimental laboratory tests. In the first case, it is necessary to develop a mathematical model and then identify its parameters, in the second – it is necessary test stand with measuring equipment.

Opportunities and benefits of computer simulations, combined with the continuous development of hardware and software computational techniques, make this way of realization research increasingly important and make from simulation programs a fundamental tool in the task of the optimal solution search [5].

The development of computational techniques allows to take into account in models new factors, by which the results of calculations can be more consistent with those that can be obtained on test stands, however, at significantly different, in favour of the first, financial expenses. The undoubted advantage of the simulation is also their repeatability and ability to perform simulations of damages, which not only would be too costly to implement in laboratory conditions, but also would impose too big a security risk.

The aim of this study is to present the computational possibilities of developed model to simulate work of toothed gear with local damage of tooth. Article presents also the impact of these simulated defects on the vibroacoustic signal, which is a good carrier of information about the technical sate. Especially interesting are changes in the noise and vibration signals from toothed gears with progressive wear [13] or with damages in the earliest stage of their occurrence [6].

2. The dynamics of toothed gear

Vibroacoustic phenomena in toothed gears have many sources. The main internal factors affecting the vibration and noise include changing the stiffness of the teeth on the line of tooth contact, impacts of the teeth at the entrance in mesh, changes of frictional force between the teeth due to slippage and insufficient lubrication. To the level of vibration and noise, also affect damages and wear of toothed gear elements. External factors affecting the vibration and noise are in the form of mechanical and acoustic extortions. Combined effect of internal and external factors is responsible for the overall level of vibrations and generated noise. Ideally, a single-stage gear having parallel axes, have in vibroacoustic signals only the meshing frequency and its harmonics:

$$x(t) = \sum_{k=1}^{K} A_k \cdot \cos\left(2 \cdot \pi \cdot k \cdot f_z \cdot t + \varphi_k\right), \tag{1}$$

where:
k – meshing frequency harmonic, $k = 1, 2, 3, \ldots, K$,
A_k – amplitude of harmonic of meshing frequency,
f_z – meshing frequency,
t – time,
φ_k – start phase of harmonic of meshing frequency.

In the real gear, power of vibroacoustic signal is mainly contained in a band of meshing frequency and due to the presence of involute errors in the bands of its first's harmonics.

In the spectrum are also visible shafts rotation frequencies and their harmonics, resulting among others the eccentric mounting or unbalance of wheels. They are visible in the low-frequency part of the spectrum. However, they will typically rise as high-frequency components in the form of sidebands resulting from the modulation of meshing frequency and its harmonics, in accordance with the following relation:

$$f = k \cdot f_z \pm k_{01} \cdot f_{01} \pm k_{02} \cdot f_{02}, \tag{2}$$

where:
k_{01}, k_{02} – harmonics of shafts rotation frequencies, k_{01}, k_{02} = 1, 2, 3, …,
f_{01} – rotation frequency of the pinion shaft,
f_{02} – rotation frequency of the wheel shaft.

With value of transverse contact ratio ε_α are connected contact frequency f_p and one-pair contact frequency f_{p1}. The step changes in the number of teeth found in the mesh is described by relations:

$$f_p = \frac{1}{T_p} = \frac{f_z}{\varepsilon_\alpha}, \tag{3}$$

$$f_{p1} = \frac{f_z}{2 - \varepsilon_\alpha}. \tag{4}$$

The spectrum of the toothed gears contains a number of harmonic frequencies associated with the cycle of teeth associations. The same pair of teeth of the pinion and the wheel comes into mesh at each z_2/z_p pinion rotation and z_1/z_p wheel rotation, where z_p is the greatest common divisor of the numbers of teeth of the pinion z_1 and wheel z_2. Assuming that i = 1, 2, 3, ..., frequency of meshing repetition f_{pz} and its harmonics can be described by formula:

$$f_{pz} = z_p \cdot \frac{i \cdot f_{01}}{z_2} = z_p \cdot \frac{i \cdot f_{02}}{z_1}. \tag{5}$$

In addition to the above-mentioned spectral components, in the toothed gear are present a number of design and operating factors, forcing vibrations that can modulate the amplitude and the frequency of signal generated in the mesh. Mention may be made at this point different deviations, the working surface wear and local damages of teeth. The lowest possible frequency f_{min} in this case would be:

$$f_{\min} = \frac{n_{1,2}}{60 \cdot z_n}, \tag{6}$$

where z_n – the least common multiple of the number of teeth of the pinion and gear.

The source of the vibration spectral components at high frequencies is irregularities working surface of the teeth.

The research presented in [1] shown, that an additional source of modulation are also rolling bearings. The causes of vibrations generated by the bearings can be divided into design, manufacturing and operating. The first group includes a variable stiffness bearing resulting from step changes of the number of rolling elements in the load-bearing zone. This is confirmed by the studies presented, among others in [12]. The production factors include the variations of shape and dimensions obtained during the production and assembly errors. Among the operational factors should be mentioned occurring in the elements bearing wear processes.

A dynamic effect in mesh is modulated by the serial combined impact of rolling bearings and is cause vibrations of housing of toothed gear. The housing has a complicated resonant structure and is the main emitter of the noise. Convolution of signal from the mesh and the transfer function from the source to the measure point is the basis for assessing the vibroactivity of toothed gear [2, 17].

Vibrations and noise is a good carrier of information about the status of the device and can be used in the vibroacoustic diagnosis. Especially interesting in this context are the works aimed at relationship of changes in the noise and vibration signals with progressive wear [13] or with damages in the earliest stage of their occurrence [6].

Vibroacoustic methods are perfectly suitable to the task of monitoring the status of devices during their normal work, what has a special advantage – do not need downtime of observed device, what usually involves some financial losses. Monitoring during operation is particular importance for industrial toothed gear as well gears working in the power transmission systems of aircrafts. In both situations, it helps prevent possible failure, which could carry very important negative consequences.

3. Dynamic model of power transmission system with toothed gear

The problem of modelling gear, although not new, is still relevant especially in from the point of view of increasing computing capabilities and requirements in relation to the quality of the developed models and validity of the results obtained from their use. In recent decades many different models of isolated toothed gear, as well as models of power transmission system with toothed gear have been developed [2, 8, 18]. Adopted schools of thought, determined the complexity and accuracy of the models, and thus their usefulness. Anyone, even a very powerful model has some limitations, which is not always fit for use in a specific task.

Based on the number of publications it was decided to build a new dynamic model of test stand with gear working in circulating power system, which is an example of a more complex system. Model includes, among others, two gears, drive motor and the shafts connecting the individual elements of the test stand. Research in this test stand can be realized at different speeds and loads.

For the development model, this test stand had chosen first of all the possibility of its identification at real stands and the ability to carry on its wide range of research that can then be repeated in the form of simulation. Comparison of the results research carried out experimentally and by simulation provides the opportunity to verify the accuracy of this model and determine the scope of its usefulness for different tests.

During the development model was assumed, that after his tuning to the real objects and after verification, it would be used to the research different toothed gears. The model allows for research-toothed gear with spur or helical wheels with unmodified or modified profile as well gears with HCR wheels (High Contact Ratio). HCR wheels have transverse contact ratio ε_α equal or greater than 2.

To simplify the model assumes that the gears interact with each other only by torsional vibrations. Simplification made it possible to introduce independent coordinate systems associated with each toothed gear. In each arrangement:

- the *x* axis has direction of the meshing force,
- *y* axis has direction of meshing friction force,
- *z* axis coincides with the direction of the axis of the test stand shafts.
- The final version of the model (Fig. 1) takes into account [9, 11, 14]:
- angular displacement of rotor, couplings, pinion and wheel of both toothed gear,
- radial displacement in bearings caused by meshing and friction forces in the mesh,
- electro-mechanical characteristics of the drive asynchronous motor powered with an PWM inverter or, in the simplified version of model, only mechanical characteristics of motor,
- moments of inertia of the rotor, couplings and wheels,
- mass of wheels and equivalent mass of bearings,
- variable stiffness of mesh along the line of tooth contact,
- damping in mesh caused by the action of the oil,
- wheel manufacturing deviations (according to PN-ISO 1328:2000),
- a non-linear stiffness of the rolling bearing, in the most advanced version of the model taking into account also the variability depending on the number and position of the rolling elements relative to the direction of the force,

– friction and damping in the bearings determined during model identification [10].

Pinions, gears and clutches are modelled as rigid bodies with known moments of inertia, combined with elements with stiffness and damping. The masses of other elements of the power transmission system have been reduced to the equivalent masses concentrated in the bearings. Vibration excitation of mass elements is the result of complex interactions, both internal sources of vibration, such as variable mesh stiffness, damping in mesh and deviations and external, resulting from the changes of engine speed and/or torque of shafts.

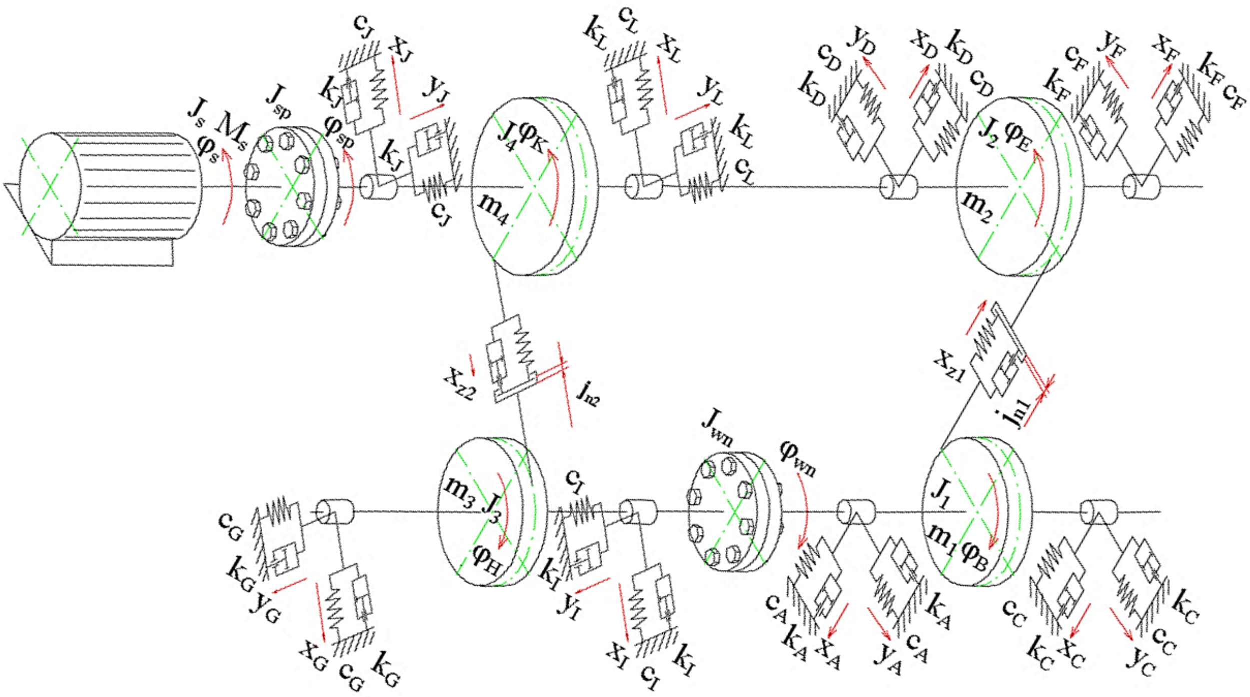

Fig. 1. Dynamic model of test stand with gear working in circulating power system [14]

4. Modelling of local damages of gears

Mesh has been modelled according to the concept of Müller spatial model in the form of spring palisade moving between the two contact surfaces. This approach permits mainly to simulate toothed gears with spur or helical wheels with unmodified or modified profile, and also to take into account deviations of teeth. In addition, it allows for the wear simulation of working surfaces of the teeth. It is also possible to include within simulations some types of local damages.

According to this concept, meshing force is the sum of the forces transmitted by each pair of teeth located in the mesh. Knowledge of teeth cooperation coordinates on the tooth contact line allows calculation of the stiffness of a single pair of teeth and allows including many different deviations and depth potential modification of the tooth.

Meshing force transmitted through a single pair of teeth is calculated as a product of the deflection of the pair of teeth and stiffness, which is variable along the line of tooth contact. For the determination of the mesh stiffness of one pair teeth from the different methods, it was decided to apply the model method described in [2], adapted by Müller from research Niemann and Baethge.

Developed dynamic model has been enhanced with the ability to simulate local damages of wheel's teeth accordance to the methods presented in [7]. Simulations of damages allow development of diagnostic algorithms that can be used for diagnostics various defects that may occur in the power transmission system.

The wheels in the model are presented as a package of small width elementary wheels. If on the selected section does not take place a cooperation of teeth (due damage), the force is equal 0. Partial breaking of the tooth was modelled by reducing the length of contact line, which translates into a reset elementary stiffness of the gear meshing at a time when there should be a cooperation of the tooth, if no failure has occurred. In the case of breaking of the tip of the driving wheel will cause a premature exit pair of teeth from contact and in the case of the driven wheel – delays the start of cooperation. The crack at the base of the tooth was modelled by reducing the stiffness of the pair of teeth.

5. Research object

The simulation research was carried out for gear in test stand with geometric parameters listed in Tab. 1. Pinion shaft rotational speed was ~1120 rpm, the static torque of the pinion shaft was $M = 493$ Nm (unit load was $Q = 3.5$ MPa). Random pitch deviation for pinion and wheel amounted to 4.5 microns (that corresponding to 28% of static deformation u_{stat}), while periodic deviations to 7 microns (44% u_{stat}).

Tab. 1. Parameters of wheels in tested toothed gear

Number of pinion teeth, z_1	19
Number of gear teeth, z_2	30
Module, m_n	3.5 mm
Pinion profile shift coefficient, x_1	0.500
Wheel profile shift coefficient, x_2	0.295
Transverse pressure angle, α_0	20º
Axis distance, a_w	91.5 mm
Gear mesh width, b	56 mm
Transverse contact ratio, ε_α	1.36

Calculations were made for two cases of damage. It was simulated work of toothed gear:
- with partial crack of the pinion (the width reduction 25 and 40% of the total wheel width),
- with chipped tooth of pinion – addendum was shortened successively at 0.5 and 1.5 mm what causing local reduction of ε_α from 1.36 to 1.11. Length of teeth contact line was shortened by less than 20%.
-

6. Results of sample simulations

The results of simulations – residual vibration velocity signals at the nodes bearing for pinion and gear shaft was analysed with use time-frequency methods. For analyses were used i.e. Wigner – Ville transform with Choi-Williams filter window, which results are below presented.

Wigner – Ville transform (WVD) is described by the formula:

$$WVD(f,t) = \int x^*(t-\frac{\tau}{2})x(t+\frac{\tau}{2})e^{-j2\pi ft}\, e^{-\left(\frac{f\tau}{\sigma}\right)^2} d\tau \ , \qquad (6)$$

where:

$x^*(t)$ – an imaginary signal conjugated with $x(t)$,

t – shift in time domain,

f – shift in frequency domain.

Effects of the partially breaking the teeth of the pinion are presented in Fig. 2. Changes are clearly visible in both presented cases; arrows show disturbances in WV distributions. They were caused due cooperation gear tooth without faults with partial broken pinion tooth. In subsequent studies, it planned to carry out simulations with the much smaller size of damage and higher value of random deviations of pinion and wheel.

Effects of pinion tooth chipping in two examined stages are presented on Fig. 3. The results show a small increase of the amplitudes of Wigner – Ville distribution, but the moment of cooperation of the damaged tooth is visible when the value of chipping was equal 1.5 mm. In first examined situation, cooperation is not clearly visible. It is necessary to use additional signals analyses that allow detection of damage at this stage of its development.

In both cases of damages, meaningful changes are visible in frequency range 500-1500 Hz for tested toothed gear. Meshing frequency f_z was ~ 900 Hz.

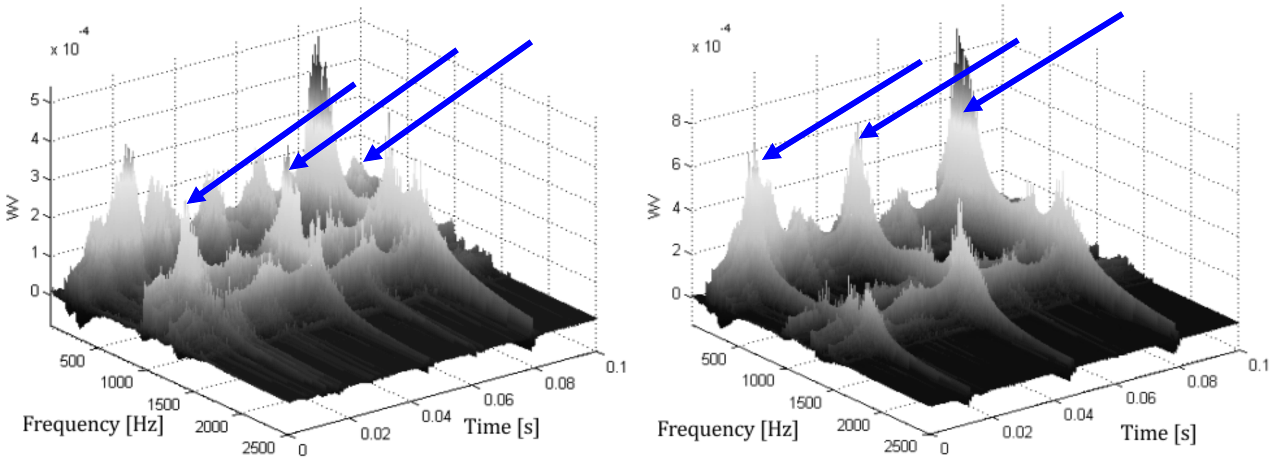

Fig. 2. Wigner-Ville distribution of residual signal of linear velocities vibrations of the pinion shaft – gear with partial break of the pinion – 25 and 40 % of the total wheel width

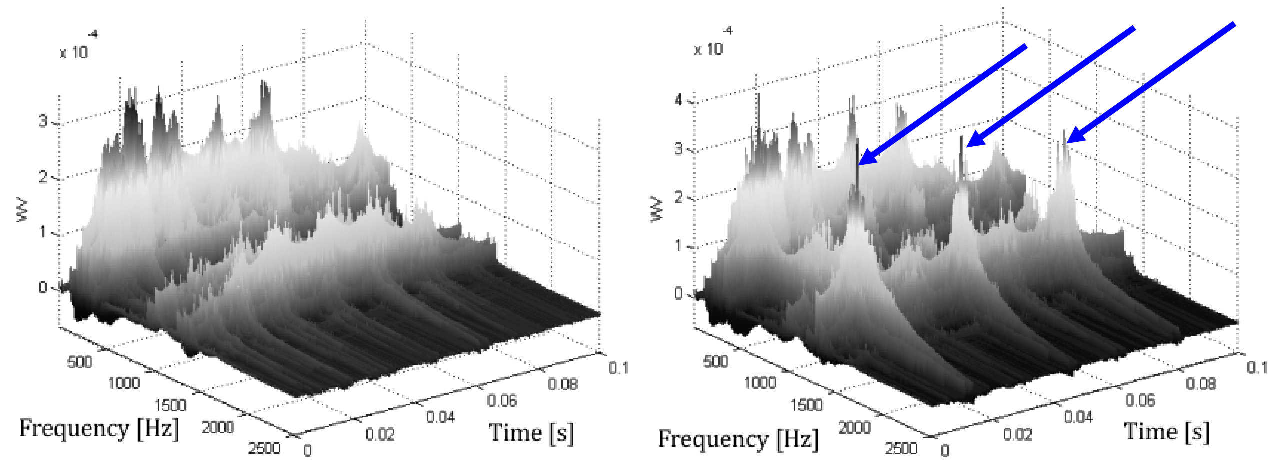

Fig. 3. Wigner-Ville distribution of residual signal of linear velocities vibrations of the pinion shaft – gear with chipped tooth of pinion, addendum was shortened at 0.5 and 1.5 mm

7. Conclusions

The study shows that local damage of the wheels affects changes in the frequency distribution of energy of vibroacoustic signals. The degree of damage significantly influences to the possibility of detecting it on the basis of analyses time signals.

In order to detect defects at early stages, it is necessary to use more advanced time-frequency methods [15, 16], as presented in the study Wigner–Ville analysis. Presented analyses were performed with use residual signals created from synchronously averaged signals. The aim of creation synchronously averaged signals was further increase the sensitivity of damage detection.

Various mesh deviations cause masking effects of the damage, perfectly it is visible in the case of impact tooth chipping, and when addendum was shortened at 0.5 mm. Due some mesh deviations, detection of small defects may be impossible.

In a further stage of the research is planned to determine the frequency band and range of scales in which changes are most important, and define measures sensitive to the presence of defects in their various stages. It is also foreseen to verify the effectiveness of other methods of analysis of signals.

It has appeared from laboratory tests, that presented dynamic model allows for correct mapping of dynamic phenomena caused by teeth damages. Further development of the model allowed simulation of bearing failures, what increases the field of its application. With described model is possible development of diagnostic algorithms, which after verification on real objects in they work conditions, can be used to improve safety of industrial toothed gears or toothed gears mounted in aircrafts. This was the first purpose of presented research.

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