

ON THE STRUCTURE OF JORDAN *-DERIVATIONS

BY

MATEJ BREŠAR (MARIBOR) AND BORUT ZALAR (LJUBLJANA)

1. Introduction. Let R be a *-ring, i.e., a ring with involution $*$. An additive mapping E from R to R is called a *Jordan *-derivation* if

$$E(x^2) = E(x)x^* + xE(x) \quad \text{for all } x \in R.$$

Note that the mapping $x \rightarrow ax^* - xa$, where a is a fixed element in R , is a Jordan *-derivation; such Jordan *-derivations are said to be *inner*.

The study of Jordan *-derivations has been motivated by the problem of the representativity of quadratic forms by bilinear forms (for the results concerning this problem we refer to [8, 12, 14–16]). It turns out that the question whether each quadratic form can be represented by some bilinear form is intimately connected with the question whether every Jordan *-derivation is inner, as shown by Šemrl [14].

In [4] Brešar and Vukman studied some algebraic properties of Jordan *-derivations. As a special case of [4; Theorem 1] we have that every Jordan *-derivation of a complex algebra A with unit element is inner. Clearly, the requirement that A must contain the unit element cannot be omitted—for example, if A is a self-adjoint ideal in an algebra B , then the mapping $x \rightarrow bx^* - xb$, where $b \in B$, is a Jordan *-derivation of A which is not necessarily inner. In this paper we prove that Jordan *-derivations of a rather wide class of complex *-algebras (in general without unit) can be represented by double centralizers (Theorem 2.1). As an application we obtain a result on automatic continuity of Jordan *-derivations (Corollary 2.3). As another application we determine the structure of Jordan *-derivations on the algebra of all compact linear operators on a complex Hilbert space (Corollary 2.4).

Roughly speaking, it is much more difficult to study Jordan *-derivations on real algebras than on complex algebras. Nevertheless, in [13] Šemrl showed that every Jordan *-derivation of $B(H)$, the algebra of all bounded linear operators on a real Hilbert space H ($\dim H > 1$), is inner. In the present paper, using a completely different approach, we give a new proof of this result. Our proof is based on two well-known results. The first is from algebra (due to Martindale, concerning Jordan derivations of the symmetric

elements of a $*$ -ring), while the second is from analysis (due to Chernoff, stating that all derivations on $B(H)$ are inner). In fact, throughout this paper we combine algebraic and analytic methods.

2. Jordan $*$ -derivations of complex $*$ -algebras. Let A be an algebra (resp. a ring). A linear (resp. additive) mapping T from A to A is called a *left centralizer* of A if $T(xy) = T(x)y$ for all $x, y \in A$. Analogously, a linear (resp. additive) mapping S from A to A satisfying $S(xy) = xS(y)$ for all $x, y \in A$ is called a *right centralizer* of A . For T a left centralizer of A and S a right centralizer of A , the pair (S, T) is called a *double centralizer* of A if $xT(y) = S(x)y$ for all $x, y \in A$.

Let A be a $*$ -ring. Note that every double centralizer (S, T) of A induces a Jordan $*$ -derivation E , defined by $E(x) = T(x^*) - S(x)$. In the following theorem we show that in certain complex $*$ -algebras all Jordan $*$ -derivations are induced in such a way.

THEOREM 2.1. *Let A be a complex $*$ -algebra such that $Aa = 0$ or $aA = 0$ (where $a \in A$) implies $a = 0$. If E is a Jordan $*$ -derivation of A then there exists a unique double centralizer (T, S) such that $E(x) = T(x^*) - S(x)$ for all $x \in A$.*

Obviously, as a special case of Theorem 2.1 we obtain the known result stating that all Jordan $*$ -derivations of a complex $*$ -algebra with unit are inner.

Proof of Theorem 2.1. Define an additive mapping S_1 of A by $S_1(x) = 2E(ix) + 2iE(x)$. We have

$$\begin{aligned} S_1(x^2) - xS_1(x) &= 2E(ix^2) + 2iE(x^2) - 2xE(ix) - 2ixE(x) \\ &= E((1+i)^2x^2) + 2iE(x)x^* + 2ixE(x) - 2xE(ix) - 2ixE(x) \\ &= E(x+ix)(x^* - ix^*) + (x+ix)E(x+ix) + 2iE(x)x^* - 2xE(ix) \\ &= \{E(x)x^* + xE(x) - iE(ix)x^* + ixE(ix)\} \\ &\quad + i\{E(x)x^* + xE(x) - iE(ix)x^* + ixE(ix)\} \end{aligned}$$

Expanding the identity $E(x^2) = -E((ix)^2)$ we obtain

$$E(x)x^* + xE(x) = iE(ix)x^* - ixE(ix),$$

and therefore $S_1(x^2) = xS_1(x)$. In a similar fashion we see that the mapping T_1 of R , defined by $T_1(x) = 2iE(x^*) - 2E(ix^*)$, satisfies $T_1(x^2) = T_1(x)x$. Now, define $T = -\frac{1}{4}iT_1$ and $S = \frac{1}{4}iS_1$. Clearly, $E(x) = T(x^*) - S(x)$ for every x in A . We claim that (T, S) is a double centralizer of A . Let us first

verify that $xT(x^*) = S(x)x^*$ for all $x \in A$. We have

$$\begin{aligned} xT(x^*) - S(x)x^* &= xE(x) + xS(x) - T(x^{*2}) + T(x^*)x^* - S(x)x^* \\ &= xE(x) - (-S(x^2) + T(x^{*2})) + (T(x^*) - S(x))x^* \\ &= xE(x) - E(x^2) + E(x)x^* = 0. \end{aligned}$$

Linearizing $xT(x^*) = S(x)x^*$ (i.e., replacing x by $x + y$) we get

$$(1) \quad xT(y^*) + yT(x^*) = S(x)y^* + S(y)x^*.$$

From the definition of S and T we see that $S(ix) = iS(x)$ and $T(ix) = iT(x)$ for all $x \in A$. Therefore, replacing y by iy in (1) we obtain

$$-ixT(y^*) + iyT(x^*) = -iS(x)y^* + iS(y)x^*.$$

Comparing this identity with (1) we see that $xT(y) = S(x)y$ for all $x, y \in A$. Consequently,

$$xT(yz) = S(x)yz = xT(y)z,$$

that is, $A(T(yz) - T(y)z) = 0$. By hypothesis, this implies that $T(yz) = T(y)z$. Similarly we see that T is linear; namely, $xT(\lambda y) = S(x)\lambda y = x\lambda T(y)$. Thus T is a left centralizer of A . Analogously one shows that S is a right centralizer of A . Thus the pair (T, S) is a double centralizer of A .

In order to prove that T and S are uniquely determined we assume that $L(x^*) = R(x)$ where L is a left and R is a right centralizer of A . Then

$$L(y^*)x^* = L(y^*x^*) = L((xy)^*) = R(xy)$$

for all $x, y \in R$. Replacing y by iy yields $-iL(y^*)x^* = iR(xy)$. But then, comparing the last two relations we obtain $L(y^*)x^* = 0$ for all $x, y \in A$, that is, $L(A)A = 0$, which yields $L = 0$, and, therefore, $R = 0$. This completes the proof of the theorem.

As an immediate consequence of Theorem 2.1 we obtain the following result which can be compared with [4; Corollary 1].

COROLLARY 2.2. *Let A be a complex *-algebra such that $Aa = 0$ or $aA = 0$ implies $a = 0$. Then every Jordan *-derivation of A is real linear.*

COROLLARY 2.3. *Let A be a complex Banach *-algebra such that $Aa = 0$ or $aA = 0$ implies $a = 0$. If the involution is continuous then every Jordan *-derivation of A is continuous.*

We remark that every semisimple Banach *-algebra satisfies the requirements of Corollary 2.3 (see [1; p. 191]).

Proof of Corollary 2.3. By Theorem 2.1, it suffices to show that every one-sided centralizer of A is continuous. Let T be a left centralizer. Suppose that $x_n, y \in A$ with $\lim_{n \rightarrow \infty} x_n = 0$, $\lim_{n \rightarrow \infty} T(x_n) = y$. By the closed graph theorem, it is enough to prove that $y = 0$. Given any $a \in A$,

we have $ay = \lim_{n \rightarrow \infty} aT(x_n) = \lim_{n \rightarrow \infty} S(a)x_n = 0$. Hence $y = 0$. In a similar fashion one shows that every right centralizer of A is continuous.

Combining Theorem 2.1 with [6; Theorem 3.9] we obtain

COROLLARY 2.4. *Let A be the algebra of all compact linear operators on a complex Hilbert space H . Then every Jordan $*$ -derivation of A is of the form $x \rightarrow ax^* - xa$ for some bounded linear operator a on H .*

It is an open question whether Corollary 2.4 remains true in the real case.

In the proof of Theorem 2.1, there occur additive mappings S, T satisfying $S(x^2) = xS(x)$, $T(x^2) = T(x)x$. The question arises whether S (resp. T) is then necessarily a right (resp. left) centralizer. Using a similar approach to [2, 3, 5], where some Jordan mappings are considered, we now prove

PROPOSITION 2.5. *Let R be a prime ring of characteristic not 2. If an additive mapping $T : R \rightarrow R$ satisfies $T(x^2) = T(x)x$ for all $x \in R$, then T is a left centralizer of R . Similarly, if an additive mapping $S : R \rightarrow R$ satisfies $S(x^2) = xS(x)$ for all $x \in R$, then S is a right centralizer of R .*

Recall that a ring R is said to be *prime* if $aRb = 0$ implies $a = 0$ or $b = 0$.

Proof of Proposition 2.5. Linearizing $T(x^2) = T(x)x$ we get

$$(2) \quad T(xy + yx) = T(x)y + T(y)x \quad \text{for all } x, y \in R.$$

In particular,

$$T(x(xy + yx) + (xy + yx)x) = T(x)(xy + yx) + (T(x)y + T(y)x)x.$$

But on the other hand,

$$\begin{aligned} T(x(xy + yx) + (xy + yx)x) &= T(x^2y + yx^2) + 2T(xyx) \\ &= T(x^2)y + T(y)x^2 + 2T(xyx) = T(x)xy + T(y)x^2 + 2T(xyx). \end{aligned}$$

Comparing the last two relations we arrive at $2T(xyx) = 2T(x)yx$. Since the characteristic of R is not 2, it follows that

$$(3) \quad T(xyx) = T(x)yx \quad \text{for all } x, y \in R.$$

A linearization of (3) gives

$$(4) \quad T(xyz + zyx) = T(x)yz + T(z)yx \quad \text{for all } x, y, z \in R.$$

Now, analogously to [2; Theorem 3], [3; Lemma 2.1] and [5; Proposition 3] we consider $W = T(xyzyx + yxzxy)$. According to (3) we have

$$W = T(x(yzy)x) + T(y(xzx)y) = T(x)yzyx + T(y)xzxy.$$

On the other hand, we see from (4) that

$$W = T((xy)z(yx) + (yx)z(xy)) = T(xy)zyx + T(yx)zxy.$$

Comparing the two expressions for W and applying (2), we then get

$$(5) \quad (T(xy) - T(x)y)z(xy - yx) = 0 \quad \text{for all } x, y, z \in R.$$

Since R is prime, for any $x, y \in R$ we have either $T(xy) = T(x)y$ or $xy = yx$. In other words, given $x \in R$, R is the union of its subsets $G_x = \{y \in R \mid T(xy) = T(x)y\}$ and $H_x = \{y \in R \mid xy = yx\}$. Clearly G_x and H_x are additive subgroups of R . However, a group cannot be the union of two proper subgroups, therefore either $G_x = R$ or $H_x = R$. Thus we have proved that R is the union of its subsets $G = \{x \in R \mid T(xy) = T(x)y \text{ for all } y \in R\}$ and $H = \{x \in R \mid xy = yx \text{ for all } y \in R\}$. Of course, G and H are also additive subgroups of R . Hence either $G = R$, i.e., T is a left centralizer, or $H = R$, i.e., R is commutative.

Thus, we may assume that R is commutative. Then, of course, R is a domain. Following the proofs of [9; Lemma 2.2] and [5; Theorem 2] we consider $V = 2T(x^2y)$. By (3) we have $V = 2T(xyx) = 2T(x)yx$. However, from (2) we see that $V = T(x^2y + yx^2) = T(x)xy + T(y)x^2$. Comparing both expressions we obtain $(T(x)y - T(y)x)x = 0$. Since R is a domain it follows that $x = 0$ or $T(x)y = T(y)x$; in any case $T(x)y = T(y)x$. Therefore, (2) yields $2T(xy) = 2T(x)y$. Since the characteristic of R is not 2, this means that T is a left centralizer.

Similarly one proves that S is a right centralizer.

3. Jordan *-derivations of $B(H)$. Throughout this section, H will be a Hilbert space such that $\dim H > 1$. We denote by $B(H)$ the algebra of all bounded linear operators on H , and by $S(H)$ the set of all self-adjoint operators in $B(H)$. Our main purpose in this section is to give a new proof of the following theorem of Šemrl [13].

THEOREM 3.1. *If H is a real Hilbert space then every Jordan *-derivation of $B(H)$ is inner.*

Recall that an additive mapping D of a ring R into itself is called a *derivation* if it satisfies $D(xy) = D(x)y + xD(y)$ for all $x, y \in R$. If R is an algebra and D is a derivation of R which is not necessarily homogeneous, then D will be called an *additive derivation*; otherwise we call D a *linear derivation*.

Outline of the proof of Theorem 3.1. Let E be a Jordan *-derivation of $B(H)$. Using the theorem of Martindale quoted below, we show that there exists an additive derivation D of $B(H)$ such that the restrictions of D and E to $S(H)$ coincide. It turns out that D is in fact

linear, therefore, by the well-known theorem of Chernoff [7], D is inner, i.e., $D(A) = TA - AT$ for some $T \in B(H)$. Finally, we show that there is a real number μ such that $E(A) = (T + \mu I)A^* - A(T + \mu I)$ for all $A \in B(H)$.

Let R be a $*$ -ring, and let S denote the set of all symmetric elements of R . A *Jordan derivation* d of S into R is an additive mapping of S into R such that $d(s^2) = d(s)s + sd(s)$ for all $s \in S$ (we will only deal with 2-torsion free rings, i.e., ones where $2a = 0$ implies $a = 0$; in such rings our definition of Jordan derivations coincides with the definition in [10]). Our proof of Theorem 3.1 is based on the fact that the restriction of a Jordan $*$ -derivation to the set of symmetric elements is a Jordan derivation.

In [10; Corollary 3, Theorem 4] Martindale proved

THEOREM M. *Let R be a 2-torsion free $*$ -ring with unit element 1. Suppose that either*

(i) *R contains nonzero orthogonal symmetric idempotents e_1, e_2 and e_3 such that $e_1 + e_2 + e_3 = 1$ and $Re_iR = R$ for $i = 1, 2, 3$, or*

(ii) *R is simple and it contains nonzero orthogonal idempotents e_1 and e_2 such that $e_1 + e_2 = 1$.*

Then every Jordan derivation of R into S can be uniquely extended to a derivation of R .

Remark 3.2. Let us show that the algebra $B(H)$ (H real or complex) satisfies the requirements of Theorem M. First, if H is finite-dimensional, then $B(H)$ satisfies (ii). Now suppose H is infinite-dimensional. Then there exists an orthonormal basis in H of the form $\{e_\alpha, f_\alpha, g_\alpha; \alpha \in A\}$. Let H_1 be the subspace generated by $\{e_\alpha; \alpha \in A\}$, and let E_1 be the orthogonal projection with range H_1 . Analogously we define the subspaces H_2, H_3 , and projections E_2, E_3 . Of course, $E_1 + E_2 + E_3 = I$, the identity on H . We claim that $B(H)E_iB(H) = B(H)$, $i = 1, 2, 3$. Indeed, there exists a one-to-one bounded linear operator B on H with range contained in H_i . Note that there is $A \in B(H)$ such that $AE_iB = AB = I$. But then $B(H)E_iB(H) = B(H)$.

In order to determine the structure of Jordan derivations of $S(H)$ into $B(H)$ we also need the following simple lemma.

LEMMA 3.3. *If $A, B \in B(H)$ are such that $ASB = 0$ for all $S \in S(H)$ then either $A = 0$ or $B = 0$.*

Proof. It suffices to prove that if a, b are nonzero vectors in H , then there exists $S \in S(H)$ such that $Sb = \lambda a$ for some nonzero scalar λ . If a and b are not orthogonal then this condition is satisfied by the operator $a \otimes a$ (we denote by $u \otimes v$ the operator $(u \otimes v)x = \langle x, v \rangle u$ where $\langle \cdot, \cdot \rangle$ is the inner product); otherwise take $S = a \otimes b + b \otimes a$.

We are now in a position to prove

THEOREM 3.4. *Let H be a (real or complex) Hilbert space. If a Jordan derivation d of $S(H)$ into $B(H)$ is real linear then there exists $T \in B(H)$ such that $d(S) = TS - ST$ for all $S \in S(H)$.*

PROOF. By Theorem M (and Remark 3.2) there is an additive derivation D of $B(H)$ such that $D|_{S(H)} = d$. Since every linear derivation of $B(H)$ is inner [7], the theorem will be proved by showing that D is linear.

Let us first show that D is real linear. For $A \in B(H)$ we may write $A = W + K$ where $W^* = W$ and $K^* = -K$. By assumption, $D(\lambda W) = \lambda D(W)$ for every real λ , therefore it suffices to show that $D(\lambda K) = \lambda D(K)$. Given any $S \in S(H)$, we have $KSK \in S(H)$. Therefore,

$$\begin{aligned} D(\lambda KSK) &= d(\lambda KSK) = \lambda d(KSK) = \lambda D(KSK) \\ &= \lambda D(K)SK + \lambda KD(S)K + \lambda KSD(K); \end{aligned}$$

on the other hand,

$$D(\lambda KSK) = D((\lambda K)SK) = D(\lambda K)SK + \lambda KD(S)K + \lambda KSD(K).$$

Comparing the above expressions for $D(\lambda KSK)$, we arrive at $(D(\lambda K) - \lambda D(K))SK = 0$ for all $S \in S(H)$. By Lemma 3.3 we conclude that $D(\lambda K) = \lambda D(K)$.

Now suppose H is a complex space. Since D is real linear it suffices to show that $D(iA) = iD(A)$ for every $A \in B(H)$. We have $D(I) = 0$. Hence

$$0 = D((iI)^2) = D(iI)iI + iID(iI) = 2iD(iI).$$

Thus $D(iI) = 0$. But then for any $A \in B(H)$ we have

$$D(iA) = D((iI)A) = D(iI)A + iID(A) = iD(A),$$

which completes the proof.

For the proof of Theorem 3.1 we also need the following lemma which is similar to [11; Theorem 1].

LEMMA 3.5. *If $A, B \in B(H)$ are such that $ABS = BSA$ for all $S \in S(H)$, and if $B \neq 0$, then $A = \lambda B$ for some scalar λ .*

PROOF. For all $x, y \in H$ we have $A(y \otimes y)Bx = B(y \otimes y)Ax$; that is, $\langle Bx, y \rangle Ay = \langle Ax, y \rangle By$. Consequently,

$$\begin{aligned} \langle Bx, y \rangle \langle By, z \rangle Az &= \langle Bx, y \rangle \langle Ay, z \rangle Bz = \langle \langle Bx, y \rangle Ay, z \rangle Bz \\ &= \langle \langle Ax, y \rangle By, z \rangle Bz = \langle Ax, y \rangle \langle By, z \rangle Bz. \end{aligned}$$

Thus $\langle By, z \rangle \{ \langle Bx, y \rangle Az - \langle Ax, y \rangle Bz \} = 0$ for all $x, y, z \in H$. Hence for any $y, z \in H$ we have either $\langle By, z \rangle = 0$ or $\langle Bx, y \rangle Az = \langle Ax, y \rangle Bz$ for all $x \in H$. Using the fact that a group cannot be the union of two proper subgroups (cf. the proof of Proposition 2.5) one can easily show that either $\langle By, z \rangle = 0$

for all $y, z \in H$ or $\langle Bx, y \rangle Az = \langle Ax, y \rangle Bz$ for all $x, y, z \in H$. Since we have assumed that $B \neq 0$ it follows at once that $A = \lambda B$ for some λ .

Remark 3.6. It is easy to see [4; Lemma 2] that every Jordan $*$ -derivation E satisfies $E(xyx) = E(x)y^*x^* + xE(y)x^* + xyE(x)$.

Proof of Theorem 3.1. Let E be a Jordan $*$ -derivation of $B(H)$. By [4; Corollary 1], E is linear. Since the restriction of E to $S(H)$ is a Jordan derivation of $S(H)$ to $B(H)$, it follows from Theorem 3.4 that there exists $T \in B(H)$ such that

$$(1) \quad E(S) = TS - ST \quad \text{for all } S \in S(H).$$

Pick $K \in B(H)$ such that $K^* = -K$. For every $S \in S(H)$ we have $KSK \in S(H)$. Therefore,

$$E(KSK) = TKSK - KSKT.$$

On the other hand, using Remark 3.6 we obtain

$$\begin{aligned} E(KSK) &= -E(K)SK - KE(S)K + KSE(K) \\ &= -E(K)SK - K(TS - ST)K + KSE(K) \end{aligned}$$

Comparing both expressions we get

$$(E(K) + KT + TK)SK = KS(E(K) + KT + TK)$$

for all $S \in S(H)$. Now Lemma 3.5 yields

$$(2) \quad E(K) + KT + TK = \lambda(K)K$$

for some real $\lambda(K)$. We claim that $\lambda(K)$ is a constant. Pick $K_1, K_2 \in B(H)$ with $K_1^* = -K_1$, $K_2^* = -K_2$. We claim that $\lambda(K_1) = \lambda(K_2)$. First assume that K_1 and K_2 are linearly independent. In view of (2) we have

$$E(K_1 + K_2) = \lambda(K_1 + K_2)(K_1 + K_2) - T(K_1 + K_2) - (K_1 + K_2)T.$$

On the other hand,

$$\begin{aligned} E(K_1 + K_2) &= E(K_1) + E(K_2) \\ &= \lambda(K_1)K_1 - TK_1 - K_1T + \lambda(K_2)K_2 - TK_2 - K_2T. \end{aligned}$$

Comparing we get

$$(\lambda(K_1 + K_2) - \lambda(K_1))K_1 + (\lambda(K_1 + K_2) - \lambda(K_2))K_2 = 0.$$

Since K_1 and K_2 are linearly independent we obtain $\lambda(K_1) = \lambda(K_1 + K_2) = \lambda(K_2)$.

If K_1 and K_2 are linearly dependent, then for any $K \in B(H)$ with $K^* = -K$ which is linearly independent from both K_1 and K_2 , we have $\lambda(K_1) = \lambda(K)$ and $\lambda(K_2) = \lambda(K)$. Thus $\lambda(K_1)$ and $\lambda(K_2)$ are also equal in this case. This means that $\lambda(K)$ is a constant λ , so that

$$(3) \quad E(K) = \lambda K - KT - TK$$

for every $K \in B(H)$ with $K^* = -K$.

Take $A \in B(H)$. We have $A = S + K$, where $S^* = S$, $K^* = -K$. Using (1) and (3) we then get

$$\begin{aligned} E(A) &= E(S) + E(K) = TS - ST + \lambda K - KT - TK \\ &= (T - \frac{1}{2}\lambda I)(S - K) - (S + K)(T - \frac{1}{2}\lambda I) \\ &= (T - \frac{1}{2}\lambda I)A^* - A(T - \frac{1}{2}\lambda I). \end{aligned}$$

Thus $E(A) = T_1 A^* - A T_1$ for all $A \in B(H)$, where $T_1 = T - \frac{1}{2}\lambda I$. This proves the theorem.

REFERENCES

- [1] F. F. Bonsall and J. Duncan, *Complete Normed Algebras*, Springer, Berlin 1973.
- [2] M. Brešar, *Jordan derivations on semiprime rings*, Proc. Amer. Math. Soc. 104 (1988), 1003–1006.
- [3] —, *Jordan mappings of semiprime rings*, J. Algebra 127 (1989), 218–228.
- [4] M. Brešar and J. Vukman, *On some additive mappings in rings with involution*, Aequationes Math. 38 (1989), 178–185.
- [5] —, —, *Jordan (Θ, φ) -derivations*, Glasnik Mat., to appear.
- [6] R. C. Busby, *Double centralizers and extensions of C^* -algebras*, Trans. Amer. Math. Soc. 132 (1968), 79–99.
- [7] P. R. Chernoff, *Representations, automorphisms and derivations of some operator algebras*, J. Funct. Anal. 12 (1973), 257–289.
- [8] S. Kurepa, *Quadratic and sesquilinear functionals*, Glasnik Mat. Fiz.-Astronom. 20 (1965), 79–92.
- [9] A. Leroy et J. Matczuk, *Quelques remarques à propos des S -dérivations*, Comm. Algebra 13 (1985), 1229–1244.
- [10] W. S. Martindale, *Jordan homomorphisms of the symmetric elements of a ring with involution*, J. Algebra 5 (1967), 232–249.
- [11] —, *Prime rings satisfying a generalized polynomial identity*, ibid. 12 (1969), 576–584.
- [12] P. Šemrl, *On quadratic functionals*, Bull. Austral. Math. Soc. 37 (1988), 27–28.
- [13] —, *On Jordan *-derivations and an application*, Colloq. Math. 59 (1990), 241–251.
- [14] —, *Quadratic functionals and Jordan *-derivations*, Studia Math. 97 (1991), 157–165.
- [15] P. Vrbová, *Quadratic functionals and bilinear forms*, Časopis Pěst. Mat. 98 (1973), 159–161.
- [16] J. Vukman, *Some functional equations in Banach algebras and an application*, Proc. Amer. Math. Soc. 100 (1987), 133–136.

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF MARIBOR
PF, KOROŠKA 160
62000 MARIBOR, SLOVENIA

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF LJUBLJANA
SF, MURNIKOVA 2
61000 LJUBLJANA, SLOVENIA

Reçu par la Rédaction le 26.11.1990